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AIRCRAFT DESIGN DATA

VOLUME I

EDITION OF 1919

DEPARTMENT OF THE NAVY
BUREAU OF CONSTRUCTION & REPAIR

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AIRCRAFT DESIGN DATA, VOL. I.

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Volume 1

NAVY DEPARTMENT

BUREAU OF CONSTRUCTION AND REPAIR

WASHINGTON
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PREFACE TO FIRST ISSUE.

Aircraft Design Data Notes were first issued in 1918 as blue-printed notes in order to secure, together with the aeronautical specifications issued by the Bureau, as far as practicable, uniformity in practice, workmanship, and procedure in the construction and performance estimates of aircraft of like types at various manufacturing companies, and to furnish engineering information on which these calculations are to be based. Future notes will be issued as they are prepared.

D. W. Taylor,
Rear Admiral, Chief Constructor, U. S. Navy.

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I.—CONVERSION FACTORS.

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FUNDAMENTAL AND DERIVED UNITS.

To change a quantity from one system of units to another: Substitute in the corresponding conversion factor from the following table the ratio of the magnitudes of the *old* units to the *new* and multiply the old quantity by the resulting number. For example: To reduce velocity in miles per hour to feet per second, the conversion factor is $l\,t^{-1}$; l=5280/1, t=3600/1, therefore the factor=5280/3600=1.467.

(*a*) Fundamental Units.

Name of unit.	Symbol.	Conversion factor.	Name of unit.	Symbol.	Conversion factor.
Length	L	l	Temperature	Θ	θ
Mass	M	m	Electric inductive capacity	K	k
Time	T	t	Magnetic inductive capacity	P	p

(*b*) Derived Units.

I. GEOMETRIC AND DYNAMIC UNITS.

Name of unit.	Conversion factor.	Name of unit.	Conversion factor.
Area	l^2	Intensity of attraction, or "force at a point"	$l\,t^{-2}$
Volume	l^3	Absolute force of a center of attraction, or "strength of a center"	$l^3\,t^{-2}$
Angle	1	Momentum	$m\,l\,t^{-1}$
Solid angle	1	Moment of momentum, or angular momentum	$m\,l^2\,t^{-1}$
Curvature	l^{-1}	Force	$m\,l\,t^{-2}$
Tortuosity	l^{-1}	Moment of a couple, or torque	$m\,l^2\,t^{-2}$
Specific curvature of a surface	l^{-2}	Intensity of stress	$m\,l^{-1}\,t^{-2}$
Angular velocity	t^{-1}	Modulus of elasticity	$m\,l^{-1}\,t^{-2}$
Angular acceleration	t^{-2}	Work and energy	$m\,l^2\,t^{-2}$
Linear velocity	$l\,t^{-1}$	Resilience	$m\,l^{-1}\,t^{-2}$
Linear acceleration	$l\,t^{-2}$	Power or activity	$m\,l^2\,t^{-3}$
Density	$m\,l^{-3}$		
Moment of inertia	$m\,l^2$		

II. HEAT UNITS.

Name of unit.	Conversion factor.	Name of unit.	Conversion factor.
Quantity of heat (thermal units)	$m\,\theta$	Conductivity (dynamical units)	$m\,l\,t^{-3}\,\theta^{-1}$
Quantity of heat (thermometric units)	$l^3\,\theta$	Thermal capacity	m
Quantity of heat (dynamical units)	$m\,l^2\,t^{-2}$	Latent heat (thermal units)	θ
Coefficient of thermal expansion	θ^{-1}	Latent heat (dynamical units)	$l^2\,t^{-2}$
Conductivity (thermal units)	$m\,l^{-1}\,t^{-1}$	Joule's equivalent	$l^2\,t^{-2}\,\theta$
Conductivity (thermometric units), or diffusivity	$l^2\,t^{-1}$	Entropy (heat measured in thermal units)	m
		Entropy (heat measured in dynamical units)	$m\,l^2\,t^{-2}\,\theta$

TABLE 1.—*System of units*—Continued.

(*b*) DERIVED UNITS—Continued.

III. MAGNETIC AND ELECTRIC UNITS.

Name of unit.	Conversion factor for electrostatic system.	Conversion factor for electromagnetic system.
Magnetic pole, or quantity of magnetism	$m^{\frac{1}{2}} l^{\frac{1}{2}} k^{-\frac{1}{2}}$	$m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-1} p^{\frac{1}{2}}$
Density of surface distribution of magnetism	$m^{\frac{1}{2}} l^{-\frac{3}{2}} k^{-\frac{1}{2}}$	$m^{\frac{1}{2}} l^{-\frac{1}{2}} t^{-1} p^{\frac{1}{2}}$
Intensity of magnetic field	$m^{\frac{1}{2}} l^{\frac{1}{2}} t^{-2} k^{\frac{1}{2}}$	$m^{\frac{1}{2}} l^{-\frac{1}{2}} t^{-1} p^{-\frac{1}{2}}$
Magnetic potential	$m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-2} k^{\frac{1}{2}}$	$m^{\frac{1}{2}} l^{\frac{1}{2}} t^{-1} p^{-\frac{1}{2}}$
Magnetic moment	$m^{\frac{1}{2}} l^{\frac{3}{2}} k^{-\frac{1}{2}}$	$m^{\frac{1}{2}} l^{\frac{5}{2}} t^{-1} p^{\frac{1}{2}}$
Intensity of magnetization	$m^{\frac{1}{2}} l^{-\frac{3}{2}} k^{-\frac{1}{2}}$	$m^{\frac{1}{2}} l^{-\frac{1}{2}} t^{-1} p^{\frac{1}{2}}$
Magnetic permeability	1	1
Magnetic susceptibility and magnetic inductive capacity	$l^{-2} t^{2} k^{-1}$	p
Quantity of electricity	$m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-1} k^{\frac{1}{2}}$	$m^{\frac{1}{2}} l^{\frac{1}{2}} p^{-\frac{1}{2}}$
Electric surface density and electric displacement	$m^{\frac{1}{2}} l^{-\frac{1}{2}} t^{-1} k^{-\frac{1}{2}}$	$m^{\frac{1}{2}} l^{-\frac{3}{2}} p^{-\frac{1}{2}}$
Intensity of electric field	$m^{\frac{1}{2}} l^{-\frac{1}{2}} t^{-1} k^{-\frac{1}{2}}$	$m^{\frac{1}{2}} l^{\frac{1}{2}} t^{-2} p^{\frac{1}{2}}$
Electric potential and e. m. f.	$m^{\frac{1}{2}} l^{\frac{1}{2}} t^{-1} k^{-\frac{1}{2}}$	$m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-2} p^{\frac{1}{2}}$
Capacity of a condenser	$l\, k$	$l^{-1} t^{2} p^{-1}$
Inductive capacity	k	$l^{-2} t^{2} p^{-1}$
Specific inductive capacity	1	1
Electric current	$m^{\frac{1}{2}} l^{\frac{1}{2}} t^{-2} k^{\frac{1}{2}}$	$m^{\frac{1}{2}} l^{\frac{1}{2}} t^{-1} p^{-\frac{1}{2}}$
Conductivity	$t^{-1} k$	$l^{-2} t\, p^{-1}$
Specific resistance	$t\, k^{-1}$	$l^{2} t^{-1} p$
Conductance	$l\, t^{-1} k$	$l^{-1} t\, p^{-1}$
Resistance	$l^{-1} t\, k^{-1}$	$l\, t^{-1} p$
Coefficient of self-induction and coefficient of mutual induction	$l^{-1} t^{2} k^{-1}$	$l\, p$
Electrokinetic momentum	$m^{\frac{1}{2}} l^{\frac{1}{2}} k^{-\frac{1}{2}}$	$m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-1} p^{\frac{1}{2}}$
Electromotive force at a point	$m^{\frac{1}{2}} l^{-\frac{1}{2}} t^{-1} k^{-\frac{1}{2}}$	$m^{\frac{1}{2}} l^{\frac{1}{2}} t^{-2} p^{\frac{1}{2}}$
Vector potential	$m^{\frac{1}{2}} l^{-\frac{1}{2}} k^{-\frac{1}{2}}$	$m^{\frac{1}{2}} l^{\frac{1}{2}} t^{-1} p^{\frac{1}{2}}$
Thermoelectric height and specific heat of electricity	$m^{\frac{1}{2}} l^{\frac{1}{2}} t^{-1} k^{-\frac{1}{2}} \theta^{-1}$	$m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-2} p^{\frac{1}{2}} \theta^{-1}$
Coefficient of Peltier effect	$m^{\frac{1}{2}} l^{-\frac{3}{2}} t\, k^{-\frac{1}{2}} \theta$	$m^{\frac{1}{2}} l^{-\frac{1}{2}} p^{\frac{1}{2}} \theta$

ENGINEERING SYSTEM OF UNITS.

Name of unit.	Dimensions of units in terms of F, L, T	British "gravitational" system, or "foot-pound-second" system.	Metric "gravitational" system, or "kilogram-meter-second" system.	Metric "absolute" system, or "C. G. S." system.	British "absolute" system (little used).
Force	F	1 lb.	1 kg.	1 dyne	1 poundal.
Length	L	1 ft.	1 m.	1 cm.	1 ft.
Time	T	1 sec.	1 sec.	1 sec.	1 sec.
Velocity	L/T	1 ft. per sec.	1 m. per sec.	1 cm. per sec.	1 ft. per sec.
Acceleration	L/T^2	1 ft. per sec.^{2}	1 m. per sec.^{2}	1 cm. per sec.^{2}	1 ft. per sec.^{2}
Pressure	F/L^2	1 lb. per ft.^{2}	1 kg. per m.^{2}	1 dyne per cm.^{2}	1 pdl. per ft.^{2}
Impulse or momentum	FT	1 lb.-sec.	1 kg.-sec.	1 dyne-sec.	1 pdl.-sec.
Work or energy	FL	1 ft.-lb.	1 kg.-m.	1 dyne-cm. = 1 "erg."	1 ft.-pdl.
Power	FL/T	1 ft.-lb. per sec.	1 kg.-m. per sec.	1 dyne-cm. per sec.	1 ft.-pdl. per sec.
Mass	$F/(L/T^2)$	1 lb. per (ft. per sec.^{2}) = 1 "slug."	1 kg. per (m. per sec.^{2}) = 1 "metric slug."	1 dyne per (cm. per sec.^{2}) = 1 gram mass.	1 pdl. per (ft. per sec.^{2}) = 1 pound mass.

NOTE.—The "slug" is also called the "geepound," or the "engineer's unit of mass."

TABLE 2.—*Length equivalents.*

Units.	Inches.	Feet.	Yards.	Fathoms.	Statute miles.	Nautical miles.	Centimeters.	Meters.	Kilometers.	Leagues.
1 inch	1	.0833	.0278	.0139	$.{}^{4}_{0}1578$	$.{}^{4}_{0}1370$	2.540	.0254	$.{}^{4}_{0}254$	$.{}^{5}_{0}4567$
1 foot	12	1	.333	.1667	$.{}^{3}_{0}1894$	$.{}^{3}_{0}1645$	30.480	.30480	$.{}^{3}_{0}3048$	$.{}^{4}_{0}548$
1 yard	36	3	1	.50	$.{}^{3}_{0}5682$	$.{}^{3}_{0}4933$	91.440	.9144	$.{}^{3}_{0}9144$	$.{}^{3}_{0}1644$
1 fathom	72	6	2	1	$.{}^{2}_{0}1136$	$.{}^{3}_{0}9868$	182.88	1.8288	$.{}^{2}_{0}1829$	$.{}^{3}_{0}329$
1 mile	63,360	5,280	1,760	880	1	.8683	160,934	1,609.35	1.60935	.2894
1 nautical mile	72,962	6,080.2	2,026.7	1,013.4	1.1516	1	185,325	1,853.25	1.85325	.333
1 centimeter	.3937	.03281	.01094	$.{}^{2}_{0}5468$	$.{}^{5}_{0}6214$	$.{}^{5}_{0}5396$	1	.01	$.{}^{4}_{0}1$	$.{}^{6}_{0}1799$
1 meter	39.370	3.281	1.0936	.5468	$.{}^{3}_{0}6214$	$.{}^{3}_{0}5396$	1^{2}_{0}	1	$.{}^{2}_{0}1$	$.{}^{3}_{0}1799$
1 kilometer	39,370	3,281	1,093.6	546.8	.6214	.53459	1^{5}_{0}	1^{3}_{0}	1	.1799
1 league	218,886	18,240.9	6,080.3	3,040.1	3.4547	3	555,975	555.975	5.55975	1

TABLE 3.—*Area equivalents.*

Units.	Square inches.	Square feet	Square yards.	Acres.	Square miles.	Square meters.	Hectares.
1 square inch	1	$.{}^{2}_{0}6944$	$.{}^{3}_{0}7716$	$.{}^{6}_{0}1594$	$.{}^{9}_{0}249$	$.{}^{3}_{0}6452$	$.{}^{7}_{0}6454$
1 square foot	144	1	.1111	$.{}^{4}_{0}2296$	$.{}^{7}_{0}3587$	.09290	$.{}^{5}_{0}9290$
1 square yard	1,296	9	1	$.{}^{3}_{0}2066$	$.{}^{6}_{0}3228$	.8361	$.{}^{4}_{0}8361$
1 acre	6,272,640	43,560	4,840	1	$.{}^{2}_{0}1563$	4,046.87	.4047
1 square mile	....	278784^{2}_{0}	309762^{2}_{0}	640	1	25.89999	259
1 square meter	1,550	10.764	1.196	$.{}^{3}_{0}24711$	$.{}^{6}_{0}3861$	1	$.{}^{3}_{0}107$
1 hectare	155^{5}_{0}	1076.4	.11960	2.4711	$.{}^{2}_{0}3861$	1^{4}_{0}	1

TABLE 4.—*Volume equivalents.*

Units.	Cubic inches.	Cubic feet.	Cubic yards.	Cubic centimeters.	Cubic meters.
1 cubic inch	1	$.{}^{3}_{0}5787$	$.{}^{4}_{0}2143$	16.39	$.{}^{4}_{0}1638$
1 cubic foot	1,728	1	.03704	28,317	.02832
1 cubic yard	46,656	27	1	764,559	.7645
1 cubic centimeter	.06102	$.{}^{4}_{0}3531$	$.{}^{5}_{0}1307$	1	$.{}^{5}_{0}1$
1 cubic meter	61,023	35.314	1.3079	1^{6}_{0}	1

TABLE 5.—*Capacity equivalents.*

Units.	Cubic inches.	Fluid ounces.	Gills.	Liquid pints.	Liquid quarts.	Gallons (U. S.).	Gallons (Imperial).	Liters.
1 cubic inch	1	.5541	.1385	.03464	.01732	$.{}^{2}_{0}4329$	$.{}^{2}_{0}36046$	.01639
1 fluid ounce	1.8047	1	.25	.0625	.03125	$.{}^{2}_{0}7813$	$.{}^{2}_{0}6506$	.02957
1 gill	7.2188	4	1	.25	.125	.03125	$.{}^{2}_{0}2602$	.118292
1 liquid pint	28.875	16	4	1	.5	.125	.10408	.473167
1 liquid quart	57.75	32	8	2	1	.25	.20822	.9463
1 gallon (U. S.)	231	128	32	8	4	1	.83265	3.785
1 gallon (Imperial)	277.41	153.718	38.429	9.607	4.804	1.201	1	4.5460
1 liter	61.023	33.814	8.453	2.113	1.0567	.2642	.21998	1

TABLE 6.—*Mass or weight equivalents.*

Units.	Kilograms.	Grains.	Ounces.		Pounds.		Tons.		
			Troy.	Avoirdupois.	Troy.	Avoirdupois.	Short.	Long.	Metric.
1 kilogram	1	15, 432	32. 151	35. 273	2. 6792	2. 2046	$.\,_0^2 1102$	$.\,_0^3 9842$	$.\,_0^2 1$
1 grain	$.\,_0^4 6480$	1	$.\,_0^2 2083$	.02286	$.\,_0^3 1736$	$.\,_0^3 1429$	$.\,_0^7 7143$	$.\,_0^7 6378$	$.\,_0^7 6480$
1 ounce (troy)	.0311	480	1	.10971	.08333	.06857	$.\,_0^4 3429$	$.\,_0^4 3061$	$.\,_0^4 3110$
1 ounce (avoirdupois)	.02835	437.5	.9115	1	.07595	.0625	$.\,_0^4 3125$	$.\,_0^4 2790$	.02835
1 pound (troy)	.3732	5, 760	12	13. 17	1	.8229	.04114	.02673	.03732
1 pound (avoirdupois)	.4536	7_0^3	14. 583	16	1. 2152	1	$.\,_0^3 5$	$.\,_0^3 4464$	$.\,_0^3 4536$
1 ton, short	907. 18	14_0^6	29, 167	32_0^3	2, 431	2_0^3	1	.8929	.9072
1 ton, long	1, 016	$15, 68_0^4$	326_0^3	35, 840	2, 722	2, 240	1. 12	1	1. 016
1 ton, metric	1_0^3	15, 432, 356	32, 151	35, 274	2, 679	2, 205	1. 102	.9842	1

MASS UNITS USED BY ENGINEERS.

A.—English systems:

Unit of mass=g pounds, where g is the acceleration due to gravity.

Hence, on foot-second system, unit of mass=32.14 pounds; give it arbitrary symbol U_1.

Hence, on mile-hour system, unit of mass=78,900 pounds; give it arbitrary symbol U_2.

B.—French systems:

Unit of mass=g kilograms.

Hence, on meter-second system, unit of mass=9.80 kilograms; give it arbitrary symbol U_3.

Hence, on kilometer-hour system, unit of mass=127,000 kilograms; give it arbitrary symbol U_4.

TABLE 7.—*Density equivalents.*

Units.	Grams per cubic centimeter.	Pounds per cubic inch.	Pounds per cubic foot.	Kilograms per cubic meter.	Pounds per United States gallon.	Pounds per British gallon.
1 gram per cubic centimeter	1	. 03613	62. 43	1_0^3	8. 345	10. 022
1 pound per cubic inch	27. 68	1	1, 728	277. 02	231	277, 431
1 pound per cubic foot	. 01602	$.\,_0^3 5787$	1	16. 02	. 1337	. 1606
.1 kilogram per cubic meter	. 09998	$.\,_0^2 3612$	. 06243	1	$.\,_0^2 8345$	. 010
1 pound per U. S. gallon	. 1198	$.\,_0^2 4329$	7. 481	119. 845	1	. 8326
1 pound per British gallon	. 0998	$.\,_0^2 3604$	6. 2266	1_0^2	1. 201	1

USING ENGINEERING UNITS OF MASS.

$$1\ \frac{\text{lb.}}{\text{ft.}^3}=0.0311\ \frac{U_1}{\text{ft.}^3};\quad 1\ \frac{U_1}{\text{ft.}^3}=32.14\ \frac{\text{lb.}}{\text{ft.}^3}$$

$$1\ \frac{\text{kg.}}{\text{m}^3}=0.0120\ \frac{U_3}{\text{m}^3};\quad 1\ \frac{U_3}{\text{m}^3}=9.80\ \frac{\text{kg.}}{\text{m}^3}$$

TABLE 8.—*Linear velocity equivalents.*

Units.	Centimeters per second.	Meters per second.	Meters per minute.	Kilometers per hour.	Feet per second.	Feet per minute.	Miles per hour.	Knots.
1 centimeter per second	1	. 01	. 6	. 036	. 03281	1. 9685	. 02237	. 01942
1 meter per second	100	1	60	3. 6	3. 281	196. 85	2. 237	1. 942
1 meter per minute	1. 667	. 01667	1	. 06	. 05468	3. 281	. 03728	. 03237
1 kilometer per hour	27. 78	. 2778	16. 67	1	. 9113	54. 68	. 6214	. 53960
1 foot per second	30. 48	. 3048	18. 29	1. 097	1	60	. 6818	. 59209
1 foot per minute	. 5080	$.\,_0^2 508$	. 3048	. 01829	. 01667	1	. 01136	$.\,_0^2 987$
1 mile per hour	44. 70	. 4470	26. 82	1. 609	1. 467	88	1	. 86839
1 knot	51. 497	. 51497	30. 898	1. 8532	1. 68894	101. 337	1. 15155	1

Table 9.—*Angular velocity equivalents.*

Units.	Degrees per second.	Grades per second.	Radians per second.	Revolutions per second.	Degrees per minute.	Grades per minute.	Radians per minute.	Revolutions per minute.
1 degree per second	1	1.111	.01745	$.{}_0^2 2778$	60	54	3,438	216_0^2
1 grade per second	.90	1	.01571	$.{}_0^2 25$	66.667	60	3,820	24_0^3
1 radian per second	57.30	63.67	1	.1592	1.047	.9425	60	376.9
1 revolution per second	360	4_0^2	6.283	1	.1667	.15	9.551	60
1 degree per minute	.01667	.01852	$.{}_0^3 2909$	$.{}_0^4 4630$	1	.90	57.30	360
1 grade per minute	.0150	.01667	$.{}_0^3 2618$	$.{}_0^4 4167$	1.111	1	63.67	4_0^2
1 radian per minute	.9549	1.061	.01667	$.{}_0^2 2653$	.01745	.01571	1	6.283
1 revolution per minute	6.0	6.667	.1047	.01667	$.{}_0^2 2778$	$.{}_0^2 25$	.1592	1

Note.—One circumference of a circle=400 grades.

Table 10.—*Linear acceleration equivalents.*

Units.	Feet per second.[2]	Miles per hour per second.	Miles per second.[2]	Kilometers per hour per second.	Standard gravity, g.
1 foot per second [2]	1	.6818	.3048	1.097	.03108
1 mile per hour per second	1.467	1	.4470	1.609	.03624
1 mile per second [2]	3.281	2.237	1	3.600	.1020
1 kilogram per hour per second	.9114	.6214	.2778	1	.0283
Standard gravity, g	32.174	21.936	9.8067	35.30	

Table 11.—*Force equivalents.*

Units.	Pound weight, avoirdupois.	Hundred-weight, (CWT.).	Poundal.	Grain, weight.	Gram, weight.	Short ton, weight.	Long ton, weight.	Dyne.	Mega-dyne.
1 pound weight, avoirdupois	1	.01	32.174	7_0^3	453.6	$.{}_0^3 50$	$.{}_0^3 4464$	4448_0^2	.4448
1 hundred weight	1_0^2	1	3,217.4	7_0^5	45,360	.05	.0446	4448_0^4	44.48
1 poundal	.03108	$.{}_0^3 3108$	1	217.591	14.10	$.{}_0^4 1554$	$.{}_0^4 1388$	13,825	.0138
1 grain, weight	$.{}_0^3 1429$	$.{}_0^5 1429$	$.{}_0^2 4595$	1	.0648	$.{}_0^7 714$	$.{}_0^7 6377$	63.54	$.{}_0^4 6354$
1 gram, weight	$.{}_0^2 2204$	$.{}_0^4 2204$	.07093	15.43	1	$.{}_0^5 1102$	$.{}_0^6 9841$	980.6	$.{}_0^3 9806$
1 short ton	2_0^3	20	64,348	14_0^6	9072_0^2	1	.8928	8896_0^5	889.6
1 long ton	2,240	22.4	72,074.4	1568_0^4	1016_0^3	1.12	1	9964_0^5	996.4
1 dyne	$.{}_0^5 2248$	$.{}_0^7 2248$	$.{}_0^4 7233$	.01574	$.{}_0^2 102$	$.{}_0^8 1124$	$.{}_0^8 1004$	1	$.{}_0^5 1$
1 megadyne	2.248	.0225	72.33	15,740	1,020	$.{}_0^2 1124$	$.{}_0^2 1004$	1_0^6	1

Table 12.—*Work or energy equivalents.*

Units.	Joules= 1_0^7 ergs.	Kilogram meters.	Foot-pounds.	Kilowatt hours.	Cheval vapeur hours.	Horse-power hours.	Calories.	Kilogram calories.	British thermal units.
1 joule	1	.10197	.7376	$.{}_0^6 2778$	$.{}_0^6 3777$	$.{}_0^6 3725$	.2390	$.{}_0^3 2390$	$.{}_0^3 9486$
1 kilogram meter	9.80665	1	7.233	$.{}_0^5 2724$	$.{}_0^5 37037$	$.{}_0^5 3653$	2.344	$.{}_0^2 2344$	$.{}_0^2 930$
1 foot-pound	1.356	.1383	1	$.{}_0^6 3766$	$.{}_0^6 51206$	$.{}_0^6 50505$	.3240	$.{}_0^3 3241$	$.{}_0^2 128$
1 kilowatt hour	36_0^5	3671_0^2	2655_0^3	1	1.3596	1.341	8605_0^2	860.5	3,415
1 cheval vapeur hour	2648_0^3	27_0^4	19529_0^2	.7355	1	.9863	6329_0^2	632.9	2,512
1 horsepower hour	26845_0^2	273,750	198_0^4	.7457	1.0139	1	6417_0^2	641.7	2,547
1 calorie	4.183	.4266	3.086	$.{}_0^5 1162$	$.{}_0^5 158$	$.{}_0^5 1558$	1	$.{}_0^2 1$	$.{}_0^2 3968$
1 kilogram calorie	4,183	426.6	3,086	$.{}_0^2 1162$	$.{}_0^2 158$	$.{}_0^2 1558$	1_0^3	1	3.968
1 British thermal unit	1,054	107.5	777.52	$.{}_0^3 2928$	$.{}_0^3 3981$	$.{}_0^3 3927$	252.2	.252	1

TABLE 13.—*Power equivalents.*

Units.	Horsepower.	Kilowatts.	Cheval vapeur metric horsepower.	Meter kilograms per second.	Foot-pounds per second.	Kilogram calories per second.	British thermal units per second.
1 horsepower	1	.7457	1.014	76.04	550	.1783	.7074
1 kilowatt	1.341	1	1.360	102.0	737.6	.2390	.9486
1 cheval vapeur metric horsepower	.9863	.7355	1	75	542.3	.1758	.6977
1 meter kilogram per second	.01315	$.^{2}_{0}9807$	.01333	1	7.233	$.^{2}_{0}2344$	$.^{2}_{0}9303$
1 foot pound per second	$.^{2}_{0}182$	$.^{2}_{0}1356$	$.^{2}_{0}184$	.1383	1	$.^{3}_{0}3241$	$.^{2}_{0}1286$
1 kilogram calorie per second	5.610	4.183	5.688	426.6	3,086	1	3.968
1 British thermal unit per second	1.414	1.054	1.433	107.5	777.5	.2520	1

TABLE 14.—*Pressure equivalents.*

Units.	Megabars or megadynes per square centimeter.	Kilograms per square centimeter.	Kilograms per square meter.	Pounds per square inch.	Pounds per square foot.	Long tons per square inch.	Long tons per square foot.	Short tons per square inch.	Short tons per square foot.
1 megabar (=18 dynes per square centimeter)	1	1.0197	10,197	14.50	2,088			$.^{2}_{0}725$	1.044
1 kilogram per square centimeter	.9807	1	1^{4}_{0}	14.22	2,047.6	$.^{2}_{0}6348$		$.^{2}_{0}711$	1.024
1 kilogram per square meter	$.^{4}_{0}9807$	$.^{3}_{0}1$	1	$.^{2}_{0}1422$	.2048	$.^{6}_{0}6348$		$.^{6}_{0}711$	$.^{3}_{0}1024$
1 pound per square inch	.06895	.07031	703.1	1	144			$.^{3}_{0}5$	.072
1 pound per square foot	$.^{3}_{0}4788$	$.^{3}_{0}4883$	4.882	$.^{2}_{0}694$	1			$.^{5}_{0}3472$	$.^{3}_{0}5$
1 long ton per square inch		157.5	1575^{3}_{0}			1			
1 long ton per square foot							1		
1 short ton per square inch	366.97	140.632	1,406,320					1	
1 short ton per square foot	.9576	.9765	9,765	13.89	2,000.16				1
1 atmosphere	1.0133	1.0333	10,333	14.70	2,116.8				1.058
Mercury 1 meter	1.333	1.3596	13,596	19.34	2,784.9				1.392
Mercury 1 inch	.03386	.03453	345.3	.4912	70.732				.03536
Water 1 meter	.09798	.09991	999.1	1.421	204.62				.1023
Water 1 inch	$.^{2}_{0}2489$	$.^{2}_{0}2538$	25.4	.03613	5.204				$.^{2}_{0}2599$
Water 1 foot	.02986	.03045	304.5	.4332	62.380				.03119

Units.	Atmospheres.	Columns of mercury at 15° C.		Columns of water at 15° C.		
		Meters.	Inches.	Meters.	Inches.	Feet.
1 megabar (=18 dynes per square centimeter)	.9869	$.75^{2}_{0}$	29.53	10.21	401.8	33.48
1 kilogram per square centimeter	.9678	.7355	28.96	10.01	394	32.84
1 kilogram per square meter	$.^{4}_{0}9678$	$.^{4}_{0}735$	$.^{2}_{0}2896$	$.^{2}_{0}1001$	.03937	$.^{2}_{0}3284$
1 pound per square inch	.06804	.05171	2.036	.7037	27.70	2.309
1 pound per square foot	$.^{3}_{0}4725$	$.^{3}_{0}3591$	.01414	$.^{2}_{0}4887$	.1922	.01602
1 short ton per square foot	.9450	.7182	28.28	9.773	384.8	32.06
1 atmosphere	1	.76	29.92	10.34	407.2	33.93
Mercury 1 meter		1	39.37	13.61	535.7	44.64
Mercury 1 inch		.0254	1	.3456	13.61	1.134
Water 1 meter		.07349	2.893	1	39.37	3.281
Water 1 inch		$.^{2}_{0}1868$	.07349	.02540	1	.08333
Water 1 foot		.02240	.8819	.3048	12	1

TABLE 15.—*Couple equivalents.*

Units.	Kilogram-meters.	Pound-feet.
1 kilogram-meter	1	7.233
1 pound-foot	.1383	1

Table 16.—*Torque equivalents.*

Units.	Pound-foot.	Gram-foot.	Dyne-foot.
1 pound-foot	1	13,840	1356_0^4
1 gram-foot	$.{}_0^4 7233$	1	980.665
1 dyne-foot	$.{}_0^7 7375$	$.{}_0^2 102$	1

Table 17.—*Loading equivalents.*

Units.	Pound per square foot.	Pound per English horsepower.	Kilogram per square meter.	Kilogram per metric horsepower.
1 pound per square foot	1		4.882	
1 pound per English horsepower		1		0.459
1 kilogram per square meter	0.2048		1	
1 kilogram per metric horsepower		2.1726		1

Table 18.—*Aerodynamic equivalents.*

Units.	Pounds per square foot per mile per hour	Pounds per square foot per foot per second.	Kilograms per square meter per kilometer per hour.	Kilograms per square meter per meter per second.
Pound per square foot per mile per hour		.4649	1.995	24.5
Pound per square foot per foot per second	2.1510		4.070	52.5
Kilograms per square meter per kilometer per hour	.5304	.2466		
Kilograms per square meter per meter per second	.04092	.01903		
Absolute units	.0251	.02237		

Table 19.—*Inclination equivalents.*

Rise.	Angle of inclination.	Tangent.	Sine.	Radians.
	° ′			
1 in 30	1 55	0.0333	0.0333	0.0333
1 in 25	2 17	.0400	.0400	.0400
1 in 20	2 52	.0500	.0500	.0500
1 in 18	3 11	.0555	.0554	.0555
1 in 16	3 35	.0625	.0624	.0625
1 in 14	4 5	.0714	.0712	.0713
1 in 12	4 46	.0833	.0831	.0832
1 in 10	5 43	.1000	.0996	.0997
1 in 9	6 21	.1111	.1103	.1105
1 in 8	7 8	.1250	.1242	.1245
1 in 7	8 8	.1429	.1415	.1419
1 in 6	9 28	.1667	.1645	.1652
1 in 5	11 19	.2000	.1962	.1975
1 in 4	14 2	.2500	.2425	.2449
1 in 3	18 26	.3333	.3162	.3217

II.—CONVERSION TABLES.

Table 1.—*Fractions of inches in millimeters.*

$\frac{1}{128}$ inch.	$\frac{1}{64}$ inch.	$\frac{1}{32}$ inch.	$\frac{1}{16}$ inch.	$\frac{1}{8}$ inch.	$\frac{1}{4}$ inch.	Millimeters.	$\frac{1}{128}$ inch.	$\frac{1}{64}$ inch.	$\frac{1}{32}$ inch.	$\frac{1}{16}$ inch.	$\frac{1}{8}$ inch.	$\frac{1}{4}$ inch.	Millimeters.
0							60	30	15				11.910
1						0.198	2	31					12.307
2	1					0.397	4	32	16	8	4	2	12.700
4	2	1				0.794	6	33					13.097
6	3					1.191	8	34	17				13.493
8	4	2	1			1.587	70	35					13.890
10	5					1.984	2	36	18	9			14.287
2	6	3				2.381	4	37					14.684
4	7					2.778	6	38	19				15.081
6	8	4	2	1		3.175	8	39					15.478
8	9					3.572	80	40	20	10	5		15.875
20	10	5				3.969	2	41					16.272
2	11					4.366	4	42	21				16.668
4	12	6	3			4.762	6	43					17.065
6	13					5.519	8	44	22	11			17.462
8	14	7				5.556	90	45					17.859
30	15					5.953	2	46	23				18.256
2	16	8	4	2	1	6.350	4	47					18.653
4	17					6.747	6	48	24	12	6	3	19.050
6	18	9				7.144	8	49					19.446
8	19					7.540	100	50	25				19.843
40	20	10	5			7.937	2	51					20.240
2	21					8.334	4	52	26	13			20.637
4	22	11				8.731	6	53					21.034
6	23					9.128	8	54	27				21.431
8	24	12	6	3		9.525	110	55					21.828
50	25					9.922	2	56	28	14	7		22.225
2	26	13				10.319	4	57					22.621
4	27					10.725	6	58	29				23.018
6	28	14	7			11.112	8	59					23.415
8	29					11.509	120	60	30	15			23.812
							2	61					24.209
							4	62	31				34.606
							6	63					25.003
							128=1″	64	32	16	8	4	25.399

Table 2.—*Inches to centimeters—1 in.=2.540005 cm.*

Units.	0	1	2	3	4	5	6	7	8	9
Tens.										
0		2.540	5.080	7.620	10.160	12.700	15.240	17.780	20.320	22.860
1	25.400	27.940	30.480	33.020	35.560	38.100	40.640	43.180	45.720	48.260
2	50.800	53.340	55.880	58.420	60.960	63.500	66.040	68.580	71.120	73.660
3	76.200	78.740	81.280	83.820	86.360	88.900	91.440	93.980	96.520	99.060
4	101.600	104.140	106.680	109.220	111.760	114.300	116.840	119.380	121.920	124.460
5	127.000	129.540	132.080	134.620	137.160	139.700	142.240	144.780	147.320	149.860
6	152.400	154.940	157.480	160.020	162.560	165.100	167.640	170.180	172.720	175.260
7	177.800	180.340	182.880	185.420	187.960	190.500	193.040	195.580	198.120	200.660
8	203.200	205.740	208.280	210.820	213.360	215.900	218.440	220.980	223.520	226.060
9	228.600	231.140	233.680	236.220	238.760	241.300	243.840	246.380	248.920	251.460

TABLE 3.—*Inches² to centimeters²—1 in.² = 6.451625 cm.²*

Units. / Tens.	0	1	2	3	4	5	6	7	8	9
	....	6.452	12.903	19.355	25.807	32.258	38.710	45.161	51.613	58.065
1	64.516	70.968	77.420	83.871	90.323	96.774	103.226	109.678	116.129	122.581
2	129.033	135.484	141.936	148.387	154.839	161.291	167.742	174.194	180.646	187.097
3	193.549	200.000	206.452	212.904	219.355	225.807	232.259	238.710	245.162	251.613
4	258.065	264.517	270.968	277.420	283.872	290.323	296.775	303.226	309.678	316.130
5	322.581	329.033	335.485	341.936	348.388	354.839	361.291	367.743	374.194	380.646
6	387.098	393.549	400.001	406.452	412.904	419.356	425.807	432.259	438.711	445.162
7	451.614	458.065	464.517	470.969	477.420	483.872	490.324	496.775	503.227	509.678
8	516.130	522.582	529.033	535.485	541.937	548.388	554.840	561.291	567.743	574.195
9	580.646	587.098	593.550	600.001	606.453	612.904	619.356	625.808	632.259	638.711

TABLE 4.—*Inches³ to centimeters³—1 in.³ = 16.38716 cm.³*

Units. / Tens.	0	1	2	3	4	5	6	7	8	9
0	....	16.39	32.77	49.16	65.55	81.94	98.32	114.71	131.10	147.48
1	163.87	180.26	196.65	213.03	229.42	245.81	262.19	278.58	294.97	311.36
2	327.74	344.13	360.52	376.90	393.29	409.68	426.07	442.45	458.84	475.23
3	491.61	508.00	524.39	540.78	557.16	573.55	589.94	606.32	622.71	639.10
4	655.49	671.87	688.26	704.65	721.04	737.42	753.81	770.20	786.58	802.97
5	819.36	835.75	852.13	868.52	884.91	901.29	917.68	934.07	950.46	966.84
6	983.23	999.62	1,016.00	1,032.39	1,048.78	1,065.17	1,081.55	1,097.94	1,114.33	1,130.71
7	1,147.10	1,163.49	1,179.88	1,196.26	1,212.65	1,229.04	1,245.42	1,261.81	1,278.20	1,294.59
8	1,310.97	1,327.36	1,343.75	1,360.13	1,376.52	1,392.91	1,409.30	1,425.68	1,442.07	1,458.46
9	1,474.84	1,491.23	1,507.62	1,524.01	1,540.39	1,556.78	1,573.17	1,589.55	1,605.94	1,622.33

TABLE 5.—*Inches⁴ to centimeters⁴—1 in.⁴ = 41.62347 cm.⁴*

Units. / Tens.	0	1	2	3	4	5	6	7	8	9
0	....	41.62	83.25	124.87	166.49	208.12	249.74	291.36	332.99	374.61
1	416.23	457.86	499.48	541.11	582.73	624.35	665.98	707.60	749.22	790.85
2	832.47	874.09	915.72	957.34	998.96	1,040.59	1,082.21	1,123.83	1,165.46	1,207.08
3	1,248.70	1,290.33	1,331.95	1,373.57	1,415.20	1,456.82	1,498.44	1,540.07	1,581.69	1,623.32
4	1,664.94	1,706.56	1,748.19	1,789.81	1,831.43	1,873.06	1,914.68	1,956.30	1,997.93	2,039.55
5	2,081.17	2,122.80	2,164.42	2,206.04	2,247.67	2,289.29	2,330.91	2,372.54	2,414.16	2,455.78
6	2,497.41	2,539.03	2,580.66	2,622.28	2,663.90	2,705.53	2,747.15	2,788.77	2,830.40	2,872.02
7	2,913.64	2,955.27	2,996.89	3,038.51	3,080.14	3,121.76	3,163.38	3,205.01	3,246.63	3,288.25
8	3,329.88	3,371.50	3,413.12	3,454.75	3,496.37	3,537.99	3,579.62	3,621.24	3,662.87	3,704.49
9	3,746.11	3,787.74	3,829.36	3,870.98	3,912.61	3,954.23	3,995.85	4,037.48	4,079.10	4,120.72

TABLE 6.—*Centimeters to inches—1 cm. = 0.3937 in.*

Units. / Tens.	0	1	2	3	4	5	6	7	8	9
0	....	0.3937	0.7874	1.1811	1.5748	1.9685	2.3622	2.7559	3.1496	3.5433
1	3.9370	4.3307	4.7244	5.1181	5.5118	5.9055	6.2992	6.6929	7.0866	7.4803
2	7.8740	8.2677	8.6614	9.0551	9.4488	9.8425	10.2362	10.6299	11.0236	11.4173
3	11.8110	12.2047	12.5984	12.9921	13.3858	13.7795	14.1732	14.5669	14.9606	15.3543
4	15.7480	16.1417	16.5354	16.9291	17.3228	17.7165	18.1102	18.5039	18.8976	19.2913
5	19.6850	20.0787	20.4724	20.8661	21.2598	21.6535	22.0472	22.4409	22.8346	23.2283
6	23.6220	24.0157	24.4094	24.8031	25.1968	25.5905	25.9842	26.3779	26.7716	27.1653
7	27.5590	27.9527	28.3464	28.7401	29.1338	29.5275	29.9212	30.3149	30.7086	31.1023
8	31.4960	31.8897	32.2834	32.6771	33.0708	33.4645	33.8582	34.2519	34.6456	35.0393
9	35.4330	35.8267	36.2204	36.6141	37.0078	37.4015	37.7952	38.1889	38.5826	38.9763

TABLE 7.—*Centimeters² to inches²—1 cm.² = 0.15499969 in.²*.

Units.	0	1	2	3	4	5	6	7	8	9
Tens.										
0	....	0. 1550	0. 3100	0. 4650	0. 6200	0. 7750	0. 9300	1. 0850	1. 2400	1. 3950
1	1. 5500	1. 7050	1. 8600	2. 0150	2. 1700	2. 3250	2. 4800	2. 6350	2. 7900	2. 9450
2	3. 1000	3. 2550	3. 4100	3. 5650	3. 7200	3. 8750	4. 0300	4. 1850	4. 3400	4. 4950
3	4. 6500	4. 8050	4. 9600	5. 1150	5. 2700	5. 4250	5. 5800	5. 7350	5. 8900	6. 0450
4	6. 2000	6. 3550	6. 5100	6. 6650	6. 8200	6. 9750	7. 1300	7. 2850	7. 4400	7. 5950
5	7. 7500	7. 9050	8. 0600	8. 2150	8. 3700	8. 5250	8. 6800	8. 8350	8. 9900	9. 1450
6	9. 3000	9. 4550	9. 6100	9. 7650	9. 9200	10. 0750	10. 2300	10. 3850	10. 5400	10. 6950
7	10. 8500	11. 0050	11. 1600	11. 3150	11. 4700	11. 6250	11. 7800	11. 9350	12. 0900	12. 2450
8	12. 4000	12. 5550	12. 7100	12. 8650	13. 0200	13. 1750	13. 3300	13. 4850	13. 6400	13. 7950
9	13. 9500	14. 1050	14. 2600	14. 4150	14. 5700	14. 7250	14. 8800	15. 0350	15. 1900	15. 3450

TABLE 8.—*Centimeters³ to inches³—1 cm.³ = 0.0610234 in.³*.

Units.	0	1	2	3	4	5	6	7	8	9
Tens.										
0	....	0. 06102	0. 12205	0. 18307	0. 24409	0. 30512	0. 36614	0. 42716	0. 48819	0. 54921
1	0. 61023	0. 67126	0. 73228	0. 79330	0. 85433	0. 91535	0. 97637	1. 03740	1. 09842	1. 15944
2	1. 22047	1. 28149	1. 34251	1. 40354	1. 46456	1. 52559	1. 58661	1. 64763	1. 70866	1. 76968
3	1. 83070	1. 89173	1. 95275	2. 01377	2. 07480	2. 13582	2. 19684	2. 25787	2. 31889	2. 37991
4	2. 44094	2. 50196	2. 56298	2. 62401	2. 68503	2. 74605	2. 80708	2. 86810	2. 92912	2. 99015
5	3. 05117	3. 11219	3. 17322	3. 23424	3. 29526	3. 35629	3. 41731	3. 47833	3. 53936	3. 60038
6	3. 66140	3. 72243	3. 78345	3. 84447	3. 90550	3. 96652	4. 02754	4. 08857	4. 14959	4. 21061
7	4. 27164	4. 33266	4. 39368	4. 45471	4. 51573	4. 57675	4. 63778	4. 69880	4. 75983	4. 82085
8	4. 88187	4. 94290	5. 00392	5. 06494	5. 12597	5. 18699	5. 24801	5. 30904	5. 37006	5. 43108
9	5. 49211	5. 55313	5. 61415	5. 67518	5. 73620	5. 79722	5. 85825	5. 91927	5. 98029	6. 04132

TABLE 9.—*Centimeters⁴ to inches⁴—1 cm.⁴ = 0.0240249 in.⁴*.

Units.	0	1	2	3	4	5	6	7	8	9
Tens.										
0	....	0. 02402	0. 04805	0. 07207	0. 09610	0. 12012	0. 14415	0. 16817	0. 19220	0. 21622
1	0. 24025	0. 26427	0. 28830	0. 31232	0. 33635	0. 36037	0. 38440	0. 40842	0. 43245	0. 45647
2	0. 48050	0. 50452	0. 52885	0. 55257	0. 57660	0. 60062	0. 62465	0. 64867	0. 67270	0. 69672
3	0. 72075	0. 74477	0. 76880	0. 79282	0. 81685	0. 84087	0. 86490	0. 88892	0. 91295	0. 93697
4	0. 96100	0 98502	1. 00905	1 03307	1. 05710	1. 08112	1. 10515	1. 12917	1. 15320	1. 17722
5	1. 20125	1. 22527	1. 24930	1. 27332	1. 29734	1. 32137	1. 34539	1. 36942	1. 39344	1. 41747
6	1. 44149	1. 46552	1. 48954	1. 51357	1. 53759	1. 56162	1. 58564	1. 60967	1. 63369	1. 65772
7	1. 68174	1. 70577	1. 72979	1. 75382	1. 77784	1. 80187	1. 82589	1. 84992	1. 87394	1. 89797
8	1. 92199	1. 94602	1. 97004	1. 99407	2. 01809	2. 04212	2. 06614	2. 09017	2. 11419	2. 13822
9	2. 16224	2. 18627	2. 21029	2. 23432	2. 25834	2. 28237	2. 30639	2. 33042	2. 35444	2. 37847

TABLE 10.—*Feet to meters—1 ft. = 0.3048006 m.*

Units.	0	1	2	3	4	5	6	7	8	9
Tens.										
0	....	0. 3048	0. 6096	0. 9144	1. 2192	1. 5240	1. 8288	2. 1336	2. 4384	2. 7432
1	3. 0480	3. 3528	3. 6576	3. 9624	4. 2672	4. 5720	4. 8768	5. 1816	5. 4864	5. 7912
2	6. 0960	6. 4008	6. 7056	7. 0104	7. 3152	7. 6200	7. 9248	8. 2296	8. 5344	8. 8392
3	9. 1440	9. 4488	9. 7536	10. 0584	10. 3632	10. 6680	10. 9728	11. 2776	11. 5824	11. 8872
4	12. 1920	12. 4968	12. 8016	13. 1064	13. 4112	13. 7160	14. 0208	14. 3256	14. 6304	14. 9352
5	15. 2400	15. 5448	15. 8496	16. 1544	16. 4592	16. 7640	17. 0688	17. 3736	17. 6784	17. 9832
6	18. 2880	18. 5928	18. 8976	19. 2024	19. 5072	19. 8120	20. 1168	20. 4216	20. 7264	21. 0312
7	21. 3360	21. 6408	21. 9456	22. 2504	22. 5552	22. 8600	23. 1648	23. 4696	23. 7744	24. 0792
8	24. 3840	24. 6888	24. 9936	25. 2984	25. 6033	25. 9081	26. 2129	26. 5177	26. 8225	27. 1273
9	27. 4321	27. 7369	28. 0417	28. 3465	28. 6513	28. 9561	29. 2609	29. 5657	29. 8705	30. 1753

TABLE 11.—*Feet² to meters²—1 ft.² = 0.0929 m.².*

Units.	0	1	2	3	4	5	6	7	8	9
Tens.										
0	....	.093	.186	.279	.372	.465	.557	.650	.743	.836
1	.929	1.022	1.115	1.208	1.301	1.394	1.486	1.579	1.672	1.765
2	1.858	1.951	2.044	2.137	2.232	2.323	2.415	2.508	2.601	2.694
3	2.787	2.88	2.973	3.066	3.159	3.251	3.344	3.437	3.530	3.623
4	3.716	3.809	3.902	3.995	4.088	4.181	4.274	4.366	4.459	4.552
5	4.645	4.738	4.831	4.924	5.016	5.110	5.203	5.296	5.388	5.487
6	5.574	5.667	5.760	5.853	5.946	6.039	6.132	6.225	6.317	6.410
7	6.503	6.596	6.689	6.782	6.875	6.968	7.061	7.154	7.246	7.339
8	7.432	7.525	7.618	7.711	7.804	7.897	7.990	8.083	8.176	8.268
9	8.361	8.454	8.547	8.640	8.733	8.826	8.919	9.011	9.105	9.197

TABLE 12.—*Feet³ to meters³—1 ft.³ = 0.028317 m.³.*

Units.	0	1	2	3	4	5	6	7	8	9
Tens.										
0	.....	.028	.057	.085	.113	.142	.170	.198	.226	.255
1	.283	.311	.340	.368	.396	.425	.453	.481	.509	.538
2	.566	.594	.623	.651	.679	.708	.736	.764	.792	.821
3	.849	.877	.906	.934	.962	.991	1.019	1.047	1.075	1.104
4	1.132	1.160	1.189	1.217	1.245	1.274	1.302	1.330	1.358	1.387
5	1.415	1.443	1.472	1.500	1.528	1.557	1.585	1.613	1.641	1.670
6	1.698	1.726	1.755	1.783	1.811	1.840	1.868	1.896	1.924	1.953
7	1.981	2.009	2.038	2.066	2.094	2.123	2.151	2.179	2.207	2.236
8	2.264	2.292	2.321	2.349	2.377	2.406	2.434	2.462	2.490	2.519
9	2.547	2.575	2.604	2.632	2.660	2.689	2.717	2.745	2.773	2.802

TABLE 13.—*Meters to feet—1 m. = 3.2808333 ft.*

Units.	0	1	2	3	4	5	6	7	8	9
Tens.										
0	....	3.281	6.562	9.843	13.123	16.404	19.685	22.966	26.247	29.528
1	32.808	36.089	39.370	42.651	45.932	49.213	52.493	55.774	59.055	62.336
2	65.617	68.898	72.178	75.459	78.740	82.021	85.302	88.583	91.863	95.144
3	98.425	101.706	104.987	108.268	111.548	114.829	118.110	121.391	124.672	127.953
4	131.233	134.514	137.795	141.076	144.357	147.638	150.918	154.199	157.480	160.761
5	164.042	167.323	170.603	173.884	177.165	180.446	183.727	187.008	190.288	193.569
6	196.850	200.131	203.412	206.693	209.973	213.254	216.535	219.816	223.097	226.378
7	229.658	232.939	236.220	239.501	242.782	246.063	249.343	252.624	255.905	259.186
8	262.467	265.748	269.028	272.309	275.590	278.871	282.152	285.433	288.713	291.994
9	295.275	298.556	301.837	305.118	308.398	311.679	314.960	318.241	321.522	324.803

TABLE 14.—*Meters² to feet²—1 m.² = 10.7636 ft.².*

Units.	0	1	2	3	4	5	6	7	8	9
Tens.										
0	....	10.764	21.528	32.292	43.056	53.820	64.584	75.348	86.111	96.875
1	107.640	118.403	129.167	139.931	150.695	161.458	172.223	182.987	193.751	204.515
2	215.280	226.043	236.808	247.570	258.334	269.098	279.862	290.626	301.390	312.154
3	322.918	333.682	344.446	355.210	365.974	376.738	387.501	398.265	409.029	419.793
4	430.557	441.321	452.085	462.849	473.613	484.377	495.141	505.905	516.669	527.433
5	538.197	548.961	559.724	570.488	581.252	592.116	602.780	613.544	624.308	635.072
6	645.836	656.600	667.364	678.128	688.892	699.655	710.419	721.183	731.947	742.711
7	753.475	764.239	775.003	785.767	796.530	807.295	818.059	828.823	839.587	850.350
8	861.114	871.878	882.642	893.406	904.170	914.934	925.698	936.462	947.226	957.990
9	968.754	979.518	990.282	1,001.045	1,011.809	1,022.573	1,033.337	1,044.101	1,054.865	1,065.629

TABLE 15.—*Meters³ to feet³—1 m.³ = 35.3147 ft.³*.

Units.	0	1	2	3	4	5	6	7	8	9
Tens.										
0	....	35.314	70.630	105.946	141.259	176.574	211.889	247.204	282.514	317.834
1	353.149	388.463	423.778	459.093	494.408	529.723	565.038	600.353	635.667	670.982
2	706.297	741.612	776.927	812.242	847.557	882.871	918.186	953.501	988.816	1,024.131
3	1,059.441	1,094.761	1,130.075	1,165.390	1,200.705	1,236.020	1,271 335	1,306.650	1,341.965	1,377.279
4	1,412.594	1,447.909	1,483.224	1,518.539	1,553.854	1,589.169	1,624.483	1,659.798	1,695.113	1,730.428
5	1,765.743	1,801.058	1,836.373	1,871.687	1,907.002	1,942.317	1,977.632	2,012.947	2,048.262	2,083.577
6	2,118.891	2,154.206	2,189.521	2,224.836	2,260.151	2,295.466	2,330.781	2,366.095	2,401.410	2,436.725
7	2,472.040	2,507.355	2,542.670	2,577.985	2,613.299	2,648.614	2,683.929	2,719.244	2,754.559	2,789.874
8	2,825.189	2,860.503	2,895.818	2,931.133	2,966.448	3,001.763	3,037.078	3,072.393	3,107.708	3,143.022
9	3,178.337	3,213.652	3,248.967	3,284.282	3,319.597	3,354.912	3,390.226	3,425.541	3,460.856	3,496.171

TABLE 16.—*Miles to kilometers—1 mi. = 1.60934 kilom.*

Units.	0	1	2	3	4	5	6	7	8	9
Tens.										
0	....	1. 609	3. 218	4. 827	6. 436	8. 045	9. 654	11. 263	12. 872	14. 481
1	16. 090	17. 699	19. 308	20. 917	22. 526	24. 315	25. 744	27. 353	28. 962	30. 571
2	32. 180	33. 789	35. 398	37. 007	38. 616	40. 225	41. 834	43. 443	45. 052	46. 661
3	48. 270	49. 879	51. 488	53. 097	54. 706	56. 315	57. 924	59. 533	61. 142	62. 751
4	64. 360	65. 969	67. 578	69. 187	70. 796	72. 405	74. 014	75. 623	77. 232	78. 841
5	80. 450	82. 059	83. 668	85. 277	86. 886	88. 495	90. 104	91. 713	93. 322	94. 931
6	96. 540	98. 149	99. 758	101. 367	102. 976	104. 585	106. 194	107. 803	109. 412	111. 021
7	112. 630	114. 239	115. 848	117. 457	119. 066	120. 675	122. 284	123. 893	125. 502	127. 111
8	128. 720	130. 329	131. 938	133. 547	135. 156	136. 765	138. 374	139. 983	141. 592	143. 201
9	144. 810	146. 419	148. 028	149. 637	151. 246	152. 855	154. 464	156. 073	157. 682	159. 291

TABLE 17.—*Kilometers to miles—1 kilom. = 0.6214 miles.*

Units.	0	1	2	3	4	5	6	7	8	9
Tens.										
0	....	0. 621	1. 242	1. 863	2. 484	3. 105	3. 726	4. 347	4. 968	5. 589
1	6. 210	6. 831	7. 452	8. 073	8. 694	9. 315	9. 936	10. 557	11. 178	11. 799
2	12. 420	13. 041	13. 662	14. 283	14. 904	15. 525	16. 146	16. 767	17. 388	18. 009
3	18. 630	19. 251	19. 872	20. 493	21. 114	21. 735	22. 356	22. 977	23. 598	24. 219
4	24. 840	25. 461	26. 082	26. 703	27. 324	27. 945	28. 566	29. 187	29. 808	30. 429
6	31. 050	31. 671	32. 292	32. 913	33. 534	34. 155	34. 776	35. 397	36. 018	36. 639
6	37. 260	37. 881	38. 502	39. 123	39. 744	40. 365	40. 986	41. 607	42. 228	42. 849
7	43. 470	44. 091	44. 712	45. 333	45. 954	46. 575	47. 196	47. 817	48. 438	49. 059
8	49. 680	50. 301	50. 922	51. 543	52. 164	52. 785	53. 406	54. 027	54. 648	55. 269
9	55. 890	56. 511	57. 132	57. 753	58. 374	58. 995	59. 616	60. 237	60. 858	61. 479

TABLE 18.—*Pounds per linear inch to kilograms per linear meter—1 lb. per in. = 17.856 kg. per m.*

Units.	0	1	2	3	4	5	6	7	8	9
Tens.										
0	....	17.856	35.712	53.568	71.424	89.280	107.136	124.992	142.848	160.704
1	178.560	196.416	214.272	232.128	249.984	267.840	285.696	303.352	321.408	339.264
2	357.120	374.976	392.832	410.688	428.544	446.400	464.256	482.112	499.968	517.824
3	535.860	553.536	571.392	589.248	607.104	624.960	642.816	660.672	678.528	696.384
4	714.240	732.096	749.952	767.808	785.664	803.520	821.376	839.232	857.088	874.944
5	892.800	910.656	928.512	946.368	964.224	982.080	999.936	1,017.792	1,035.648	1,053.504
6	1,071.360	1,089.216	1,107.072	1,124.928	1,142.784	1,160.640	1,178.496	1,196.352	1,214.208	1,232.064
7	1,249.920	1,267.776	1,285.632	1,303.488	1,321.344	1,339.200	1,357,056	1,374.912	1,392.768	1,410.624
8	1,428.480	1,446.336	1,464.192	1,482.048	1,499.904	1,517.760	1,535.616	1,553.472	1,571.328	1,589.184
9	1,607.040	1,624.896	1,642.752	1,660.080	1,678.464	1,696.320	1,714.176	1,732.032	1,749.888	1,767.744

TABLE 19.—*Kilograms per linear meter to pounds per linear inch—1 kg. per m. = .055997 lbs. per in.*

Kilograms per meter.	Pounds per linear inch.									
	0	10	20	30	40	50	60	70	80	90
500	27.998	28.558	29.118	29.678	30.238	30.798	31.358	31.918	32.478	33.038
600	33.598	34.158	34.718	35.278	35.838	36.398	36.958	37.518	38.078	38.638
700	39.198	39.758	40.318	40.878	41.438	41.998	42.558	43.118	43.678	44.238
800	44.798	45.358	45.918	46.478	47.038	47.597	48.157	48.717	49.277	49.837
900	50.397	50.957	51.517	52.077	52.637	53.197	53.757	54.317	54.877	55.437
1,000	55.997	56.557	57.117	57.677	58.237	58.797	59.357	59.917	60.477	61.037
1,100	61.597	62.157	62.717	63.277	63.837	64.397	64.957	65.517	66.077	66.637
1,200	67.197	67.757	68.317	68.877	69.437	69.997	70.557	71.117	71.677	72.237
1,300	72.797	73.357	73.917	74.477	75.037	75.597	76.157	76.717	77.277	77.837
1,400	78.397	78.957	79.517	80.077	80.637	81.197	81.757	82.317	82.877	83.437
1,500	83.997	....	....	....	....	....	....	....	....	....

TABLE 20.—*Inch-pounds to kilogram-centimeters—1 in-lb. = 1.152127 kg-cm.*

Units.	0	1	2	3	4	5	6	7	8	9
Tens.										
0	....	1.152	2.304	3.456	4.609	5.761	6.913	8.065	9.217	10.369
1	11.521	12.763	13.826	14.978	16.130	17.282	18.434	19.586	20.738	21.890
2	23.043	24.195	25.347	26.499	27.651	28.803	29.955	31.107	32.260	33.412
3	34.564	35.716	36.868	38.020	39.172	40.324	41.477	42.629	43.781	44.933
4	46.085	47.237	48.389	49.541	50.694	51.846	52.998	54.150	55.302	56.454
5	57.606	58.758	59.911	61.063	62.215	63.367	64.519	65.671	66.823	67.975
6	69.128	70.280	71.432	72.584	73.736	74.888	76.040	77.193	78.345	79.497
7	80.649	81.801	82.953	84.105	85.257	86.410	87.562	88.714	89.866	91.018
8	92.170	93.322	94.474	95.627	96.779	97.931	99.083	100.235	101.387	102.539
9	103.691	104.844	105.996	107.148	108.300	109.452	110.604	111.756	112.908	114.061

TABLE 21.—*Kilogram-centimeters to inch-pounds—1 kg-cm = 0.86796 in-lb.*

Units.	0	1	2	3	4	5	6	7	8	9
Tens.										
0	....	0.8680	1.7359	2.6039	3.4718	4.3398	5.2078	6.0757	6.9437	7.8116
1	8.6796	9.5476	10.4155	11.2835	12.1514	13.0194	13.8874	14.7553	15.6233	16.4912
2	17.3592	18.2272	19.0951	19.9631	20.8310	21.6990	22.5670	23.4349	24.3029	25.1708
3	26.0388	26.9068	27.7747	28.6427	29.5106	30.3786	31.2466	32.1145	32.9825	33.8504
4	34.7184	35.5864	36.4543	37.3223	38.1902	39.0582	39.9262	40.7941	41.6621	42.5300
5	43.3980	44.2660	45.1339	46.0019	46.8698	47.7378	48.6058	49.4737	50.3417	51.2096
6	52.0776	52.9456	53.8135	54.6815	55.5494	56.4174	57.2854	58.1533	59.0213	59.8892
7	60.7572	61.6252	62.4931	63.3611	64.2290	65.0970	65.9650	66.8329	67.7009	68.5688
8	69.4368	70.3048	71.1727	72.0407	72.9086	73.7766	74.6446	75.5125	76.3805	77.2484
9	78.1164	78.9844	79.8523	80.7203	81.5882	82.4562	83.3242	84.1921	85.0601	85.9280

Table 22.—*Pounds per square inch to kilograms per square centimeter—1 lb-in.^{2}=0.0703067 kg-cm.^{2}.*

Units.	0	1	2	3	4	5	6	7	8	9
Tens.										
0	....	0. 07031	0. 14061	0. 21092	0. 28123	0. 35153	0. 42184	0. 49215	0. 56245	0. 63276
1	0. 70307	. 77337	. 84368	. 91399	. 98429	1. 05460	1. 12491	1. 19521	1. 26552	1. 33583
2	1. 40613	1. 47644	1. 54675	1. 61705	1. 68736	1. 75767	1. 82797	1. 89828	1. 96859	2. 03889
3	2. 10920	2. 17951	2. 24981	2. 32012	2. 39043	2. 46073	2. 53104	2. 60135	2. 67165	2. 74196
4	2. 81227	2. 88257	2. 95288	3. 02319	3. 09349	3. 16380	3. 23411	3. 30441	3. 37472	3. 44503
5	3. 51534	3. 58564	3. 65595	3. 72626	3. 79656	3. 86687	3. 93718	4. 00748	4. 07779	4. 14810
6	4. 21840	4. 28871	4. 35902	4. 42932	4. 49963	4. 56994	4. 64024	4. 71055	4. 78086	4. 85116
7	4. 92147	4. 99178	5. 06208	5. 13239	5. 20270	5. 27300	5. 34331	5. 41362	5. 48392	5. 55423
8	5. 62454	5. 69484	5. 76515	5. 83546	5. 90576	5. 97607	6. 04638	6. 11668	6. 18699	6. 25730
9	6. 32760	6. 39791	6. 46822	6. 53852	6. 60883	6. 67914	6. 74944	6. 81975	6. 89006	6. 96036

Table 23.—*Kilograms per square centimeter to pounds per square inch—1 kg-cm.^{2}=14.2234 lbs-in.^{2}.*

Units.	0	1	2	3	4	5	6	7	8	9
Tens.										
0	....	14. 22	28. 45	42. 67	56. 89	71. 12	85. 34	99. 56	113. 79	128. 01
1	142. 23	156. 46	170. 68	184. 90	199. 13	213. 35	227. 57	241. 80	256. 02	270. 24
2	284. 47	298. 69	312. 91	327. 14	341. 36	355. 59	369. 81	384. 03	398. 26	412. 48
3	426. 70	440. 93	455. 15	469. 37	483. 60	497. 82	512. 04	526. 27	540. 49	554. 71
4	568. 94	583. 16	597. 38	611. 61	625. 83	640. 05	654. 28	668. 50	682. 72	696. 95
5	711. 17	725. 39	739. 62	753. 84	768. 06	782. 29	796. 51	810. 73	824. 96	839. 18
6	853. 40	867. 63	881. 85	896. 07	910. 30	924. 52	938. 74	952. 97	967. 19	981. 41
7	995. 64	1, 009. 86	1, 024. 08	1, 038. 31	1, 052. 53	1, 066. 76	1, 080. 98	1, 095. 20	1, 109. 43	1, 123. 65
8	1, 137. 87	1, 152. 10	1, 166. 32	1, 180. 54	1, 194. 77	1, 208. 99	1, 223. 21	1, 237. 44	1, 251. 66	1, 265. 88
9	1, 280. 11	1, 294. 33	1, 308. 55	1, 322. 78	1, 337. 00	1, 351. 22	1, 365. 45	1, 379. 67	1, 393. 89	1, 408. 12

Table 24.—*Grams per square meter to ounces per square yard.*

Grams per square meter.	Ounces per square yard.									
	0	1	2	3	4	5	6	7	8	9
0	....	0. 030	0. 059	0. 089	0. 118	0. 148	0. 177	0. 206	0. 236	0. 265
10	0. 295	. 324	. 354	. 383	. 413	. 442	. 472	. 501	. 531	. 560
20	. 590	. 619	. 649	. 678	. 708	. 737	. 767	. 796	. 826	. 855
30	. 885	. 914	. 944	. 973	1. 003	1. 032	1. 062	1. 091	1. 121	1. 150
40	1. 180	1. 209	1. 239	1. 268	1. 298	1. 327	1. 357	1. 386	1. 416	1. 445
50	1. 475	1. 504	1. 533	1. 563	1. 592	1. 622	1. 651	1. 681	1. 710	1. 740
60	1. 769	1. 799	1. 828	1. 858	1. 887	1. 917	1. 946	1. 976	2. 005	2. 035
70	2. 064	2. 094	2. 123	2. 153	2. 182	2. 212	2. 241	2. 271	2. 300	2. 330
80	2. 359	2. 389	2. 418	2. 448	2. 477	2. 507	2. 536	2. 566	2. 595	2. 625
90	2. 654	2. 684	2. 713	2. 743	2. 772	2. 802	2. 831	2. 860	2. 890	2. 919
100	2. 949	2. 978	3. 008	3. 037	3. 067	3. 096	3. 126	3. 155	3. 185	3. 214
110	3. 244	3. 273	3. 303	3. 332	3. 362	3. 391	3. 421	3. 450	3. 480	3. 509
120	3. 539	3. 568	3. 598	3. 627	3. 657	3. 686	3. 716	3. 745	3. 775	3. 804
130	3. 834	3. 863	3. 893	3. 922	3. 952	3. 981	4. 011	4. 040	4. 070	4. 099
140	4. 129	4. 158	4. 187	4. 217	4. 246	4. 276	4. 305	4. 335	4. 364	4. 394
150	4. 423	....	....	....	....	....	....	....	....	....

TABLE 25.—*Ounces per square yard to pounds per square feet.*

Ounces per square yard.	Pounds per square foot.									
	0	1	2	3	4	5	6	7	8	9
0	....	0.00695	0.014	0.021	0.028	0.035	0.042	0.049	0.056	0.063
10	0.0695	.076	.083	.090	.097	.104	.111	.118	.125	.132
20	.139	.146	.153	.160	.167	.174	.181	.188	.195	.202
30	.209	.215	.222	.229	.236	.243	.250	.257	.264	.271
40	.278	.285	.292	.299	.306	.313	.320	.327	.334	.341
50	.348	.354	.361	.368	.375	.382	.389	.396	.403	.410
60	.417	.424	.431	.438	.445	.452	.459	.466	.473	.480
70	.487	.493	.500	.507	.514	.521	.528	.535	.542	.549
80	.556	.563	.570	.577	.584	.591	.598	.605	.612	.619
90	.626	.632	.639	.646	.653	.660	.667	.674	.681	.688
100	.695	.702	.709	.716	.723	.730	.737	.744	.750	.758
110	.764	.771	.778	.785	.792	.799	.806	.813	.820	.826
120	.834	.840	.848	.855	.861	.869	.876	.883	.890	.897
130	.902	.910	.917	.924	.931	.938	.945	.952	.959	.966
140	.973	.980	.986	.994	1.001	1.007	1.015	1.022	1.029	1.036
150	1.043	1.049	1.056	1.063	1.070	1.077	1.084	1.091	1.098	1.114

III.—COMPARATIVE SCALES.

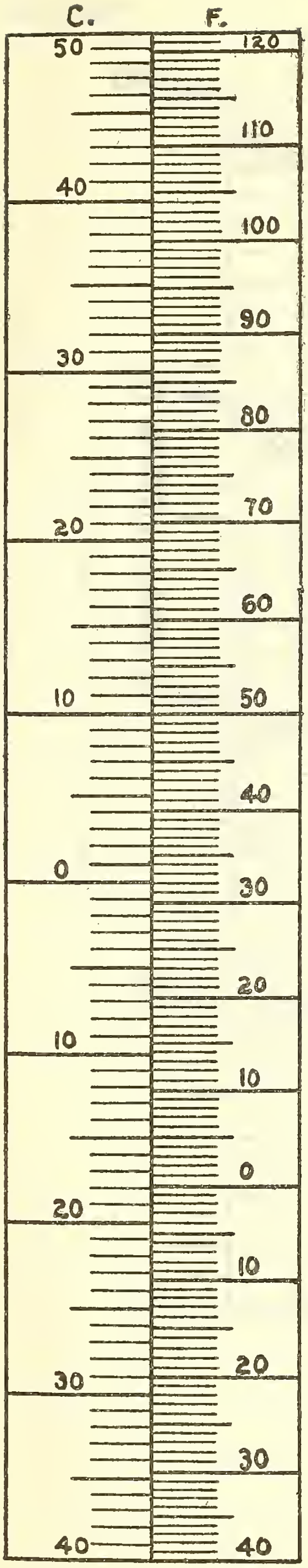


CHART 1.—*Thermometric degrees—Centigrade and Fahrenheit.*

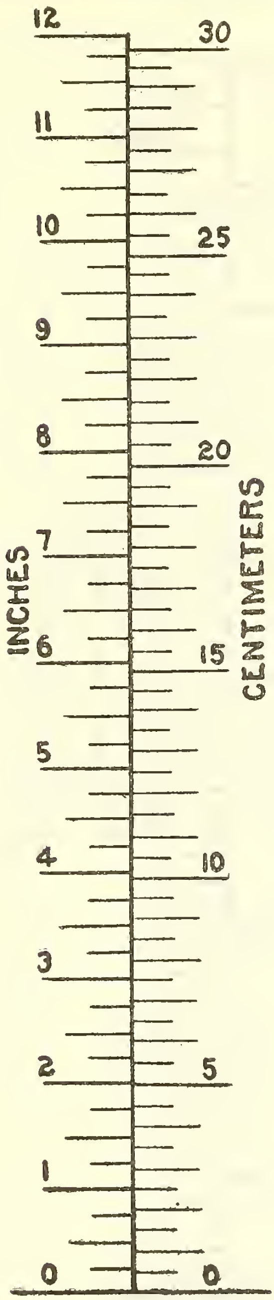


CHART 2.—*Inches—Centimeters.*

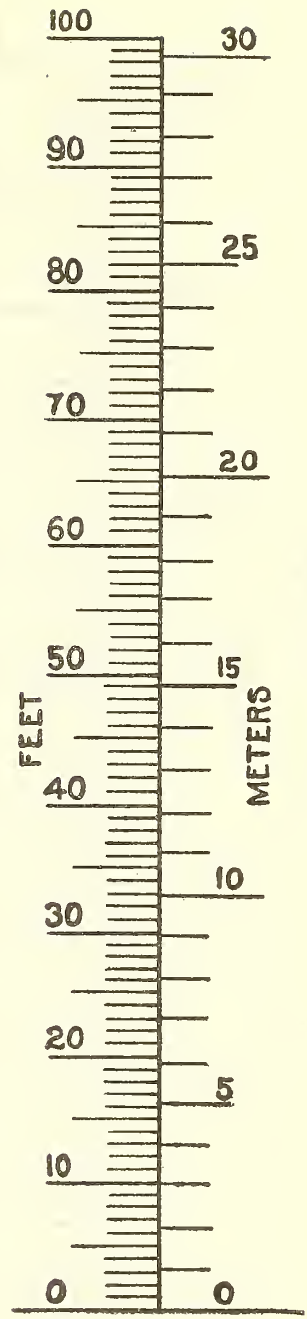


CHART 3.—*Feet—Meters.*

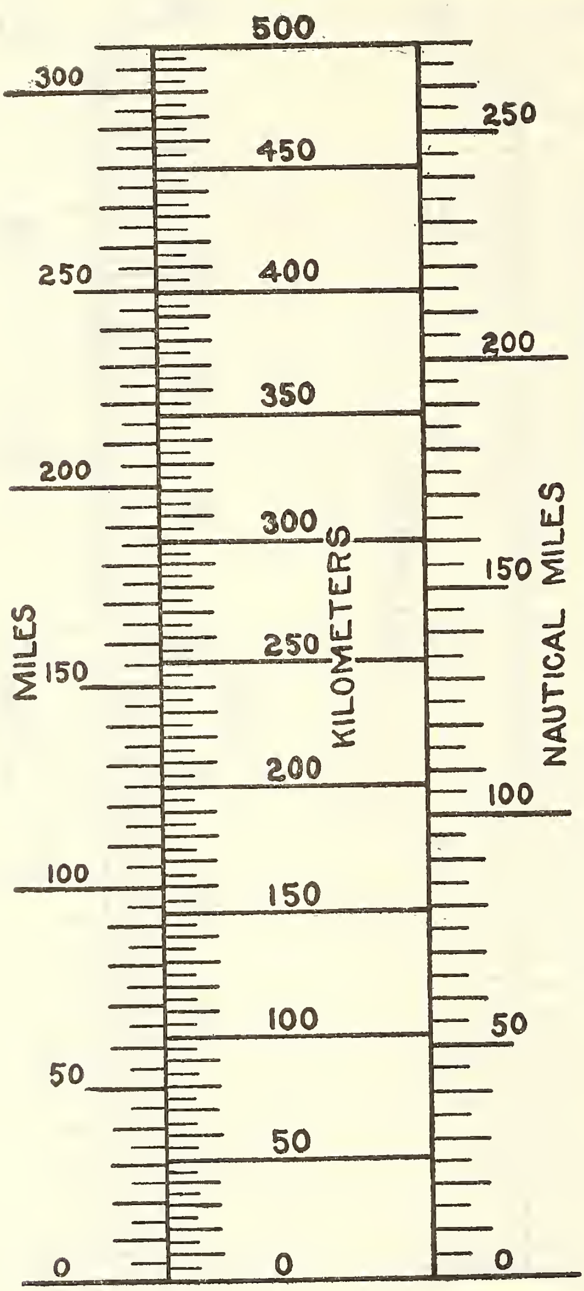


CHART 4.—*Miles—kilometers. Kilometers—nautical miles.*

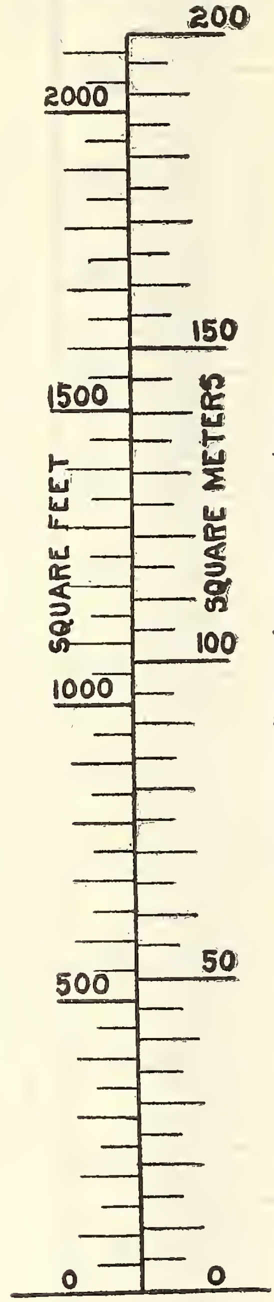


CHART 5.—*Square feet—square meters.*

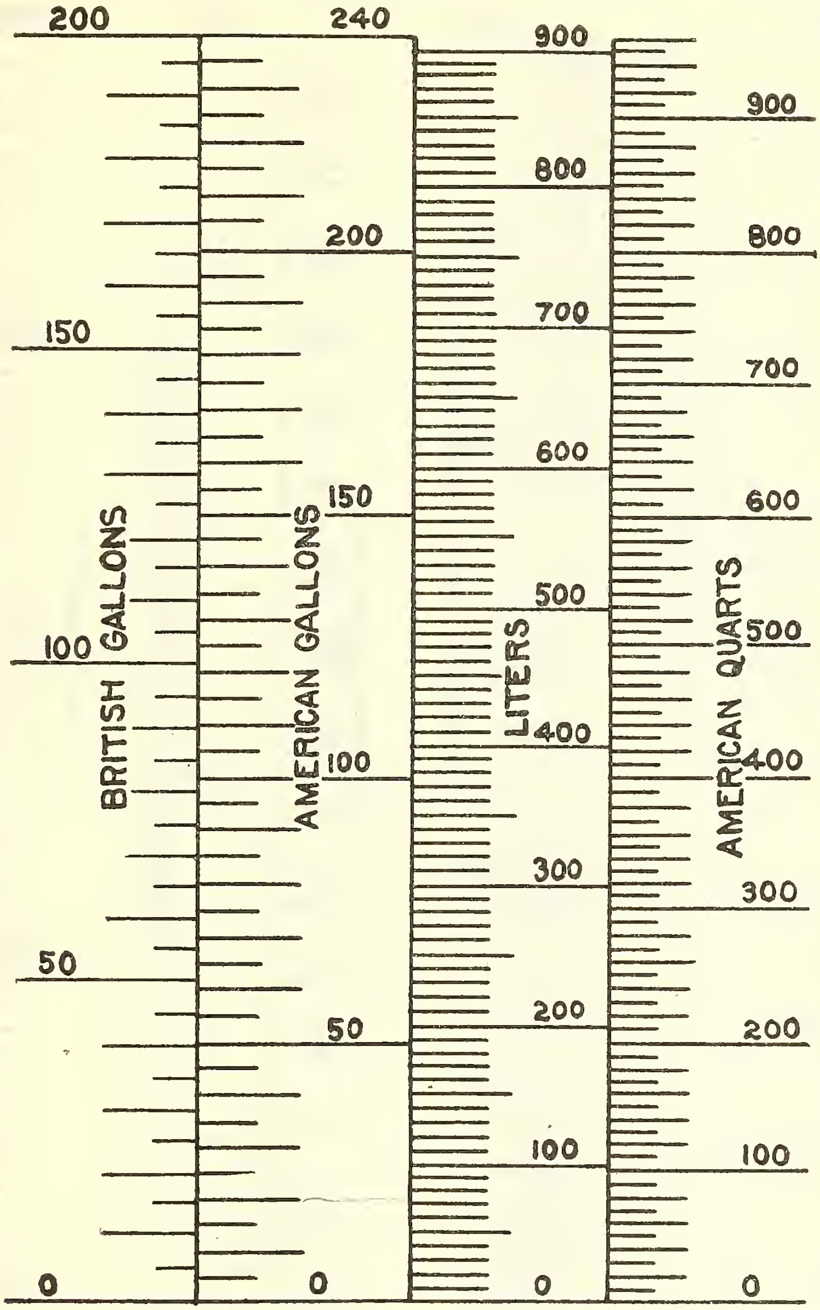


CHART 6.—*British gallons—American gallons; American gallons—liters; liters—American quarts.*

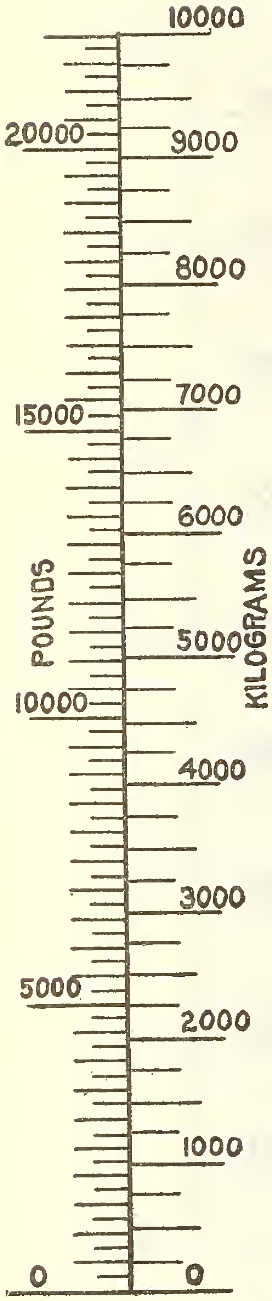


CHART 7.—*Pounds—Kilograms.*

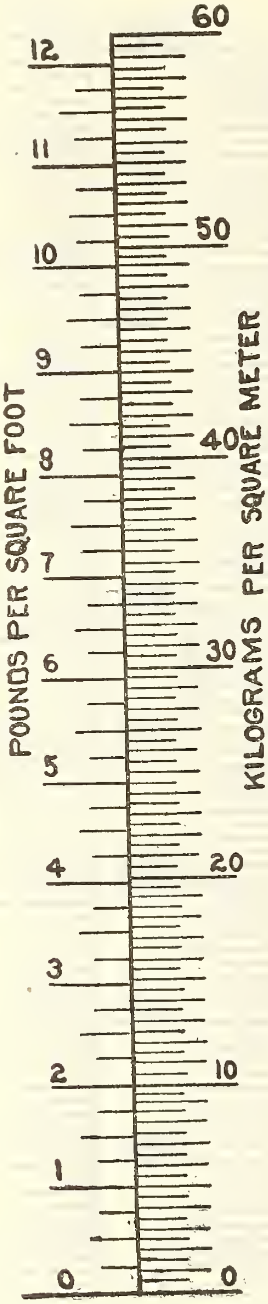


CHART 8.—*Pounds per square foot—Kilograms per square meter.*

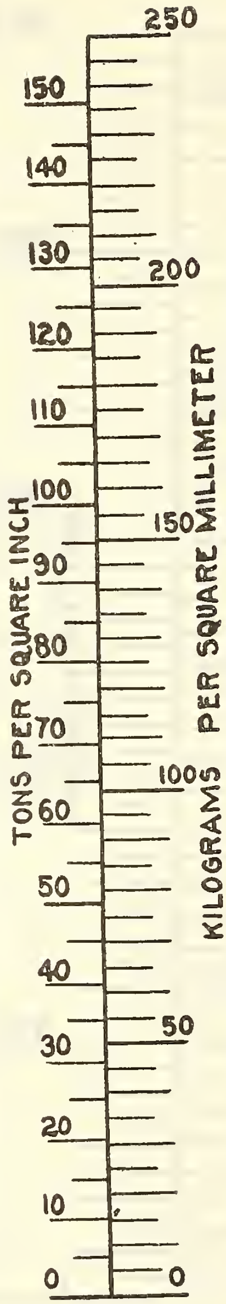


CHART 9.—*Tons per square inch—Kilograms per square millimeter.*

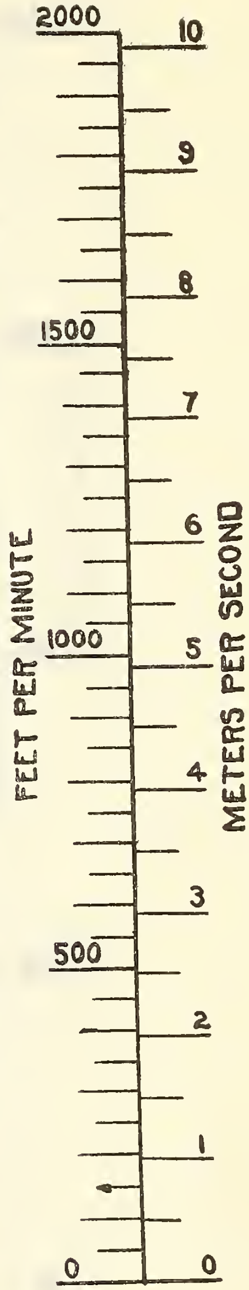


CHART 10.—*Feet per minute—Meters per second.*

IV.—FUNCTIONS OF NUMBERS.

TABLE 1.—*Powers and roots.*

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
1	1	1	1.000 0000	1.000 0000	1.000 000 000
2	4	8	1.414 2136	1.259 9210	.500 000 000
3	9	27	1.732 0508	1.442 2496	.333 333 333
4	16	64	2.000 0000	1.587 4011	.250 000 000
5	25	125	2.236 0680	1.709 9759	.200 000 000
6	36	216	2.449 4897	1.817 1206	.166 666 667
7	49	343	2.645 7513	1.912 9312	.142 857 143
8	64	512	2.828 4271	2.000 0000	.125 000 000
9	81	729	3.000 0000	2.080 0837	.111 111 111
10	100	1,000	3.162 2777	2.154 4347	.100 000 000
11	121	1,331	3.316 6248	2.223 9801	.090 909 091
12	144	1,728	3.464 1016	2.289 4286	.083 333 333
13	169	2,197	3.605 5513	2.351 3347	.076 923 077
14	196	2,744	3.741 6574	2.410 1422	.071 428 571
15	225	3,375	3.872 9833	2.466 2121	.066 666 667
16	256	4,096	4.000 0000	2.519 8421	.062 500 000
17	289	4,193	4.123 1056	2.571 2816	.058 823 529
18	324	5,832	4.242 6407	2.620 7414	.055 555 556
19	361	6,859	4.358 8989	2.668 4016	.052 631 579
20	400	8,000	4.472 1360	2.714 4177	.050 000 000
21	441	9,261	4.582 5757	2.758 9243	.047 619 048
22	484	10,648	4.690 4158	2.802 0393	.045 454 545
23	529	12,167	4.795 8315	2.843 8670	.043 478 261
24	576	13,824	4.898 9795	2.884 4991	.041 666 667
25	625	15,625	5.000 0000	2.924 0177	.040 000 000
26	676	17,576	5.099 0195	2.962 4960	.038 461 538
27	729	19,683	5.196 1524	3.000 0000	.037 037 037
28	784	21,952	5.291 5026	3.036 5889	.035 714 286
29	841	24,389	5.385 1648	3.072 3168	.034 482 759
30	900	27,000	5.477 2256	3.107 2325	.033 333 333
31	961	29,791	5.567 7644	3.141 3806	.032 258 065
32	1,024	32,768	5.656 8542	3.174 8021	.031 250 000
33	1,089	35,937	5.744 5626	3.207 5343	.030 303 030
34	1,156	39,304	5.830 9519	3.239 6118	.029 411 765
35	1,225	42,875	5.916 0798	3.271 0663	.028 571 429
36	1,296	46,656	6.000 0000	3.301 9272	.027 777 778
37	1,369	50,653	6.082 7625	3.332 2218	.027 027 027
38	1,444	54,872	6.164 4140	3.361 9754	.026 315 789
39	1,521	59,319	6.244 9980	3.391 2114	.025 641 026
40	1,600	64,000	6.324 5553	3.419 9519	.025 000 000
41	1,681	68,921	6.403 1242	3.448 2172	.024 390 244
42	1,764	74,088	6.480 7407	3.476 0266	.023 809 524
43	1,849	79,507	6.557 4385	3.503 3981	.023 255 814
44	1,936	85,184	6.633 2496	3.530 3483	.022 727 273
45	2,025	91,125	6.708 2039	3.556 8933	.022 222 222
46	2,116	97,336	6.782 3300	3.583 0479	.021 739 130
47	2,209	103,823	6.855 6546	3.608 8261	.021 276 600
48	2,304	110,592	6.928 2032	3.634 2411	.020 833 333
49	2,401	117,649	7.000 0000	3.659 3057	.020 408 163
50	2,500	125,000	7.071 0678	3.684 0314	.020 000 000
51	2,601	132,651	7.141 4284	3.708 4298	.019 607 843
52	2,704	140,608	7.211 1026	3.732 5111	.019 230 769
53	2,809	148,877	7.280 1099	3.756 2858	.018 867 925
54	2,916	157,464	7.348 4692	3.779 7631	.018 518 519
55	3,025	166,375	7.416 1985	3.802 9525	.018 181 818
56	3,136	175,616	7.483 3148	3.825 8624	.017 857 143
57	3,249	185,193	7.549 8344	3.848 5011	.017 543 860
58	3,364	195,112	7.615 7731	3.870 8766	.017 241 379
59	3,481	205,379	7.681 1457	3.892 9965	.016 949 153
60	3,600	216,000	7.745 9667	3.914 8676	.016 666 667
61	3,721	226,981	7.810 2497	3.930 4972	.016 393 443
62	3,844	238,328	7.874 0079	3.957 8915	.016 129 032

TABLE 1.—*Powers and roots*—Continued.

Number.	Squares.	Cubes.	√Roots.	√Roots.	Reciprocals.
63	3, 969	250, 047	7. 937 2539	3. 979 0571	. 015 873 016
64	4, 096	262, 144	8. 000 0000	4. 000 0000	. 015 625 000
65	4, 225	274, 625	8. 062 2577	4. 020 7256	. 015 384 615
66	4, 356	287, 496	8. 124 0384	4. 041 2401	. 015 151 515
67	4, 489	300, 763	8. 185 3528	4. 061 5480	. 014 925 373
68	4, 624	314, 432	8. 246 2113	4. 081 6551	. 014 705 882
69	4, 761	328, 509	8. 306 6239	4. 101 5661	. 014 492 754
70	4, 900	343, 000	8. 366 6003	4. 121 2853	. 014 285 714
71	5, 041	357, 911	8. 426 1498	4. 140 8178	. 014 084 517
72	5, 184	373, 248	8. 485 2814	4. 160 1676	. 013 888 889
73	5, 329	389, 017	8. 544 0037	4. 179 3390	. 013 698 630
74	5, 476	405, 224	8. 602 3253	4. 198 3364	. 013 513 514
75	5, 625	421, 875	8. 660 2540	4. 217 1633	. 013 333 333
76	5, 776	438, 976	8. 717 7979	4. 235 8236	. 013 157 895
77	5, 929	456, 533	8. 774 9644	4. 254 3210	. 012 987 013
78	6, 084	474, 552	8. 831 7609	4. 272 6586	. 012 820 513
79	6, 241	493, 039	8. 888 1944	4. 290 8404	. 012 658 228
80	6, 400	512, 000	8. 944 2719	4. 308 8695	. 012 500 000
81	6, 561	531, 441	9. 000 0000	4. 326 7487	. 012 345 679
82	6, 724	551, 368	9. 055 3851	4. 344 4815	. 012 195 122
83	6, 889	571, 787	9. 110 4336	4. 362 0707	. 012 048 193
84	7, 056	592, 704	9. 165 1514	4. 379 5191	. 011 904 762
85	7, 225	614, 125	9. 219 5445	4. 396 8296	. 011 764 706
86	7, 396	636, 056	9. 273 6185	4. 414 0049	. 011 627 907
87	7, 569	658, 503	9. 327 3791	4. 431 0476	. 011 494 253
88	7, 744	681, 472	9. 380 8315	4. 447 9692	. 011 363 636
89	7, 921	704, 969	9. 433 9811	4. 464 7451	. 011 235 955
90	8, 100	729, 000	9. 486 8330	4. 481 4047	. 011 111 111
91	8, 281	753, 571	9. 539 3920	4. 497 9414	. 010 989 011
92	8, 464	778, 688	9. 591 6630	4. 514 3574	. 010 869 565
93	8, 649	804, 357	9. 643 6508	4. 530 6549	. 010 752 688
94	8, 836	830, 584	9. 695 3597	4. 546 8359	. 010 638 298
95	9, 025	857, 375	9. 746 7943	4. 562 9026	. 010 526 316
96	9, 216	884, 736	9. 797 9590	4. 578 8570	. 010 416 667
97	9, 409	912, 673	9. 848 8578	4. 594 7009	. 010 309 278
98	9, 604	941, 192	9. 899 4949	4. 610 4363	. 010 204 082
99	9, 801	970, 299	9. 949 8744	4. 626 0650	. 010 101 010
100	10, 000	1, 000, 000	10. 000 0000	4. 641 5888	. 010 000 000
101	10, 201	1, 030, 301	10. 049 8756	4. 657 0095	. 009 900 990
102	10, 404	1, 061, 208	10. 099 5049	4. 672 3287	. 009 803 922
103	10, 609	1, 092, 727	10. 148 8916	4. 687 5482	. 009 708 738
104	10, 816	1, 124, 864	10. 198 0390	4. 702 6694	. 009 615 385
105	11, 025	1, 157, 625	10. 246 9508	4. 717 6940	. 009 523 810
106	11, 236	1, 191, 016	10. 295 6301	4. 732 6235	. 009 433 962
107	11, 449	1, 225, 043	10. 344 0804	4. 747 4594	. 009 345 794
108	11, 664	1, 259, 712	10. 392 3048	4. 762 2032	. 009 259 259
109	11, 881	1, 295, 029	10. 440 3065	4. 776 8562	. 009 174 312
110	12, 100	1, 331, 000	10. 488 0885	4. 791 4199	. 009 090 909
111	12, 321	1, 367, 631	10. 535 6538	4. 805 8995	. 009 009 009
112	12, 544	1, 404, 928	10. 583 0052	4. 820 2845	. 008 928 571
113	12, 769	1, 442, 897	10. 630 1458	4. 834 5881	. 008 849 558
114	12, 996	1, 481, 544	10. 677 0783	4. 848 8076	. 008 771 930
115	13, 225	1, 520, 875	10. 723 8053	4. 862 9442	. 008 695 652
116	13, 456	1, 560, 896	10. 770 3296	4. 876 9990	. 008 620 690
117	13, 689	1, 601, 613	10. 816 6538	4. 890 9732	. 008 547 009
118	13, 924	1, 643, 032	10. 862 7805	4. 904 8681	. 008 474 576
119	14, 161	1, 685, 159	10. 908 7121	4. 918 6847	. 008 403 361
120	14, 400	1, 728, 000	10. 954 4512	4. 932 4242	. 008 333 333
121	14, 641	1, 771, 561	11. 000 0000	4. 946 0874	. 008 264 463
122	14, 884	1, 815, 848	11. 045 3610	4. 959 6757	. 008 196 721
123	15, 129	1, 860, 867	11. 090 5365	4. 973 1898	. 008 130 081
124	15, 376	1, 906, 624	11. 135 5287	4. 986 6310	. 008 064 516
125	15, 625	1, 953, 125	11. 180 3399	5. 000 0000	. 008 000 000
126	15, 876	2, 000, 376	11. 224 9722	5. 013 2979	. 007 936 508
127	16, 129	2, 048, 383	11. 269 4277	5. 026 5257	. 007 874 016

TABLE 1.—*Powers and roots*—Continued.

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
128	16, 384	2, 097, 152	11. 313 7085	5. 039 6842	. 007 812 500
129	16, 641	2, 146, 689	11. 357 8167	5. 052 7743	. 007 751 938
130	16, 900	2, 197, 000	11. 401 7543	5. 065 7970	. 007 692 308
131	17, 161	2, 248, 091	11. 445 5231	5. 078 7531	. 007 633 588
132	17, 424	2, 299, 968	11. 489 1253	5. 091 6434	. 007 575 758
133	17, 689	2, 352, 637	11. 532 5626	5. 104 4687	. 007 518 797
134	17, 956	2, 406, 104	11. 575 8369	5. 117 2299	. 007 462 687
135	18, 225	2, 460, 375	11. 618 9500	5. 129 9278	. 007 407 407
136	18, 496	2, 515, 456	11. 661 9038	5. 142 5632	. 007 352 941
137	18, 769	2, 571, 353	11. 704 6999	5. 155 1367	. 007 299 270
138	19, 044	2, 628, 072	11. 747 3401	5. 167 6493	. 007 246 377
139	19, 321	2, 685, 619	11. 789 8261	5. 180 1015	. 007 194 245
140	19, 600	2, 744, 000	11. 832 1596	5. 192 4941	. 007 142 857
141	19, 881	2, 803, 221	11. 874 3421	5. 204 8279	. 007 092 199
142	20, 164	2, 863, 288	11. 916 3753	5. 217 1034	. 007 042 254
143	20, 449	2, 924, 207	11. 958 2607	5. 229 3215	. 006 993 007
144	20, 736	2, 985, 984	12. 000 0000	5. 241 4828	. 006 944 444
145	21, 025	3, 048, 625	12. 041 5946	5. 253 5879	. 006 896 552
146	21, 316	3, 112, 136	12. 083 0460	5. 265 6374	. 006 849 315
147	21, 609	3, 176, 523	12. 124 3557	5. 277 6321	. 006 802 721
148	21, 904	3, 241, 792	12. 165 5251	5. 289 5725	. 006 756 757
149	22, 201	3, 307, 949	12. 206 5556	5. 301 4592	. 006 711 409
150	22, 500	3, 375, 000	12. 247 4487	5. 313 2928	. 006 666 667
151	22, 801	3, 442, 951	12. 288 2057	5. 325 0740	. 006 622 517
152	23, 104	3, 511, 008	12. 328 8280	5. 336 8033	. 006 578 947
153	23, 409	3, 581, 577	12. 369 3169	5. 348 4812	. 006 535 948
154	23, 716	3, 652, 264	12. 409 6736	5. 360 1084	. 006 493 506
155	24, 025	3, 723, 875	12. 449 8996	5. 371 6854	. 006 451 613
156	24, 336	3, 796, 416	12. 489 9960	5. 383 2126	. 006 410 256
157	24, 649	3, 869, 893	12. 529 9641	5. 394 6907	. 006 369 427
158	24, 964	3, 944, 312	12. 569 8051	5. 406 1202	. 006 329 114
159	25, 281	4, 019, 679	12. 609 5202	5. 417 5015	. 006 289 308
160	25, 600	4, 096, 000	12. 649 1106	5. 428 8352	. 006 250 000
161	25, 921	4, 173, 281	12. 688 5775	5. 440 1218	. 006 211 180
162	26, 244	4, 251, 528	12. 727 9221	5. 451 3618	. 006 172 840
163	36, 569	4, 330, 747	12. 767 1453	5. 462 5556	. 006 134 969
164	26, 896	4, 410, 944	12. 806 2485	5. 473 7037	. 006 097 561
165	27, 225	4, 492, 125	12. 845 2326	5. 484 8066	. 006 060 606
166	27, 556	4, 574, 296	12. 884 0987	5. 495 8647	. 006 024 096
167	27, 889	4, 657, 463	12. 922 8480	5. 506 8784	. 005 988 024
168	28, 224	4, 741, 632	12. 961 4814	5. 517 8484	. 005 952 381
169	28, 561	4, 826, 809	13. 000 0000	5. 528 7748	. 005 917 160
170	28, 900	4, 913, 000	13. 038 4048	5. 539 6583	. 005 882 353
171	29, 241	5, 000, 211	13. 076 6968	5. 550 4991	. 005 847 953
172	29, 584	5, 088, 448	13. 114 8770	5. 561 2978	. 005 813 953
173	29, 929	5, 177, 717	13. 152 9464	5. 572 0546	. 005 780 347
174	30, 276	5, 268, 024	13. 190 9060	5. 582 7702	. 005 747 126
175	30, 625	5, 359, 375	13. 228 7566	5. 593 4447	. 005 714 286
176	30, 976	5, 451, 776	13. 266 4992	5, 604 0787	. 005 681 818
177	31, 329	5, 545, 233	13. 304 1347	5. 614 6724	. 005 649 718
178	31, 684	5, 639, 752	13. 341 6641	5. 625 2263	. 005 617 978
179	32, 041	5, 735, 339	13. 379 0882	5. 635 7408	. 005 586 592
180	32, 400	5, 832, 000	13. 416 4079	5. 646 2162	. 005 555 556
181	32, 761	5, 929, 741	13. 453 6240	5. 656 6528	. 005 524 862
182	33, 124	6, 028, 568	13. 490 7376	5. 667 0511	. 005 494 505
183	33, 489	6, 128, 487	13. 527 7493	5. 677 4114	. 005 464 481
184	33, 856	6, 229, 504	13. 564 6600	5. 687 7340	. 005 434 783
185	34, 225	6, 331, 625	13. 601 4705	5. 698 0192	. 005 405 405
186	34, 596	6, 434, 856	13. 638 1817	5. 708 2675	. 005 376 344
187	34, 969	6, 539, 203	13. 674 7943	5. 718 4791	. 005 347 594
188	35, 344	6, 644, 672	13. 711 3092	5. 728 6543	. 005 319 149
189	35, 721	6, 751, 269	13. 747 7271	5. 738 7936	. 005 291 005
190	36, 100	6, 859, 000	13. 784 0488	5. 748 8971	. 005 263 158
191	36, 481	6, 967, 871	13. 820 2750	5. 758 9652	. 005 235 602
192	36, 864	7, 077, 888	13. 856 4065	5. 768 9982	. 005 208 333

TABLE 1.—*Powers and roots*—Continued.

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
193	37, 249	7, 189, 517	13. 892 4400	5. 778 9966	. 005 181 347
194	37, 636	7, 301, 384	13. 928 3883	5. 788 9604	. 005 154 639
195	38, 025	7, 414, 875	13. 964 2400	5. 798 8900	. 005 128 205
196	34, 416	7, 529, 536	14. 000 0000	5. 808 7857	. 005 102 041
197	38, 809	7, 645, 373	14. 035 6688	5. 818 6479	. 005 076 142
198	39, 204	7, 762, 392	14. 071 2473	5. 828 4867	. 005 050 505
199	39, 601	7, 880, 599	14. 106 7360	5. 838 2725	. 005 025 126
200	40, 000	8, 000, 000	14. 142 1356	5. 848 0355	. 005 000 000
201	40, 401	8, 120, 601	14. 177 4469	5. 857 7660	. 004 975 124
202	40, 804	8, 242, 408	14. 212 6704	5. 867 4673	. 004 950 495
203	41, 209	8, 365, 427	14. 247 8068	5. 877 1307	. 004 926 108
204	41, 616	8, 489, 664	14. 282 8569	5. 886 7653	. 004 901 961
205	42, 025	8, 615, 125	14. 317 8211	5. 896 3685	. 004 878 049
206	42, 436	8, 741, 816	14. 352 7001	5. 905 9406	. 004 854 369
207	42, 849	8, 869, 743	14. 387 4946	5. 915 4817	. 004 830 918
208	43, 264	8, 998, 912	14. 422 2051	5. 924 9921	. 004 807 692
209	43, 681	9, 129, 329	14. 456 8323	5. 934 4721	. 004 784 689
210	44, 100	9, 261, 000	14. 491 3767	5. 943 9220	. 004 761 905
211	44, 521	9, 393, 931	14. 525 8390	5. 953 3418	. 004 739 336
212	44, 944	9, 528, 128	14. 560 2198	5. 962 7320	. 004 716 981
213	45, 369	9, 663, 597	14. 594 5195	5. 972 0926	. 004 694 836
214	45, 796	9, 800, 344	14. 628 7388	5. 981 4240	. 004 672 897
215	46, 225	9, 938, 375	14. 662 8783	5. 990 7264	. 004 651 163
216	46, 656	10, 077, 696	14. 696 9385	6. 000 0000	. 004 629 630
217	47, 089	10, 218, 313	14. 730 9199	6. 009 2450	. 004 608 295
218	47, 524	10, 360, 232	14. 764 8231	6. 018 4617	. 004 587 156
219	47, 961	10, 503, 459	14. 798 6486	6. 027 6502	. 004 566 210
220	48, 400	10, 648, 000	14. 832 3970	6. 036 8107	. 004 545 455
221	48, 841	10, 793, 861	14. 866 0687	6. 045 9435	. 004 524 887
222	49, 284	10, 941, 048	14. 899 6644	6. 055 0489	. 004 504 505
223	49, 729	11, 089, 567	14. 933 1845	6. 064 1270	. 004 484 305
224	50, 176	11, 239, 424	14. 966 6295	6. 073 1779	. 004 464 286
225	50, 625	11, 390, 625	15. 000 0000	6. 082 4020	. 004 444 444
226	51, 076	11, 543, 176	15. 003 2964	6. 099 1994	. 004 424 779
227	51, 529	11, 697, 083	15. 066 5192	6. 100 1702	. 004 405 286
228	51, 984	11, 852, 352	15. 099 6689	6. 109 1147	. 004 385 965
229	52, 441	12, 008, 989	15. 132 7460	6. 118 0332	. 004 366 812
230	52, 900	12, 167, 000	15. 165 7509	6. 126 9257	. 004 347 826
231	53, 361	12, 326, 391	15. 198 6842	6. 135 7924	. 004 329 004
232	53, 824	12, 487, 168	15. 231 5462	6. 144 6337	. 004 310 345
233	54, 289	12, 649, 337	15. 264 3375	6. 153 4495	. 004 291 845
234	54, 756	12, 812, 904	15. 297 0585	6. 162 2401	. 004 273 504
235	55, 225	12, 977, 875	15. 329 7097	6. 171 0058	. 004 255 319
236	55, 696	13, 144, 256	15. 362 2915	6. 179 7466	. 004 237 288
237	56, 169	13, 312, 053	15. 394 8043	6. 188 4628	. 004 219 409
238	56, 644	13, 481, 272	15. 427 2486	6. 197 1544	. 004 201 681
239	57, 121	13, 651, 919	15. 459 6248	6. 205 8218	. 004 184 100
240	57, 600	13, 824, 000	15. 491 9334	6. 214 4650	. 004 166 667
241	58, 081	13, 997, 521	15. 524 1747	6. 223 0843	. 004 149 378
242	58, 564	14, 172, 488	15. 556 3492	6. 231 6797	. 004 132 231
243	59, 049	14, 348, 907	15. 588 4573	6. 240 2515	. 004 115 226
244	59, 536	14, 526, 784	15. 620 4994	6. 248 7998	. 004 098 361
245	60, 025	14, 706, 125	15. 652 4758	6. 257 3248	. 004 081 633
246	60, 516	14, 886, 936	15. 684 3871	6. 265 8266	. 004 065 041
247	61, 009	15, 069, 223	15. 716 2336	6. 274 3054	. 004 048 583
248	61, 504	15, 252, 992	15. 748 0157	6. 282 7613	. 004 032 258
249	62, 001	15, 438, 249	15. 779 7338	6. 291 1946	. 004 016 064
250	62, 500	15, 625, 000	15. 811 3883	6. 299 6053	. 004 000 000
251	63, 001	15, 813, 251	15. 842 9795	6. 307 9935	. 003 984 064
252	63, 504	16, 003, 008	15. 874 5079	6. 316 3596	. 003 968 254
253	64, 009	16, 194, 277	15. 905 9737	6. 324 7035	. 003 952 569
254	64, 516	16, 387, 064	15. 937 3775	6. 333 0256	. 003 937 008
255	65, 025	16, 581, 375	15. 968 7194	6. 341 3257	. 003 921 569
256	65, 536	16, 777, 216	16. 000 0000	6. 349 6042	. 003 906 250
257	66, 049	16, 974, 593	16. 031 2195	6. 357 8611	. 003 891 051

TABLE 1.—*Powers and roots*—Continued.

Number.	Squares.	Cubes.	√Roots.	∛Roots.	Reciprocals.
258	66, 564	17, 173, 512	16. 062 3784	6. 366 0968	. 003 875 969
259	67, 081	17, 373, 979	16. 093 4769	6. 374 3111	. 003 861 004
260	67, 600	17, 576, 000	16. 124 5155	6. 382 5043	. 003 846 154
261	68, 121	17. 779, 581	16. 155 4944	6. 390 6765	. 003 831 418
262	68, 644	17, 984, 728	16. 186 4141	6. 398 8279	. 003 816 794
263	69, 169	18, 191, 447	16. 217 2747	6. 406 9585	. 003 802 281
264	69, 696	18, 399, 744	16. 248 0768	6. 415 0687	. 003 787 879
265	70, 225	18, 609, 625	16. 278 8206	6. 423 1583	. 003 773 585
266	70, 756	18, 821, 096	16. 309 5064	6. 431 2276	. 003 759 398
267	71, 289	19, 034, 163	16. 340 1346	6. 439 2767	. 003 745 318
268	71, 824	19, 248, 832	16. 370 7055	6. 447 3057	. 003 731 343
269	72, 361	19, 465, 109	16. 401 2195	6. 455 3148	. 003 717 472
270	72, 900	19, 683, 000	16. 431 6767	6. 463 3041	. 003 703 704
271	73, 441	19, 902, 511	16. 462 0776	6. 471 2736	. 003 690 037
272	73, 984	20, 123, 643	16. 492 4225	6. 479 2236	. 003 676 471
273	74, 529	20, 346, 417	16. 522 7116	6. 487 1541	. 003 663 004
274	75, 076	20, 570, 824	16. 552 9454	6. 495 0653	. 003 649 635
275	75, 625	20, 796, 875	16. 583 1240	6. 502 9572	. 003 636 364
276	76, 176	21, 024, 576	16. 613 2477	6. 510 8300	. 003 623 188
277	76, 729	21, 253, 933	16. 643 3170	6. 518 6839	. 003 610 108
278	77, 284	21, 484, 952	16. 673 3320	6. 526 5189	. 003 597 122
279	77, 841	21, 717, 639	16. 703 2931	6. 534 3351	. 003 584 229
280	78, 400	21, 952, 000	16. 733 2005	6. 542 1326	. 003 571 429
281	78, 961	22, 188, 041	16. 763 0546	6. 549 9116	. 003 558 719
282	79, 524	22, 425, 768	16. 792 8556	6. 557 6722	. 003 546 099
283	80, 089	22, 665, 187	16. 822 6038	6. 565 4144	. 003 533 569
284	80, 656	22, 906, 304	16. 852 2995	6. 573 1385	. 003 521 127
285	81, 225	23, 149, 125	16. 881 9430	6. 580 8443	. 003 508 772
286	81, 796	23, 393, 656	16. 911 5345	6. 588 5323	. 003 496 503
287	82, 369	23, 639, 903	16. 941 0743	6. 596 2023	. 003 484 321
288	82, 944	23, 887, 872	16. 970 5627	6. 603 8545	. 003 472 222
289	83, 521	24, 137, 569	17. 000 0000	6. 611 4890	. 003 460 208
290	84, 100	24, 389, 000	17. 029 3864	6. 619 1060	. 003 448 276
291	84, 681	24, 642, 171	17. 058 7221	6. 626 7054	. 003 436 426
292	85, 264	24, 897, 088	17. 088 0075	6. 634 2874	. 003 424 658
293	85, 849	25, 153, 757	17. 117 2428	6. 641 8522	. 003 412 969
294	86, 436	25, 412, 184	17. 146 4282	6. 649 3998	. 003 401 361
295	87, 025	25, 672, 375	17. 175 5640	6. 656 9302	. 003 389 831
296	87, 616	25, 934, 336	17. 204 6505	6. 664 4437	. 003 378 378
297	88, 209	26, 198, 073	17. 233 6879	6. 671 9403	. 003 367 003
298	88, 804	26, 463, 592	17. 262 6765	6. 679 4200	. 003 355 705
299	89, 401	26, 730, 899	17. 291 6165	6. 686 8831	. 003 344 482
300	90, 000	27, 000, 000	17. 320 5081	6. 694 3295	. 003 333 333
301	90, 601	27, 270, 901	17. 349 3516	6. 701 7593	. 003 322 259
302	91, 204	27, 543, 608	17. 378 1472	6. 709 1729	. 003 311 258
303	91, 809	27, 818, 127	17. 406 8952	6. 716 5700	. 003 301 330
304	92, 416	28, 094, 464	17. 435 5958	6. 723 9508	. 003 289 474
305	93, 025	28, 372, 625	17. 464 2492	6. 731 3155	. 003 278 689
306	93, 636	28, 652, 616	17. 492 8557	6. 738 6641	. 003 267 974
307	94, 249	28, 934, 443	17. 521 4155	6. 745 9967	. 003 257 329
308	94, 864	29, 218, 112	17. 549 9288	6. 753 3134	. 003 246 753
309	95, 481	29, 503, 609	17. 578 3958	6. 760 6143	. 003 236 246
310	96, 100	29, 791, 000	17. 606 8169	6. 767 8995	. 003 225 806
311	96, 721	30, 080, 231	17. 635 1921	6. 775 1690	. 003 215 434
312	97, 344	30, 371, 328	17. 663 5217	6. 782 4229	. 003 205 128
313	97, 969	30, 664, 297	17. 691 8060	6. 789 6613	. 003 194 888
314	98, 596	30, 959, 144	17. 720 0451	6. 796 8844	. 003 184 713
315	99, 225	31, 255, 875	17. 748 2393	6. 804 0921	. 003 174 603
316	99, 856	31, 554, 496	17. 776 3888	6. 811 2847	. 003 164 557
317	100, 489	31, 855, 013	17. 804 4938	6. 818 4620	. 003 154 574
318	101, 124	32, 157, 432	17. 832 5545	6. 825 6242	. 003 144 654
319	101, 761	32, 461, 759	17. 860 5711	6. 832 7714	. 003 134 796
320	102, 400	32, 768, 000	17. 888 5438	6. 839 9037	. 003 125 000
321	103, 041	33, 076, 161	17. 916 4729	6. 847 0213	. 003 115 265
322	103, 684	33, 386, 248	17. 944 3584	6. 854 1240	. 003 105 590

TABLE 1.—*Powers and roots*—Continued.

Number.	Squares.	Cubes.	√Roots.	∛Roots.	Reciprocals.
323	104, 329	33, 698, 267	17. 972 2008	6. 861 2120	. 003 095 975
324	104, 976	34, 012, 224	18. 000 0000	6. 868 2855	. 003 086 420
325	105, 625	34, 328, 125	18. 027 7564	6. 875 3433	. 003 076 923
326	106, 276	34, 645, 976	18. 055 4701	6. 882 3888	. 003 067 485
327	106, 929	34, 965, 783	18. 083 1413	6. 889 4188	. 003 058 104
328	107, 584	35, 287, 552	18. 110 7703	6. 896 4345	. 003 048 780
329	108, 241	35, 611, 289	18. 138 3571	6. 903 4359	. 003 039 514
330	108, 900	35, 937, 000	18. 165 9021	6. 910 4232	. 003 030 303
331	109, 561	36, 264, 691	18. 193 4054	6. 917 3964	. 003 021 148
332	110, 224	36, 594, 368	18. 220 8672	6. 924 3556	. 003 012 048
333	110, 889	36, 926, 037	18. 248 2876	6. 931 3088	. 003 003 003
334	111, 556	37, 259, 704	18. 275 6669	6. 938 2321	. 002 994 012
335	112, 225	37, 595, 375	18. 303 0052	6. 945 1496	. 002 985 075
336	112, 896	37, 933, 056	18. 330 3028	6. 952 0533	. 002 976 190
337	113, 569	38, 272, 753	18. 357 5598	6. 958 9434	. 002 967 359
338	114, 244	38, 614, 472	18. 384 7763	6. 965 8198	. 002 958 580
339	114, 921	38, 958, 219	18. 411 9526	6. 972 6826	. 002 949 853
340	115, 600	39, 304, 000	18. 439 0889	6. 979 5321	. 002 941 176
341	116, 281	39, 651, 821	18. 466 1853	6. 986 3681	. 002 932 551
342	116, 964	40, 001, 688	18. 493 2420	6. 993 1906	. 002 923 977
343	117, 649	40, 353, 607	18. 520 2592	7. 000 0000	. 002 915 452
344	118, 336	40, 707, 584	18. 547 2370	7. 006 7962	. 002 906 977
345	119, 025	41, 063, 625	18. 574 1756	7. 013 5791	. 002 898 551
346	119, 716	41, 421, 736	18. 601 0752	7. 020 3490	. 002 890 173
347	120, 409	41, 781, 923	18. 627 9360	7. 027 1058	. 002 881 844
348	121, 104	42, 144, 192	18. 654 7581	7. 033 8497	. 002 873 563
349	121, 801	42, 508, 549	18. 681 5417	7. 040 5860	. 002 865 330
350	122, 500	42, 875, 000	18. 708 2869	7. 047 2987	. 002 857 143
351	123, 201	43, 243, 551	18. 734 9940	7. 054 0041	. 002 849 003
352	123, 904	43, 614, 208	18. 761 6630	7. 060 6967	. 002 840 909
353	124, 609	43, 986, 977	18. 788 2942	7. 067 3767	. 002 832 861
354	125, 316	44, 361, 864	18. 814 8877	7. 074 0440	. 002 824 859
355	126, 025	44, 738, 875	18. 841 4437	7. 080 6988	. 002 816 901
356	126, 736	45, 118, 016	18. 867 9623	7. 087 3411	. 002 808 989
357	127, 449	45, 499, 293	18. 894 4436	7. 093 9709	. 002 801 120
358	128, 164	45, 882, 712	18. 920 8879	7. 100 5885	. 002 793 296
359	128, 881	46, 268, 279	18. 947 2953	7. 107 1937	. 002 785 515
360	129, 600	46, 656, 000	18. 973 6660	7. 113 7866	. 002 777 778
361	130, 321	47, 045, 881	19. 000 0000	7. 120 3674	. 002 770 083
362	131, 044	47, 437, 928	19. 026 2976	7. 126 9360	. 002 762 431
363	131, 769	47, 832, 147	19. 052 5589	7. 133 4925	. 002 754 821
364	132, 496	48, 228, 544	19. 078 7840	7. 140 0370	. 002 747 253
365	133, 225	48, 627, 125	19. 104 9732	7. 146 5695	. 002 739 726
366	133, 956	49, 027, 896	19. 131 1265	7. 153 0901	. 002 732 240
367	134, 689	49, 430, 863	19. 157 2441	7. 159 5988	. 002 724 796
368	135, 424	49, 836, 032	19. 183 3261	7. 166 0957	. 002 717 391
369	136, 161	50, 243, 409	19. 209 3727	7. 172 5809	. 002 710 027
370	136, 900	50, 653, 000	19. 235 3841	7. 179 0544	. 002 702 703
371	137, 641	51, 064, 811	19. 261 3603	7. 185 5162	. 002 695 418
372	138, 384	51, 478, 848	19. 287 3015	7. 191 9663	. 002 688 172
373	139, 129	51, 895, 117	19. 313 2079	7. 198 4050	. 002 680 965
374	139, 876	52, 313, 624	19. 339 0796	7. 204 8322	. 002 673 797
375	140, 625	52, 734, 375	19. 364 9167	7. 211 2479	. 002 666 667
376	141, 376	53, 157, 376	19. 390 7194	7. 217 6522	. 002 659 574
377	142, 129	53, 582, 633	19. 416 4878	7. 224 0450	. 002 652 520
378	142, 884	54, 010, 152	19. 442 2221	7. 230 4268	. 002 645 503
379	143, 641	54, 439, 939	19. 467 9223	7. 236 7972	. 002 638 521
380	144, 400	54, 872, 000	19. 493 5887	7. 243 1565	. 002 631 579
381	145, 161	55, 306, 341	19. 519 2213	7. 249 5045	. 002 624 672
382	145, 924	55, 742, 968	19. 544 8203	7. 255 8415	. 002 617 801
383	146, 689	56, 181, 887	19. 570 3858	7. 262 1675	. 002 610 966
384	147, 456	56, 623, 104	19. 595 9179	7. 268 4824	. 002 604 167
385	148, 225	57, 066, 625	19. 621 4169	7. 274 7864	. 002 597 403
386	148, 996	57, 512, 456	19. 646 8827	7. 281 0794	. 002 590 674
387	149, 769	57, 960, 603	19. 672 3156	7. 287 3617	. 002 583 972

TABLE 1.—*Powers and roots*—Continued.

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
388	150, 544	58, 411, 072	19. 697 7156	7. 293 6330	. 002 577 320
389	151, 321	58, 863, 869	19. 723 0829	7. 299 8936	. 002 570 694
390	152, 100	59, 319, 000	19. 748 4177	7. 306 1436	. 002 564 108
391	152, 881	59, 776, 471	19. 773 7199	7. 312 3828	. 002 557 546
392	153, 664	60, 236, 288	19. 798 9899	7. 318 6114	. 002 551 020
393	154, 449	60, 698, 457	19. 824 2276	7. 324 8295	. 002 544 529
394	155, 236	61, 162, 984	19. 849 4332	7. 331 0369	. 002 538 071
395	156, 025	61, 629, 875	19. 874 6069	7. 337 2339	. 002 531 646
396	156, 816	62, 099, 136	19. 899 7487	7. 343 4205	. 002 525 253
397	157, 609	62, 570, 773	19. 924 8588	7. 349 5966	. 002 518 892
398	158, 404	63, 044, 792	19. 949 9373	7. 355 7624	. 002 512 563
399	159, 201	63, 521, 199	19. 974 9844	7. 361 9178	. 002 506 266
400	160, 000	64, 000, 000	20. 000 0000	7. 368 0630	. 002 500 000
401	160, 801	64, 481, 201	20. 024 9844	7. 374 1979	. 002 493 766
402	161, 604	64, 964, 808	20. 049 9377	7. 380 3227	. 002 487 562
403	162, 409	65, 450, 827	20. 074 8599	7. 386 4373	. 002 481 390
404	163, 216	65, 939, 264	20. 099 7512	7. 392 5418	. 002 475 248
405	164, 025	66, 430, 125	20. 124 6118	7. 398 6363	. 002 469 136
406	164, 836	66, 923, 416	20. 149 4417	7. 404 7206	. 002 463 054
407	165, 649	67, 419, 143	20. 174 2410	7. 410 7950	. 002 457 002
408	166, 464	67, 917, 312	20. 199 0099	7. 416 8595	. 002 450 980
409	167, 281	68, 417, 929	20. 223 7484	7. 422 9142	. 002 444 988
410	168, 100	68, 921, 000	20. 248 4567	7. 428 9589	. 002 439 024
411	168, 921	69, 426, 531	20. 273 1349	7. 434 9938	. 002 433 090
412	169, 744	69, 934, 528	20. 297 7831	7. 441 0189	. 002 427 184
413	170, 569	70, 444, 997	20. 322 4014	7. 447 0343	. 002 421 308
414	171, 396	70, 957, 944	20. 346 9899	7. 453 0399	. 002 415 459
415	172, 225	71, 473, 375	20. 371 5488	7. 459 0359	. 002 409 639
416	173, 056	71, 991, 296	20. 396 0781	7. 465 0223	. 002 406 846
417	173, 889	72, 511, 713	20. 420 5779	7. 470 9991	. 002 398 082
418	174, 724	73, 034, 632	20. 445 0483	7. 476 9664	. 002 392 344
419	175, 561	73, 560, 059	20. 469 4895	7. 482 9242	. 002 386 635
420	176, 400	74, 088, 000	20. 493 9015	7. 488 8724	. 002 380 952
421	177, 241	74, 618, 461	20, 518 2845	7. 494 8113	. 002 375 297
422	178, 084	75, 151, 448	20. 542 6386	7. 500 7406	. 002 369 668
423	178, 929	75, 686, 967	20. 566 9638	7. 506 6607	. 002 364 066
424	179, 776	76, 225, 024	20. 591 2603	7. 512 5715	. 002 358 491
425	180, 625	76, 765, 625	20. 615 5281	7. 518 4730	. 002 352 941
426	181, 476	77, 308, 776	20. 639 7674	7. 524 3652	. 002 347 418
427	182, 329	77, 854, 483	20. 663 9783	7. 530 2482	. 002 341 920
428	183, 184	78, 402, 752	20. 688 1609	7. 536 1221	. 002 336 449
429	184, 041	78, 953, 589	20. 712 3152	7. 541 9867	. 002 331 002
430	184, 900	79, 507, 000	20. 736 4414	7. 547 8423	. 002 325 581
431	185, 761	80, 062, 991	20. 760 5395	7. 553 6888	. 002 320 186
432	186, 624	80, 621, 568	20. 784 6097	7. 559 5263	. 002 314 815
433	187, 489	81, 182, 737	20. 808 6520	7. 565 3548	. 002 309 469
434	188, 356	81, 746, 504	20. 832 6667	7. 571 1743	. 002 304 147
435	189, 225	82, 312, 875	20. 856 6536	7. 576 9849	. 002 298 851
436	190, 096	82, 881, 856	20. 880 6130	7. 582 7865	. 002 293 578
437	190, 969	83, 453, 453	20. 904 5450	7. 588 5793	. 002 288 330
438	191, 844	84, 027, 672	20. 928 4495	7. 594 3633	. 002 283 105
439	192, 721	84, 604, 519	20. 952 3268	7. 600 1385	. 002 277 904
440	193, 600	85, 184, 000	20. 976 1770	7. 605 9049	. 002 272 727
441	194, 481	85, 766, 121	21. 000 0000	7. 611 6626	. 002 267 574
442	195, 364	86, 350, 888	21. 023 7960	7. 617 4116	. 002 262 443
443	196, 249	86, 938, 307	21. 047 5652	7. 623 1519	. 002 257 336
444	197, 136	87, 528, 384	21. 071 3075	7. 628 8837	. 002 252 252
445	198, 025	88, 121, 125	21. 095 0231	7. 634 6067	. 002 247 191
446	198, 916	88, 716, 536	21. 118 7121	7. 640 3213	. 002 242 152
447	199, 809	89, 314, 623	21. 142 3745	7. 646 0272	. 002 237 136
448	200, 704	89, 915, 392	21. 166 0105	7. 651 7247	. 002 232 143
449	201, 601	90, 518, 849	21. 189 6201	7. 657 4138	. 002 227 171
450	202, 500	91, 125, 000	21. 213 2034	7. 663 0943	. 002 222 222
451	203, 401	91, 733, 851	21. 236 7606	7. 668 7665	. 002 217 295
452	204, 304	92, 345, 408	21. 260 2916	7. 674 4303	. 002 212 389

TABLE 1.—*Powers and roots*—Continued.

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
453	205, 209	92, 959, 677	21. 283 7967	7. 680 0857	. 002 207 506
454	206, 116	93, 576, 664	21. 307 2758	7. 685 7328	. 002 202 643
455	207, 025	94, 196, 375	21. 330 7290	7. 691 3717	. 002 197 802
456	207, 936	94, 818, 816	21. 354 1565	7. 697 0023	. 002 192 982
457	208, 849	95, 443, 993	21. 377 5583	7. 702 6246	. 002 188 184
458	209, 764	96, 071, 912	21. 400 9346	7. 708 2388	. 002 183 406
459	210, 681	96, 702, 579	21. 424 2853	7. 713 8448	. 002 178 649
460	211, 600	97, 336, 000	21. 447 6106	7. 719 4426	. 002 173 913
461	212, 521	97, 972, 181	21. 470 9106	7. 725 0325	. 002 169 197
462	213, 444	98, 611, 128	21. 494 1853	7. 730 6141	. 002 164 502
463	214, 369	99, 252, 847	21. 517 4348	7. 736 1877	. 002 159 827
464	215, 296	99, 897, 344	21. 540 6592	7. 741 7532	. 002 155 172
465	216, 225	100, 544, 625	21. 563 8587	7. 747 3109	. 002 150 538
466	217, 156	101, 194, 696	21. 587 0331	7. 752 8606	. 002 145 923
467	218, 089	101, 847, 563	21. 610 1828	7. 758 4023	. 002 141 328
468	219, 024	102, 503, 232	21. 633 3077	7. 763 9361	. 002 136 752
469	219, 961	103, 161, 709	21. 656 4078	7. 769 4620	. 002 132 196
470	220, 900	103, 823, 000	21. 679 4834	7. 774 9801	. 002 127 660
471	221, 841	104, 487, 111	21. 702 5344	7. 780 4904	. 002 123 142
472	222, 784	105, 154, 048	21. 725 5610	7. 785 9928	. 002 118 644
473	223, 729	105, 828, 817	21. 748 5632	7. 791 4875	. 002 114 165
474	224, 676	106, 496, 424	21. 771 5411	7. 796 9745	. 002 109 705
475	225, 625	107, 171, 875	21. 794 4947	7. 802 4538	. 002 105 263
476	226, 576	107, 850, 176	21. 817 4242	7. 807 9254	. 002 100 840
477	227, 529	108, 531, 333	21. 840 3297	7. 813 3892	. 002 096 436
478	228, 484	109, 215, 352	21. 863 2111	7. 818 8456	. 002 092 050
479	229, 441	109, 902, 239	21. 886 0686	7. 824 2942	. 002 087 683
480	230, 400	110, 592, 000	21. 908 9023	7. 829 7353	. 002 083 333
481	231, 361	111, 284, 641	21. 931 7122	7. 835 1688	. 002 079 002
482	232, 324	111, 980, 168	21. 954 4984	7. 840 5949	. 002 074 689
483	233, 289	112, 678, 587	21. 977 2610	7. 846 0134	. 002 070 393
484	234, 256	113, 379, 904	22. 000 0000	7. 851 4244	. 002 066 116
485	235, 225	114, 084, 125	22. 022 7155	7. 856 8281	. 002 061 856
486	236, 196	114, 791, 256	22. 045 4077	7. 862 2242	. 002 057 613
487	237, 169	115, 501, 303	22. 068 0765	7. 867 6130	. 002 053 388
488	238, 144	116, 214, 272	22. 090 7220	7. 872 9944	. 002 049 180
489	239, 121	116, 930, 169	22. 113 3444	7. 878 3684	. 002 044 990
490	240, 100	117, 649, 000	22. 135 9436	7. 883 7352	. 002 040 816
491	241, 081	118, 370, 771	22. 158 5198	7. 889 0946	. 002 036 660
492	242, 064	119, 095, 488	22. 181 0730	7. 894 4468	. 002 032 520
493	243, 049	119, 823, 157	22. 203 6033	7. 899 7917	. 002 028 398
494	244, 036	120, 553, 784	22. 226 1108	7. 905 1294	. 002 024 291
495	245, 025	121, 287, 375	22. 248 5955	7. 910 4599	. 002 020 202
496	246, 016	122, 023, 936	22. 271 0575	7. 915 7832	. 002 016 129
497	247, 009	122, 763, 473	22. 293 4968	7. 921 0994	. 002 012 072
498	248, 004	123, 505, 992	22. 315 9136	7. 926 4085	. 002 008 032
499	249, 001	124, 251, 499	22. 338 3079	7. 931 7104	. 002 004 008
500	250, 000	125, 000, 000	22. 360 6798	7. 937 0053	. 002 000 000
501	251, 001	125, 751, 501	22. 383 0293	7. 942 2931	. 001 996 008
502	252, 004	126, 506, 008	22. 405 3565	7. 947 5739	. 001 992 032
503	253, 009	127, 263, 527	22. 427 6615	7. 952 8477	. 001 988 072
504	254, 016	128, 024, 064	22. 449 9443	7. 958 1144	. 001 984 127
505	255, 025	128, 787, 625	22. 472 2051	7. 963 3743	. 001 980 198
506	256, 036	129, 554, 216	22. 494 4438	7. 968 6271	. 001 976 285
507	257, 049	130, 323, 843	22. 516 6605	7. 973 8731	. 001 972 387
508	258, 064	131, 096, 512	22. 538 8553	7. 979 1122	. 001 968 504
509	259, 081	131, 872, 229	22. 561 0283	7. 984 3444	. 001 964 637
510	260, 100	132, 651, 000	22. 583 1796	7. 989 5697	. 001 960 784
511	261, 121	133, 432, 831	22. 605 3091	7. 994 7883	. 001 956 947
512	262, 144	134, 217, 728	21. 627 4170	8. 000 0000	. 001 953 125
513	263, 169	135, 005, 697	22. 649 5033	8. 005 2049	. 001 949 318
514	264, 196	135, 796, 744	22. 671 5681	8. 010 4032	. 001 945 525
515	265, 225	136, 590, 875	22. 693 6114	8. 015 5946	. 001 941 748
516	266, 256	137, 388, 096	22. 715 6334	8. 020 7794	. 001 937 984
517	267, 289	138, 188, 413	22. 737 6341	8. 025 9574	. 001 934 236

TABLE 1.—*Powers and roots*—Continued.

Number.	Squares.	Cubes.	√Roots.	√Roots.	Reciprocals.
518	268, 324	138, 991, 832	22. 759 6134	8. 031 1287	. 001 930 502
519	269, 361	139, 798, 359	22. 781 5715	8. 036 2935	. 001 926 782
520	270, 400	140, 608, 000	22. 803 5085	8. 041 4515	. 001 923 077
521	271, 441	141, 420, 761	22. 825 4244	8. 046 6030	. 001 919 386
522	272, 484	142, 236, 648	22. 847 3193	8. 051 7479	. 001 915 709
523	273, 529	143, 055, 667	22. 869 1933	8. 056 8862	. 001 912 046
524	274, 576	143, 877, 824	22. 891 0463	8. 062 0180	. 001 908 397
525	275, 625	144, 703, 125	22. 912 8785	8. 067 1432	. 001 904 762
526	276, 676	145, 531, 576	22. 934 6899	8. 072 2620	. 001 901 141
527	277, 729	146, 363, 183	22. 956 4806	8. 077 3743	. 001 897 533
528	278, 784	147, 197, 952	22. 978 2506	8. 082 4800	. 001 893 939
529	279, 841	148, 035, 889	23. 000 0000	8. 087 5794	. 001 890 359
530	280, 900	148, 877, 000	23. 021 7289	8. 092 6723	. 001 886 792
531	281, 961	149, 721, 291	23. 043 4372	8. 097 7589	. 001 883 239
532	283, 024	150, 568, 768	23. 065 1252	8. 102 8390	. 001 879 699
533	284, 089	151, 419, 437	23. 086 7928	8. 107 9128	. 001 876 173
534	285, 156	152, 273, 304	23. 108 4400	8. 112 9803	. 001 872 659
535	286, 225	153, 130, 375	23. 130 0670	8. 118 0414	. 001 869 159
536	287, 296	153, 990, 656	23. 151 6738	8. 123 0962	. 001 865 672
537	288, 369	154, 854, 153	23. 173 2605	8. 128 1447	. 001 862 197
538	289, 444	155, 720, 872	23. 194 8270	8. 133 1870	. 001 858 736
539	290, 521	156, 590, 819	23. 216 3735	8. 138 2230	. 001 855 288
540	291, 600	157, 464, 000	23. 237 9001	8. 143 2529	. 001 851 852
541	292, 681	158, 340, 421	23. 259 4067	8. 148 2765	. 001 848 429
542	293, 764	159, 220, 088	23. 280 8935	8. 153 2939	. 001 845 018
543	294, 849	160, 103, 007	23. 302 3604	8. 158 3051	. 001 841 621
544	295, 936	160, 989, 184	23. 323 8076	8. 163 3102	. 001 838 235
545	297, 025	161, 878, 625	23. 345 2351	8. 168 3092	. 001 834 862
546	298, 116	162, 771, 336	23. 366 6429	8. 173 3020	. 001 831 502
547	229, 209	163, 667, 323	23. 388 0311	8. 178 2888	. 001 828 154
548	300, 304	164, 566, 592	23. 409 3998	8. 183 2695	. 001 824 818
549	301, 401	165, 469, 149	23. 430 7490	8. 188 2441	. 001 821 494
550	302, 500	166, 375, 000	23. 452 0788	8. 193 2127	. 001 818 182
551	303, 601	167, 284, 151	23. 473 3892	8. 198 1753	. 001 814 882
552	304, 704	168, 196, 608	23. 494 6802	8. 203 1319	. 001 811 594
553	305, 809	169, 112, 377	23. 515 9520	8. 208 0825	. 001 808 318
554	306, 916	170, 031, 464	23. 537 2046	8. 213 0271	. 001 805 054
555	308, 025	170, 953, 875	23. 558 4380	8. 217 9657	. 001 801 802
556	309, 136	171, 879, 616	23. 579 6522	8. 222 8985	. 001 798 561
557	310, 249	172, 808, 693	23. 600 8474	8. 227 8254	. 001 795 332
558	311, 364	173, 741, 112	23. 622 0236	8. 232 7463	. 001 792 115
559	312, 481	174, 676, 879	23. 643 1808	8. 237 6614	. 001 788 909
560	313, 600	175, 616, 000	23. 664 3191	8. 242 5706	. 001 785 714
561	314, 721	176, 558, 481	23. 685 4386	8. 247 4740	. 001 782 531
562	315, 844	177, 504, 328	23. 706 5392	8. 252 3715	. 001 779 359
563	316, 969	178, 453, 547	23. 727 6210	8. 257 2635	. 001 776 199
564	318, 096	179, 406, 144	23. 748 6842	8. 262 1492	. 001 773 050
565	319, 225	180, 362, 125	23. 769 7286	8. 267 0294	. 001 769 912
566	320, 356	181, 321, 496	23. 790 7545	8. 271 9039	. 001 766 784
567	321, 489	182, 284, 263	23. 811 7618	8. 276 7726	. 001 763 668
568	322, 624	183, 250, 432	23. 832 7506	8. 281 6255	. 001 760 563
569	323, 761	184, 220, 009	23. 853 7209	8. 286 4928	. 001 757 469
570	324, 900	185, 193, 000	23. 874 6728	8. 291 3444	. 001 754 386
571	326, 041	186, 169, 411	23. 895 6063	8. 296 1903	. 001 751 313
572	327, 184	187, 149, 248	23. 916 5215	8. 301 0304	. 001 748 252
573	328, 329	188, 132, 517	23. 937 4184	8. 305 8651	. 001 745 201
574	329, 476	189, 119, 224	23. 958 2971	8. 310 6941	. 001 742 160
575	330, 625	190, 109, 375	23. 979 1576	8. 315 5175	. 001 739 130
576	331, 776	191, 102, 976	24. 000 0000	8. 320 3353	. 001 736 111
577	332, 927	192, 100, 033	24. 020 8243	8. 325 1475	. 001 733 102
578	334, 084	193, 100, 552	24. 041 6306	8. 329 9542	. 001 730 104
579	335, 241	194, 104, 539	24. 062 4188	8. 334 7553	. 001 727 116
580	336, 400	195, 112, 000	24. 083 1891	8. 339 5509	. 001 724 138
581	337, 561	196, 122, 941	24. 103 9416	8. 344 3410	. 001 721 170
582	338, 724	197, 137, 368	24. 124 6762	8. 349 1256	. 001 718 213

TABLE 1.—*Powers and roots*—Continued.

Number.	Squares.	Cubes.	√Roots.	∛Roots.	Reciprocals.
583	339, 889	198, 155, 287	24. 145 3929	8. 353 9047	. 001 715 266
584	341, 056	199, 176, 704	24. 166 0919	8. 358 6784	. 001 712 329
585	342, 225	200, 201, 625	24. 186 7732	8. 363 4466	. 001 709 402
586	343, 396	201, 230, 056	24, 207 4369	8. 368 2095	. 001 706 485
587	344, 569	202, 262, 003	24. 228 0829	8. 372 9668	. 001 703 578
588	345, 744	203, 297, 472	24. 248 7113	8. 377 7188	. 001 700 680
589	346, 921	204, 336, 469	24. 269 3222	8. 382 4653	. 001 697 793
590	348, 100	205, 379, 000	24. 289 9156	8. 387 2065	. 001 694 915
591	349, 281	206, 425, 071	24. 310 4996	8. 391 9428	. 001 692 047
592	350, 464	207, 474, 688	24. 331 0501	8. 396 6729	. 001 689 189
593	351, 649	208, 527, 857	24. 351 5913	8. 401 3981	. 001 686 341
594	352, 836	209, 584, 584	24. 372 1152	8. 406 1180	. 001 683 502
595	354, 025	210, 644, 875	24. 392 6218	8. 410 8326	. 001 680 672
596	355, 216	211, 708, 736	24. 413 1112	8. 415 5419	. 001 677 852
597	356, 409	212, 776, 173	24. 433 5834	8. 420 2460	. 001 675 042
598	357, 604	213, 847, 192	24. 454 0385	8. 424 9448	. 001 672 241
599	358, 801	214, 921, 799	24. 474 4765	8. 429 6383	. 001 669 449
600	360, 000	216, 000, 000	24. 494 8974	8. 434 3267	. 001 666 667
601	361, 201	217, 081, 801	24. 515 3013	8. 439 0098	. 001 663 894
602	362, 404	218, 167, 208	24. 535 6883	8. 443 6877	. 001 661 130
603	363, 609	219, 256, 227	24. 556 0583	8. 448 3605	. 001 658 375
604	364, 816	220, 348, 864	24. 576 4115	8. 453 0281	. 001 655 629
605	366, 025	221, 445, 125	24. 596 7478	8. 457 6906	. 001 652 893
606	367, 236	222, 545, 016	24. 617 0673	8. 462 3479	. 001 650 165
607	368, 449	223, 648, 543	24. 637 3700	8. 467 0001	. 001 647 446
608	369, 664	224, 755, 712	24. 657 6560	8. 471 6471	. 001 644 737
609	370, 881	225, 866, 529	24. 677 9254	8. 476 2892	. 001 642 036
610	372, 100	226, 981, 000	24. 698 1781	8. 480 9261	. 001 639 344
611	373, 321	228, 099, 131	24. 718 4142	8. 485 5579	. 001 636 661
612	374, 544	229, 220, 928	24. 738 6338	8. 490 1848	. 001 633 987
613	375, 769	230, 346, 397	24. 758 8368	8. 494 8065	. 001 631 321
614	376, 996	231, 475, 544	24. 779 0234	8. 499 4233	. 001 628 664
615	378, 225	232, 608, 375	24. 799 1935	8. 504 0350	. 001 626 016
616	379, 456	233, 744, 896	24. 819 3473	8. 508 6417	. 001 623 377
617	380, 689	234, 885, 113	24. 839 4847	8. 513 2435	. 001 620 746
618	381, 924	236, 029, 032	24. 859 6058	8. 517 8403	. 001 618 123
619	383, 161	237, 176, 659	24. 879 7106	8. 522 4331	. 001 615 509
620	384, 400	238, 328, 000	24. 899 7992	8. 527 0189	. 001 612 903
621	385, 641	239, 483, 061	24. 919 8716	8. 531 6009	. 001 610 306
622	386, 884	240, 641, 848	24. 939 9278	8. 536 1780	. 001 607 717
623	388, 129	241, 804, 367	24. 959 9679	8. 540 7501	. 001 605 136
624	389, 376	242, 970, 624	24. 979 9920	8. 545 3173	. 001 602 564
625	390, 625	244, 140, 625	25. 000 0000	8. 549 8797	. 001 600 000
626	391, 876	245, 134, 376	25. 019 9920	8. 554 4372	. 001 597 444
627	393, 129	246, 491, 883	25. 039 9681	8. 558 9899	. 001 594 896
628	394, 384	247, 673, 152	25. 059 9282	8. 563 5377	. 001 592 357
629	395, 641	248, 858, 189	25. 079 8724	8. 568 0807	. 001 589 825
630	396, 900	250, 047, 000	25. 099 8008	8. 572 6189	. 001 587 302
631	398, 161	251, 239, 591	25. 119 7134	8. 577 1523	. 001 584 786
632	399, 424	252, 435, 968	25. 139 6102	8. 581 6809	. 001 582 278
633	400, 689	253, 636, 137	25. 159 4913	8. 586 2247	. 001 579 779
634	401, 956	254, 840, 104	25. 179 3566	8. 590 7238	. 001 577 287
635	403, 225	256, 047, 875	25. 199 2063	8. 595 2380	. 001 574 803
636	404, 496	257, 259, 456	25. 219 0404	8. 599 7476	. 001 572 327
637	405, 769	258, 474, 853	25. 238 8589	8. 604 2525	. 001 569 859
638	407, 044	259, 694, 072	25. 258 6619	8. 608 7526	. 001 567 398
639	408, 321	260, 917, 119	25. 278 4493	8. 613 2480	. 001 564 945
640	409, 600	262, 144, 000	25. 298 2213	8. 617 7388	. 001 562 500
641	410, 881	263, 374, 721	25. 317 9778	8. 622 2248	. 001 560 062
642	412, 164	264, 609, 288	25. 337 7189	8. 626 7063	. 001 557 632
643	413, 449	265, 847, 707	25. 357 4447	8. 631 1830	. 001 555 210
644	414, 736	267, 089, 984	25. 377 1551	8. 635 6551	. 001 552 795
645	416, 025	268, 336, 125	25. 396 8502	8. 640 1226	. 001 550 388
646	417, 316	269, 585, 136	25. 416 5302	8. 644 5855	. 001 547 988
647	418, 609	270, 840, 023	25. 436 1947	8. 649 0437	. 001 545 595

TABLE 1.—*Powers and roots*—Continued.

Number.	Squares.	Cubes.	√Roots.	∛Roots.	Reciprocals.
648	419, 904	272, 097, 792	25. 455 8441	8. 653 4974	. 001 543 210
649	421, 201	273, 359, 449	25. 475 4784	8. 657 9465	. 001 540 832
650	422, 500	274, 625, 000	25. 495 0976	8. 662 3911	. 001 538 462
651	423, 801	275, 894, 451	25. 514 7013	8. 666 8310	. 001 536 098
652	425, 104	277, 167, 808	25. 594 2907	8. 671 2665	. 001 533 742
653	426, 409	278, 445, 077	25. 553 8647	8. 675 6974	. 001 531 394
654	427, 716	279, 726, 264	25. 573 4237	8. 680 1237	. 001 529 052
655	429, 025	281, 011, 375	25. 592 9678	8. 684 5456	. 001 526 718
656	430, 336	282, 300, 416	25. 612 4969	8. 688 9630	. 001 524 390
657	431, 649	283, 593, 393	25. 632 0112	8. 693 3759	. 001 522 070
658	432, 964	284, 890, 312	25. 651 5107	8. 697 7843	. 001 519 757
659	434, 281	286, 191, 179	25. 670 9953	8. 702 1882	. 001 517 451
660	435, 600	287, 496, 000	25. 690 4652	8. 706 5877	. 001 515 152
661	436, 921	288, 804, 781	25. 709 9203	8. 710 9827	. 001 512 859
662	438, 244	290, 117, 528	25. 729 3607	8. 715 3734	. 001 510 574
663	439, 569	291, 434, 247	25. 748 7864	8. 719 7596	. 001 508 296
664	440, 896	292, 754, 944	25. 768 1975	8. 724 1414	. 001 506 024
665	442, 225	294, 079, 625	25. 787 5939	8. 728 5187	. 001 503 759
666	443, 556	295, 408, 296	25. 806 9758	8. 732 8918	. 001 501 502
667	444, 889	296, 740, 963	25. 826 3431	8. 737 2604	. 001 499 250
668	446, 224	298, 077, 632	25. 845 6960	8. 741 6246	. 001 497 006
669	447, 561	299, 418, 309	25. 865 0343	8. 745 9846	. 001 494 768
670	448, 900	300, 763, 000	25. 884 3582	8. 750 3401	. 001 492 537
671	450, 241	302, 111, 711	25. 903 6677	8. 754 6913	. 001 490 313
672	451, 584	303, 464, 448	25. 922 9628	8. 759 0383	. 001 488 095
673	452, 929	304, 821, 217	25. 942 2435	8. 763 3809	. 001 485 884
674	454, 276	306, 182, 024	25. 961 5100	8. 767 7192	. 001 483 680
675	455, 625	307, 546, 875	25. 980 7621	8. 772 0532	. 001 481 481
676	456, 976	308, 915, 776	26. 000 0000	8. 776 3830	. 001 479 290
677	458, 329	310, 288, 733	26. 019 2237	8. 780 7084	. 001 477 105
678	459, 684	311, 665, 752	26. 038 4331	8. 785 0296	. 001 474 926
479	461, 041	313, 046, 839	26. 057 6284	8. 789 3466	. 001 472 754
680	462, 400	314, 432, 000	26. 076 8096	8. 793 6593	. 001 470 588
681	463, 761	315, 821, 241	26. 095 9767	8. 797 9679	. 001 468 429
682	465, 124	317, 214, 568	26. 115 1297	8. 802 2721	. 001 466 276
683	466, 489	318, 611, 987	26. 134 2687	8. 806 5722	. 001 464 129
684	467, 856	320, 013, 504	26. 153 3937	8. 810 8681	. 001 461 988
685	469, 225	321, 419, 125	26. 172 5047	8. 815 1598	. 001 459 854
686	470, 596	322, 828, 856	26. 191 6017	8. 819 4474	. 001 457 726
687	471, 969	324, 242, 703	26. 210 6848	8. 823 7307	. 001 455 604
688	473, 344	325, 660, 672	26. 229 7541	8. 828 0099	. 001 453 488
689	474, 721	327, 082, 769	26. 248 8095	8. 832 2850	. 001 451 379
690	476, 100	328, 509, 000	26. 267 8511	8. 836 5559	. 001 449 275
691	477, 481	329, 939, 371	26. 286 8789	8. 840 8227	. 001 447 178
692	478, 864	331, 373, 888	26. 305 8929	8. 845 0854	. 001 445 087
693	480, 249	332, 812, 557	26. 324 8932	8. 849 3440	. 001 443 001
694	481, 636	334, 255, 384	26. 343 8797	8. 853 5985	. 001 440 922
695	483, 025	335, 702, 375	26. 362 8527	8. 857 8489	. 001 438 849
696	484, 416	337, 153, 536	26. 381 8119	8. 862 0952	. 001 436 782
697	485, 809	338, 608, 873	26. 400 7576	8. 866 3375	. 001 434 720
698	487, 204	340, 068, 392	26. 419 6896	8. 870 5757	. 001 432 665
699	488, 601	341, 532, 099	26. 438 6081	8. 874 8099	. 001 430 615
700	490, 000	343, 000, 000	26. 457 5131	8. 879 0400	. 001 428 571
701	491, 401	344, 472, 101	26. 476 4046	8. 883 2661	. 001 426 534
702	492, 804	345, 948, 408	26. 495 2826	8. 887 4882	. 001 424 501
703	494, 209	347, 428, 927	26. 514 1472	8. 891 7063	. 001 422 475
704	495, 616	348, 913, 664	26. 532 9983	8. 895 9204	. 001 420 455
705	497, 025	350, 402, 625	26. 551 8361	8. 900 1304	. 001 418 440
706	498, 436	351, 895, 816	26. 570 6605	8. 904 3366	. 001 416 431
707	499, 849	353, 393, 243	26. 589 4716	8. 908 5387	. 001 414 427
708	501, 264	354, 894, 912	26. 608 2694	8. 912 7369	. 001 412 429
709	502, 681	356, 400, 829	26. 627 0539	8. 916 9311	. 001 410 437
710	504, 100	357, 911, 000	26. 645 8252	8. 921 1214	. 001 408 451
711	505, 521	359, 425, 431	26. 664 5833	8. 925 3078	. 001 406 470
712	506, 944	360, 944, 128	26. 683 3281	8. 929 4902	. 001 404 494

TABLE 1.—*Powers and roots*—Continued.

Number.	Squares.	Cubes.	√Roots.	∛Roots.	Reciprocals.
713	508, 369	362, 467, 097	26. 702 0598	8. 933 6687	. 001 402 525
714	509, 796	363, 994, 344	26. 720 7784	8. 937 8433	. 001 400 560
715	511, 225	365, 525, 875	26. 739 4839	8. 942 0140	. 001 398 601
716	512, 656	367, 061, 696	26. 758 1763	8. 946 1809	. 001 396 648
717	514, 089	368, 601, 813	26. 776 8557	8. 950 3438	. 001 394 700
718	515, 524	370, 146, 232	26. 795 5220	8. 954 5029	. 001 392 758
719	516, 961	371, 694, 959	26. 814 1754	8. 959 6581	. 001 390 821
720	518, 400	373, 248, 000	26. 832 8157	8. 962 8095	. 001 388 889
721	519, 841	374, 805, 361	26. 851 4432	8. 966 9570	. 001 386 963
722	521, 284	376, 367, 048	26. 870 0577	8. 971 1007	. 001 385 042
723	522, 729	377, 933, 067	26. 888 6593	8. 975 2406	. 001 383 126
724	524, 176	379, 503, 424	26. 907 2481	8. 979 3766	. 001 381 215
725	525, 625	381, 078, 125	26. 925 8240	8. 983 5089	. 001 379 310
726	527, 076	382, 657, 176	26. 944 3872	8. 987 6373	. 001 377 410
727	528, 529	384, 240, 583	26. 692 9375	8. 991 7620	. 001 375 516
728	529, 984	385, 828, 352	26. 981 4751	8. 995 8899	. 001 373 626
729	531, 441	387, 420, 489	27. 000 0000	9. 000 0000	. 001 371 742
730	532, 900	389, 017, 000	27. 018 5122	9. 004 1134	. 001 369 863
731	534, 361	390, 617, 891	27. 037 0117	9. 008 2229	. 001 367 989
732	535, 824	392, 223, 168	27. 055 4985	9. 012 3288	. 001 366 120
733	537, 289	393, 832, 837	27. 073 9727	9. 016 4309	. 001 364 256
734	538, 756	395, 446, 904	27. 092 4344	9. 020 5293	. 001 362 398
735	540, 225	397, 065, 375	27. 110 8834	9. 024 6239	. 001 360 544
736	541, 696	398, 688, 256	27. 129 3199	9. 028 7149	. 001 358 696
737	543, 169	400, 315, 553	27. 147 7149	9. 032 8021	. 001 356 852
738	544, 644	401, 947, 272	27. 166 1554	9. 036 8857	. 001 355 014
739	546, 121	403, 583, 419	27. 184 5544	9. 040 9655	. 001 353 180
740	547, 600	405, 224, 000	27. 202 9140	9. 045 0419	. 001 351 351
741	549, 081	406, 869, 021	27. 221 3152	9. 049 1142	. 001 349 528
742	550, 564	408, 518, 488	27. 239 6769	9. 053 1831	. 001 347 709
743	552, 049	410, 172, 407	27. 258 0263	9. 057 2482	. 001 345 895
744	553, 536	411, 830, 784	27. 276 3634	9. 061 3098	. 001 344 086
745	555, 025	413, 493, 625	27. 294 6881	9. 065 3677	. 001 342 282
746	556, 516	415, 160, 936	27. 313 0006	9. 069 4220	. 001 340 483
747	558, 009	416, 832, 723	27. 331 3007	9. 073 4726	. 001 338 688
748	559, 504	418, 508, 992	27. 349 5887	9. 077 5197	. 001 336 898
749	561, 001	420, 189, 749	27. 367 8644	9. 081 5631	. 001 335 113
750	562, 500	421, 875, 000	27. 386 1279	9. 085 6030	. 001 333 333
751	564, 001	423, 564, 751	27. 404 3792	9. 089 6352	. 001 331 558
752	565, 504	425, 259, 008	27. 422 6184	9. 093 6719	. 001 329 787
753	567, 009	426, 957, 777	27. 440 8455	9. 097 7010	. 001 328 021
754	568, 516	428, 661, 064	27. 459 0604	9. 101 7265	. 001 326 260
755	570, 025	430, 368, 875	27. 477 2633	9. 105 7485	. 001 324 503
756	571, 536	432, 081, 216	27. 495 4542	9. 109 7669	. 001 322 751
757	573, 049	438, 798, 093	27. 513 6330	9. 113 7818	. 001 321 004
758	574, 564	435, 519, 512	27. 531 7998	9. 117 7931	. 001 319 261
759	576, 081	437, 245, 479	27. 549 9546	9. 121 8010	. 001 317 523
760	577, 600	438, 976, 000	27. 568 0975	9. 125 8053	. 001 315 789
761	579, 121	440, 711, 081	27. 586 2284	9. 129 8061	. 001 314 060
762	580, 644	442, 450, 728	27. 604 3475	9. 133 8034	. 001 312 336
763	582, 169	444, 194, 947	27. 622 4546	9. 137 7971	. 001 310 616
764	583, 696	445, 943, 744	27. 640 5499	9. 141 7874	. 001 308 901
765	585, 225	447, 697, 125	27. 658 6334	9. 145 7742	. 001 307 190
766	586, 756	449, 455, 096	27. 676 7050	9. 149 7576	. 001 305 483
767	588, 289	451, 217, 663	27. 694 7648	9. 153 7375	. 001 303 781
768	589, 824	452, 984, 832	27. 712 8129	9. 157 7139	. 001 302 083
769	591, 361	454, 756, 609	27. 730 8492	9. 161 6869	. 001 300 390
770	592, 900	456, 533, 000	27. 748 8739	9. 165 6565	. 001 298 701
771	594, 441	458, 314, 011	27. 766 8868	9. 169 6225	. 001 297 017
772	595, 984	460, 099, 648	27. 784 8880	9. 173 5852	. 001 295 337
773	597, 529	461, 889, 917	27. 802 8775	9. 177 5445	. 001 293 661
774	599, 076	463, 684, 824	27. 820 8555	9. 181 5003	. 001 291 990
775	600, 625	465, 484, 375	27. 838 8218	9. 185 4527	. 001 290 323
776	602, 176	467, 288, 576	27. 856 7766	9. 189 4018	. 001 288 660
777	603, 729	469, 097, 433	27. 874 7197	9. 193 3474	. 001 287 001
778	605, 284	470, 910, 952	27. 892 6514	9. 197 2897	. 001 285 347

TABLE 1.—*Powers and roots*—Continued.

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
779	606, 841	472, 729, 139	27. 910 5715	9. 201 2286	. 001 283 697
780	608, 400	474, 552, 000	27. 928 4801	9. 205 1641	. 001 282 051
781	609, 961	476, 379, 541	27. 946 3772	9. 209 0962	. 001 280 410
782	611, 524	478, 211, 768	27. 964 2629	9. 213 0250	. 001 278 772
783	613, 089	480, 048, 687	27. 982 1372	9. 216 9505	. 001 277 139
784	614, 656	481, 890, 304	28. 000 0000	9. 220 8726	. 001 275 510
785	616, 225	483, 736, 625	28. 017 8515	9. 224 7914	. 001 273 885
786	617, 796	485, 587, 656	28. 035 6915	9. 228 7068	. 001 272 265
787	619, 369	487, 443, 403	28. 053 5203	9. 232 6189	. 001 270 648
788	620, 944	489, 303, 872	28. 071 3377	9. 236 5277	. 001 269 036
789	622, 521	491, 169, 069	28. 089 1438	9. 240 4333	. 001 267 427
790	624, 100	493, 039, 000	28. 106 9386	9. 244 3355	. 001 265 823
791	625, 681	494, 913, 671	28. 124 7222	9. 248 2344	. 001 264 223
792	627, 264	496, 793, 088	28. 142 4946	9. 252 1300	. 001 262 626
793	628, 849	498, 677, 257	28. 160 2557	9. 256 0224	. 001 261 034
794	630, 436	500, 566, 184	28. 178 0056	9. 259 9114	. 001 259 446
795	632, 025	502, 459, 875	28. 195 7444	9. 263 7973	. 001 257 862
796	633, 616	504, 358, 336	28. 213 4720	9. 267 6798	. 001 256 281
797	635, 209	506, 261, 573	28. 231 1884	9. 271 5592	. 001 254 705
798	636, 804	508, 169, 592	28. 248 8938	9. 275 4352	. 001 253 133
799	638, 401	510, 082, 399	28. 266 5881	9. 279 3081	. 001 251 564
800	640, 000	512, 000, 000	28. 284 2712	9. 283 1777	. 001 250 000
801	641, 601	513, 922, 401	28. 301 9434	9. 287 0444	. 001 248 439
802	643, 204	515, 849, 608	28. 319 6045	9. 290 9072	. 001 246 883
803	644, 809	517, 781, 627	28. 337 2546	9. 294 7671	. 001 245 330
804	646, 416	519, 718, 464	28. 354 8938	9. 298 6239	. 001 243 781
805	648, 025	521, 660, 125	28. 372 5219	9. 302 4775	. 001 242 236
806	649, 636	523, 606, 616	28. 390 1391	9. 306 3278	. 001 240 695
807	651, 249	525, 557, 943	28. 407 7454	9. 310 1750	. 001 239 157
808	652, 864	527, 514, 112	28. 425 3408	9. 314 0190	. 001 237 624
809	654, 481	529, 475, 129	28. 442 9253	9. 317 8599	. 001 236 094
810	656, 100	531, 441, 000	28. 460 4989	9. 321 6975	. 001 234 568
811	657, 721	533, 411, 731	28. 478 0617	9. 325 5320	. 001 233 046
812	659, 344	535, 387, 328	28. 495 6137	9. 329 3634	. 001 231 527
813	660, 969	537, 367, 797	28. 513 1549	9. 333 1916	. 001 230 012
814	662, 596	539, 353, 144	28. 530 6852	9. 337 0167	. 001 228 501
815	664, 225	541, 343, 375	28. 548 2048	9. 340 8386	. 001 226 994
816	665, 856	543, 338, 496	28. 565 7137	9. 344 6575	. 001 225 499
817	667, 489	545, 338, 513	28. 583 2119	9. 348 4731	. 001 223 990
818	669, 124	547, 343, 432	28. 600 6993	9. 352 2857	. 001 222 494
819	670, 761	549, 353, 259	28. 618 1760	9. 356 0952	. 001 221 001
820	672, 400	551, 368, 000	28. 635 6421	9. 359 9016	. 001 219 512
821	674, 041	553, 387, 661	28. 653 0976	9. 363 7049	. 001 218 027
822	675, 684	555, 412, 248	28. 670 5424	9. 367 5051	. 001 216 545
823	677, 329	557, 441, 767	28. 687 9716	9. 371 3022	. 001 215 067
824	678, 976	559, 476, 224	28. 705 4002	9. 375 0963	. 001 213 592
825	680, 625	561, 515, 625	28. 722 8132	9. 378 8873	. 001 212 121
826	682, 276	563, 559, 976	28. 740 2157	9. 382 6752	. 001 210 654
827	683, 929	565, 609, 283	28. 757 6077	9. 386 4600	. 001 209 190
828	685, 584	567, 663, 552	28. 774 9891	9. 390 2419	. 001 207 729
829	687, 241	569, 722, 789	28. 792 3601	9. 394 0206	. 001 206 273
830	688, 900	571, 787, 000	28. 809 7206	9. 397 7964	. 001 204 819
831	690, 561	573, 856, 191	28. 827 0706	9. 401 5691	. 001 203 369
832	692, 224	575, 930, 368	28. 844 4102	9. 405 3387	. 001 201 923
833	693, 889	578, 009, 537	28. 861 7394	9. 409 1054	. 001 200 480
834	695, 556	580, 093, 704	28. 879 0582	9. 412 8690	. 001 199 041
835	697, 225	582, 182, 875	28. 896 3666	9. 416 6297	. 001 197 605
836	698, 896	584, 277, 056	28. 913 6646	9. 420 3873	. 001 196 172
837	700, 569	586, 376, 253	28. 930 9523	9. 424 1420	. 001 194 743
838	702, 244	588, 480, 472	28. 948 2297	9. 427 8936	. 001 193 317
839	703, 921	590, 589, 719	28. 965 4967	9. 431 6423	. 001 191 895
840	705, 600	592, 704, 000	28. 982 7535	9. 435 3800	. 001 190 476
841	707, 281	594, 823, 321	29. 000 0000	9. 439 1307	. 001 189 061
842	708, 964	596, 947, 688	29. 017 2363	9. 442 8704	. 001 187 648
843	710, 649	599, 077, 107	29. 034 4623	9. 446 6072	. 001 186 240
844	712, 336	601, 211, 584	29. 051 6781	9. 450 3410	. 001 184 834

TABLE 1.—*Powers and roots*—Continued.

Number.	Squares.	Cubes.	√Roots.	∛Roots.	Reciprocals.
845	714, 025	603, 351, 125	29. 068 8837	9. 454 0719	. 001 183 432
846	715, 716	605, 495, 736	29. 086 0791	9. 457 7999	. 001 182 033
847	717, 409	607, 645, 423	29. 103 2644	9. 461 5249	. 001 180 638
848	719, 104	609, 800, 192	29. 120 4396	9. 465 2470	. 001 179 245
849	720, 801	611, 960, 049	29. 137 6046	9. 468 9661	. 001 177 856
850	722, 500	614, 125, 000	29. 154 7595	9. 472 6824	. 001 176 471
851	724, 201	616, 295, 051	29. 171 9043	9. 476 3957	. 001 175 088
852	725, 904	618, 470, 208	29. 189 0390	9. 480 1061	. 001 173 709
853	727, 609	620, 650, 477	29. 206 1637	9. 483 8136	. 001 172 333
854	729, 316	622, 835, 864	29. 223 2784	9. 487 5182	. 001 170 960
855	731, 025	625, 026, 375	29. 240 3830	9. 491 2200	. 001 169 591
856	732, 736	627, 222, 016	29. 257 4777	9. 494 9188	. 001 168 224
857	734, 449	629, 422, 793	29. 274 5623	9. 498 6147	. 001 166 861
858	736, 164	631, 628, 712	29. 291 6370	9. 502 3078	. 001 165 501
859	737, 881	633, 839, 779	29. 308 7018	9. 505 9980	. 001 164 144
860	739, 600	636, 056, 000	29. 325 7566	9. 509 6854	. 001 162 791
861	741, 321	638, 277, 381	29. 342 8015	9. 513 3699	. 001 161 440
862	743, 044	640, 503, 928	29. 359 8365	9. 517 0515	. 001 160 093
863	744, 769	642, 735, 647	29. 376 8616	9. 520 7303	. 001 158 749
864	746, 496	644, 972, 544	29. 393 8769	9. 524 4063	. 001 157 407
865	748, 225	647, 214, 625	29. 410 8823	9. 528 0794	. 001 156 069
866	749, 956	649, 461, 896	29. 427 8779	9. 531 7497	. 001 154 734
867	751, 689	651, 714, 363	29. 444 8637	9. 535 4172	. 001 153 403
868	753, 424	653, 972, 032	29. 461 8397	9. 539 0818	. 001 152 074
869	755, 161	656, 234, 909	29. 478 8059	9. 542 7437	. 001 150 748
870	756, 900	658, 503, 000	29. 495 7624	9. 546 4027	. 001 149 425
871	758, 641	660, 776, 311	29. 512 7091	9. 550 0589	. 001 148 106
872	760, 384	663, 054, 848	29. 529 6461	9. 553 7123	. 001 146 789
873	762, 129	665, 338, 617	29. 546 5734	9. 557 3630	. 001 145 475
874	763, 876	667, 627, 624	29. 563 4910	9. 561 0108	. 001 144 165
875	765, 625	669, 921, 875	29. 580 3989	9. 564 6559	. 001 142 857
876	767, 376	672, 221, 376	29. 597 2972	9. 568 2782	. 001 141 553
877	769, 129	674, 526, 133	29. 614 1858	9. 571 9377	. 001 140 251
878	770, 884	676, 836, 152	29. 631 0648	9. 575 5745	. 001 138 952
879	772, 641	679, 151, 439	29. 647 9342	9. 579 2085	. 001 137 656
880	774, 400	681, 472, 000	29. 664 7939	9. 582 8397	. 001 136 364
881	776, 161	683, 797, 841	29. 681 6442	9. 586 4682	. 001 135 074
882	777, 924	686, 128, 968	29. 698 4848	9. 590 0937	. 001 133 787
883	779, 689	688, 465, 387	29. 715 3159	9. 593 7169	. 001 132 503
884	781, 456	690, 807, 104	29. 732 1375	9. 597 3373	. 001 131 222
885	783, 225	693, 154, 125	29. 748 9496	9. 600 9548	. 001 129 944
886	784, 996	695, 506, 456	29. 765 7521	9. 604 5696	. 001 128 668
887	786, 769	697, 864, 103	29. 782 5452	9. 608 1817	. 001 127 396
888	788, 544	700, 227, 072	29. 799 3289	9. 611 7911	. 001 126 126
889	790, 321	702, 595, 369	29. 816 1030	9. 615 3977	. 001 124 859
890	792, 100	704, 969, 000	29. 832 8678	9. 619 0017	. 001 123 596
891	793, 881	707, 347, 971	29. 849 6231	9. 622 6030	. 001 122 334
892	795, 664	707, 932, 288	29. 866 3690	9. 626 2016	. 001 121 076
893	797, 449	712, 121, 957	29. 883 1056	9. 629 7975	. 001 119 821
894	799, 236	714, 516, 984	29. 899 8328	9. 633 3907	. 001 118 568
895	801, 025	716, 917, 375	29. 916 5506	9. 636 9812	. 001 117 818
896	802, 816	719, 323, 136	29. 933 2591	9. 640 5690	. 001 116 071
897	804, 609	721, 734, 273	29. 949 9583	9. 644 1542	. 001 114 827
898	806, 404	724, 150, 792	29. 966 6481	9. 647 7367	. 001 113 586
899	808, 201	726, 572, 699	29. 983 3287	9. 651 3166	. 001 112 347
900	810, 000	729, 000, 000	30. 000 0000	9. 654 8938	. 001 111 111
901	811, 801	731, 432, 701	30. 016 6621	9. 658 4684	. 001 109 878
902	813, 604	733, 870, 808	30. 093 3148	9. 662 0403	. 001 108 647
903	815, 409	736, 314, 327	30. 049 9584	9. 665 6096	. 001 107 420
904	817, 216	738, 763, 264	30. 066 5928	9. 669 1762	. 001 106 195
905	819, 025	741, 217, 625	30. 083 2179	9. 672 7403	. 001 104 972
906	820, 836	743, 677, 416	30. 099 8339	9. 676 3017	. 001 103 753
907	822, 649	746, 142, 643	30. 116 4407	9. 679 8604	. 001 102 536
908	824, 464	748, 613, 312	30. 133 0383	9. 683 4166	. 001 101 322
909	826, 281	751, 089, 429	30. 149 6269	9. 686 9701	. 001 100 110
910	828, 100	753, 571, 000	30. 166 2063	9. 690 5211	. 001 098 901

TABLE 1.—*Powers and roots*—Continued.

Number.	Squares.	Cubes.	$\sqrt{\text{Roots}}$.	$\sqrt[3]{\text{Roots}}$.	Reciprocals.
911	829, 921	756, 058, 031	30. 182 7765	9. 694 0694	. 001 097 695
912	831, 744	758, 550, 828	30. 199 3377	9. 697 6151	. 001 096 491
913	833, 569	761, 048, 497	30. 215 8899	9. 701 1583	. 001 095 290
914	835, 396	763, 551, 944	30. 232 4329	9. 704 6989	. 001 094 092
915	837, 225	766, 060, 875	30. 248 9669	9. 708 2369	. 001 092 896
916	839, 056	768, 575, 296	30. 265 4919	9. 711 7723	. 001 091 703
917	840, 889	771, 095, 213	30. 282 0079	9. 715 3051	. 001 090 513
918	842, 724	773, 620, 632	30. 298 5148	9. 718 8354	. 001 089 325
919	844, 561	776, 151, 559	30. 315 0128	9. 722 3631	. 001 088 139
920	846, 400	778, 688, 000	30. 331 5018	9. 725 8883	. 001 086 957
921	848, 241	781, 229, 961	30. 347 9818	9. 729 4109	. 001 085 776
922	850, 084	783, 777, 448	30. 364 4529	9. 732 9309	. 001 084 599
923	851, 929	786, 330, 467	30. 380 9151	9. 736 4484	. 001 083 423
924	853, 776	788, 889, 024	30. 397 3683	9. 739 9634	. 001 082 251
925	855, 625	791, 453, 125	30. 413 8127	9. 743 4758	. 001 081 081
926	857, 476	794, 022, 776	30. 430 2481	9. 746 9857	. 001 079 914
927	859, 329	796, 597, 983	30. 446 6747	9. 750 4930	. 001 078 749
928	861, 184	799, 178, 752	30. 463 0924	9. 753 9979	. 001 077 586
929	863, 041	801, 765, 089	30. 479 5013	9. 757 5002	. 001 076 426
930	864, 900	804, 357, 000	30. 495 9014	9. 761 0001	. 001 075 269
931	866, 761	806, 954, 491	30. 512 2926	9. 764 4974	. 001 074 114
932	868, 624	809, 557, 568	30. 528 6750	9. 767 9922	. 001 072 961
933	870, 489	812, 166, 237	30. 545 0487	9. 771 4845	. 001 071 811
934	872, 356	814, 780, 504	30. 561 4136	9. 774 9743	. 001 070 664
935	874, 225	817, 400, 375	30. 577 7697	9. 778 4616	. 001 069 519
936	876, 096	820, 025, 856	30. 594 1171	9. 781 9466	. 001 068 376
937	877, 969	822, 656, 953	30. 610 4557	9. 785 4288	. 001 067 236
938	879, 844	825, 293, 672	30. 626 7857	9. 788 9087	. 001 066 098
939	881, 721	827, 936, 019	30. 643 1069	9. 792 3861	. 001 064 963
940	883, 600	830, 584, 000	30. 659 4194	9. 795 8611	. 001 063 830
941	885, 481	833, 237, 621	30. 675 7233	9. 799 3336	. 001 062 699
942	887, 364	835, 896, 888	30. 692 0185	9. 802 8036	. 001 061 571
943	889, 249	838, 561, 807	30. 708 3051	9. 806 2711	. 001 060 445
944	891, 136	841, 232, 384	30. 724 5830	9. 809 7362	. 001 059 322
745	893, 025	843, 908, 625	30. 740 8523	9. 813 1989	. 001 058 201
946	894, 916	846, 590, 536	30. 757 1130	9. 816 6591	. 001 057 082
947	896, 809	849, 278, 123	30. 773 3651	9. 820 1169	. 001 055 966
948	898, 704	851, 971, 392	30. 789 6086	9. 823 5723	. 001 054 852
949	900, 601	854, 670, 349	30. 805 8436	9. 827 0252	. 001 053 741
950	902, 500	857, 375, 000	30. 822 0700	9. 830 4757	. 001 052 632
951	904, 401	860, 085, 351	30. 838 2879	9. 833 9238	. 001 051 525
952	906, 304	862, 801, 408	30. 854 4972	9. 837 3695	. 001 050 420
953	908, 209	865, 523, 177	30. 870 6981	9. 840 8127	. 001 049 318
954	910, 116	868, 250, 664	30. 886 8904	9. 844 2536	. 001 048 218
955	912, 025	870, 983, 875	30. 903 0743	9. 847 6920	. 001 047 120
956	913, 936	873, 722, 816	30. 919 2477	9. 851 1280	. 001 046 025
957	915, 849	876, 467, 493	30. 935 4166	9. 854 5617	. 001 044 932
958	917; 764	879, 217, 912	30. 951 5751	9. 857 9929	. 001 043 841
959	919, 681	881, 974, 079	30. 967 7251	9. 861 4218	. 001 042 753
960	921, 600	884, 736, 000	30. 983 8668	9. 864 8483	. 001 041 667
961	923, 521	887, 503, 681	31. 000 0000	9. 868 2724	. 001 040 583
962	925, 444	890, 277, 128	31. 016 1248	9. 871 6941	. 001 039 501
963	927, 369	893, 056, 347	31. 032 2413	9. 875 1135	. 001 038 422
964	929, 296	895, 841, 344	31. 048 3494	9. 878 5305	. 001 037 344
965	931, 225	898, 632, 125	31. 064 4491	9. 881 9451	. 001 036 269
966	933, 156	901, 428, 696	31. 080 5405	9. 885 3574	. 001 035 197
967	935, 089	904, 231, 063	31. 096 6236	9. 888 7673	. 001 034 126
968	937, 024	907, 039, 232	31. 112 6984	9. 892 1749	. 001 033 058
969	938, 961	909, 853, 209	31. 128 7648	9. 895 5801	. 001 031 992
970	940, 900	912, 673, 000	31. 144 8230	9. 898 9830	. 001 030 928
971	942, 841	915, 498, 611	31. 160 8729	9. 902 3835	. 001 029 866
972	944, 784	918, 330, 048	31. 176 9145	9. 905 7817	. 001 028 807
973	946, 729	921, 167, 317	31. 192 9479	9. 909 1776	. 001 027 749
974	948, 676	924, 010, 424	31. 208 9731	9. 912 5712	. 001 026 694
975	950, 625	926, 859, 375	31. 224 9900	9. 915 9624	. 001 025 641

TABLE 1.—*Powers and roots*—Continued.

Number.	Squares.	Cubes.	√Roots.	∛Roots.	Reciprocals.
976	952, 576	929, 714, 176	31. 240 9987	9. 919 3513	. 001 024 590
977	954, 529	932, 574, 833	31. 256 9992	9. 922 7379	. 001 023 541
978	956, 484	935, 441, 352	31. 272 9915	9. 926 1222	. 001 022 495
979	958, 441	938, 313, 739	31. 288 9757	9. 929 5042	. 001 021 450
980	960, 400	941, 192, 000	31. 304 9517	9. 932 8839	. 001 020 408
981	962, 361	944, 076, 141	31. 320 9195	9. 936 2613	. 001 019 168
982	964, 324	946, 966, 168	31. 336 8792	9. 939 6363	. 001 018 330
983	966, 289	949, 862, 087	31. 352 8308	9. 943 0092	. 001 017 294
984	968, 256	952, 763, 904	31. 368 7743	9. 946 3797	. 001 016 260
985	970, 225	955, 671, 625	31. 384 7097	9. 949 7479	. 001 015 228
986	972, 196	958, 585, 256	31. 400 6369	9. 953 1138	. 001 014 199
987	974, 169	961, 504, 803	31. 416 5561	9. 956 4775	. 001 013 171
988	976, 144	964, 430, 272	31. 432 4673	9. 959 8389	. 001 012 146
989	978, 121	967, 361, 669	31. 448 3704	9. 963 1981	. 001 011 122
990	980, 100	970, 299, 000	31. 464 2654	9. 966 5549	. 001 010 101
991	982, 081	973, 242, 271	31. 480 1525	9. 969 9055	. 001 009 082
992	984, 064	976, 191, 488	31. 496 0315	9. 973 2619	. 001 008 065
993	986, 049	979, 146, 657	31. 511 9025	9. 976 6120	. 001 007 049
994	988, 036	982, 107, 784	31. 527 7655	9. 979 9599	. 001 006 036
995	990, 025	985, 074, 875	31. 543 6206	9. 983 3055	. 001 005 025
996	992, 016	988, 047, 936	31. 559 4677	9. 986 6488	. 001 004 016
997	994, 009	991, 026, 973	31. 575 3068	9. 989 9900	. 001 003 009
998	996, 004	994, 011, 992	31. 591 1380	9. 993 3289	. 001 002 004
999	998, 001	997, 002, 999	31. 606 9613	9. 996 6656	. 001 001 001
1, 000	1, 000, 000	1, 000, 000, 000	31. 622 7766	10. 000 0000	. 001 000 000
1, 001	1, 002, 001	1, 003, 003, 001	31. 638 5840	10. 003 3222	. 000 999 0010
1, 002	1, 004, 004	1, 006, 012, 008	31. 654 3866	10. 006 6622	. 000 998 0040
1, 003	1, 006, 009	1, 009, 027, 027	31. 670 1752	10. 009 9899	. 000 997 0090
1, 004	1, 008, 016	1, 012, 048, 064	31. 685 9590	10. 013 3155	. 000 996 0159
1, 005	1, 010, 025	1, 015, 075, 125	31. 701 7349	10. 016 6389	. 000 995 0249
1, 006	1, 012, 036	1, 018, 108, 216	31. 717 5030	10. 019 9601	. 000 994 0358
1, 007	1, 014, 049	1, 021, 147, 343	31. 733 2633	10. 023 2791	. 000 993 0487
1, 008	1, 016, 064	1, 024, 192, 512	31. 749 0157	10. 026 5958	. 000 992 0635
1, 009	1, 018, 081	1, 027, 243, 729	31. 764 7603	10. 029 9104	. 000 991 0803
1, 010	1, 020, 100	1, 030, 301, 000	31. 780 4972	10. 033 2228	. 000 990 0990
1, 011	1, 022, 121	1, 033, 364, 331	31. 796 2262	10. 036 5330	. 000 989 1197
1, 012	1, 024, 144	1, 036, 433, 728	31. 811 9474	10. 039 8410	. 000 988 1423
1, 013	1, 026, 169	1, 039, 509, 197	31. 827 6609	10. 043 1469	. 000 987 1668
1, 014	1, 028, 196	1, 042, 590, 744	31. 843 3666	10. 046 4506	. 000 986 1933
1, 015	1, 030, 225	1, 045, 678, 375	31. 859 0646	10. 049 7521	. 000 985 2217
1, 016	1, 032, 256	1, 048, 772, 096	31. 874 7549	10. 053 0514	. 000 984 2520
1, 017	1, 034, 289	1, 051, 871, 913	31. 890 4374	10. 056 3485	. 000 983 2842
1, 018	1, 036, 324	1, 054, 977, 832	31. 906 1123	10. 059 6435	. 000 982 3183
1, 019	1, 038, 361	1, 058, 089, 859	31. 921 7794	10. 062 9364	. 000 981 3543
1, 020	1, 040, 400	1, 061, 208, 000	31. 937 4388	10. 066 2271	. 000 980 3922
1, 021	1, 042, 441	1, 064, 332, 261	31. 953 0906	10. 069 5156	. 000 979 4319
1, 022	1, 044, 484	1, 067, 462, 648	31. 968 7347	10. 072 8020	. 000 978 4736
1, 023	1, 046, 529	1, 070, 599, 167	31. 984 3712	10. 076 0863	. 000 977 5171
1, 024	1, 048, 576	1, 073, 741, 824	32. 000 0000	10. 079 3684	. 000 976 5625
1, 025	1, 050, 625	1, 076, 890, 625	32. 015 6212	10. 082 6484	. 000 975 6098
1, 026	1, 052, 676	1, 080, 045, 576	32. 031 2348	10. 085 9262	. 000 974 6589
1, 027	1, 054, 729	1, 083, 206, 683	32. 046 8407	10. 089 2019	. 000 973 7098
1, 028	1, 056, 784	1, 086, 373, 952	32. 062 4391	10. 092 4755	. 000 972 7626
1, 029	1, 058, 841	1, 089, 547, 389	32. 078 0298	10. 095 7469	. 000 971 8173
1, 030	1, 060, 900	1, 092, 727, 000	32. 093 6131	10. 099 0163	. 000 970 8738
1, 031	1, 062, 961	1, 095, 912, 791	32. 109 1887	10. 102 2835	. 000 969 9321
1, 032	1, 065, 024	1, 099, 104, 768	32. 124 7568	10. 105 5487	. 000 968 9922
1, 033	1, 067, 089	1, 102, 302, 937	32. 140 3173	10. 108 8117	. 000 968 0542
1, 034	1, 069, 156	1, 105, 507, 304	32. 155 8704	10. 112 0726	. 000 967 1180
1, 035	1, 071, 225	1, 108, 717, 875	32. 171 4159	10. 115 3314	. 000 966 1836
1, 036	1, 073, 296	1, 111, 934, 656	32. 186 9539	10. 118 5882	. 000 965 2510
1, 037	1, 075, 369	1, 115, 157, 653	32. 202 4844	10. 121 8428	. 000 964 3202
1, 038	1, 077, 444	1, 118, 386, 872	32. 218 0074	10. 125 0953	. 000 963 3911
1, 039	1, 079, 521	1, 121, 622, 319	32. 233 5229	10. 128 3457	. 000 962 4639
1, 040	1, 081, 600	1, 124, 864, 000	32. 249 0310	10. 131 5941	. 000 961 5385

TABLE 1—Continued.—*Square roots and cube roots of fractions.*

N.	Square root.	Cube root.	Reciprocal.	N.	Square root.	Cube root.	Reciprocal.
$\frac{1}{32}$	.17678	.31498	32.00000	$\frac{17}{32}$	.72887	.80990	1.88235
$\frac{1}{16}$	.25000	.39685	16.00000	$\frac{9}{16}$	.75000	.82548	1.77778
$\frac{3}{32}$	.30619	.45428	10.66667	$\frac{19}{32}$	.77055	.84049	1.68421
$\frac{1}{8}$	.35355	.50000	8.00000	$\frac{5}{8}$	.79057	.85499	1.60000
$\frac{5}{32}$	.39528	.53861	6.40000	$\frac{21}{32}$	.81509	.86901	1.52381
$\frac{3}{16}$	.43301	.57236	5.33333	$\frac{11}{16}$	.82916	.88259	1.45455
$\frac{7}{32}$	.46771	.60254	4.57143	$\frac{23}{32}$	.84779	.89576	1.39130
$\frac{1}{4}$	.50000	.62996	4.00000	$\frac{3}{4}$	.86603	.90856	1.33333
$\frac{9}{32}$	.53033	.65519	3.55556	$\frac{25}{32}$	.88388	.92101	1.28000
$\frac{5}{16}$	.55902	.67860	3.20000	$\frac{13}{16}$	.90139	.93313	1.23077
$\frac{11}{32}$	.58630	.70051	2.90909	$\frac{27}{32}$	.91856	.94494	1.18519
$\frac{3}{8}$	.61237	.72112	2.66667	$\frac{7}{8}$	.93541	.95647	1.14286
$\frac{13}{32}$	.63738	.74062	2.46154	$\frac{29}{32}$	.95197	.96772	1.10345
$\frac{7}{16}$	.66144	.75915	2.28571	$\frac{15}{16}$	.96825	.97872	1.06667
$\frac{15}{32}$	.68465	.77681	2.13333	$\frac{31}{32}$	.98425	.98947	1.03226
$\frac{1}{2}$	.70711	.79370	2.00000				

TABLE 2.—*Powers and roots of useful factors.*

n	$\frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\frac{1}{\sqrt{n}}$	$\sqrt[3]{n}$	$\frac{1}{\sqrt[3]{n}}$
$\pi=$ 3.142	0.318	9.870	31.006	1.772	0.564	1.465	0.683
$2\pi=$ 6.283	0.159	39.478	248.050	2.507	0.399	1.845	0.542
$\frac{\pi}{2}=$ 1.571	0.637	2.467	3.878	1.253	0.798	1.162	0.860
$\frac{\pi}{3}=$ 1.047	0.955	1.097	1.148	1.023	0.977	1.016	0.985
$\frac{4\pi}{3}=$ 4.189	0.239	17.546	73.496	2.047	0.489	1.612	0.622
$\frac{\pi}{4}=$ 0.785	1.274	0.617	0.484	0.886	1.128	0.923	1.084
$\frac{\pi}{6}=$ 0.524	1.910	0.274	0.144	0.724	1.382	0.806	1.241
$\pi^2=$ 9.870	0.101	97.409	961.390	3.142	0.318	2.145	0.466
$\pi^3=$31.006	0.032	961.390	29,809.910	5.568	1.796	3.142	0.318
$\frac{\pi}{32}=$ 0.098	10.186	0.0095	0.001	0.313	3.192	0.461	2.168
g=32.2	0.031	1,036.84	33,386.24	5.674	0.176	3.181	0.314
$2g$=64.4	0.015	4,147.36	267,090	8.025	0.125	4.007	0.249

TABLE 3.—*Useful functions of* π.

$\pi=3.1415926+$

$\text{Log } \pi = 0.49714987$	$\pi\sqrt{2}=4.44288$	$\sqrt{\pi}= 1.77245385$
$\frac{360}{\pi}=114.59156$	$\sqrt{\frac{2}{\pi}}=0.7978846$	$\sqrt[3]{\pi}= 1.46459189$
$\frac{\pi}{16}= .19634954$	$\frac{\pi}{\sqrt{2}}=2.2214415$	$\pi^2= 9.86960440$
$\frac{\pi}{24}= .13089969$	$\sqrt{\frac{\pi}{2}}=1.2533$	$\pi^3=31.00627668$
$\frac{\pi}{32}= .09817477$	$\frac{\sqrt{2}}{\pi}=0.4501582$	$\frac{1}{\pi}= .31830989$
$\frac{\pi}{180}= .01745329$		$\frac{1}{\pi^2}= .10132118$
$\frac{\pi}{360}= .00872664$		$\frac{1}{\pi^3}= .03225153$
$\frac{180}{\pi}= 57.29578$		$\frac{1}{\sqrt{\pi}}= .56418958$
$36\pi=113.0973$		$\frac{1}{\sqrt[3]{\pi}}= .68278406$

n	$\pi\times n$	$\frac{\pi}{n}$	$\frac{n}{\pi}$	$\pi^2\times n$	$\frac{\pi^2}{n}$	$n\sqrt{\pi}$
1	3.14159	3.14159265	.31831	9.8696	9.8696	1.7725
2	6.28318	1.57079633	.63662	19.7392	4.9348	3.5449
3	9.42478	1.04719755	.95493	29.6088	3.2898	5.3174
4	12.56637	.78539816	1.27324	39.4784	2.4674	7.0898
5	15.70796	.62831853	1.59155	49.3480	1.9739	8.8623
6	18.84956	.52359878	1.90986	59.2176	1.6449	10.6347
7	21.99115	.44879895	2.22817	69.0872	1.4099	12.4072
8	25.13274	.39269908	2.54648	78.9568	1.2337	14.1796
9	28.27433	.34906585	2.86479	88.8264	1.0956	15.9521

TABLE 4.—*Areas and circumferences of circles.*

For diameters from $\frac{1}{10}$ to 10, advancing by tenths.

Diameter.	Area.	Circumference.	Diameter.	Area.	Circumference.	Diameter.	Area.	Circumference.
.1	0.007854	0.31416	3.4	9.0792	10.6814	6.7	35.2565	21.0487
.2	.031416	.62832	.5	9.6211	10.9956	.8	36.3168	21.3628
.3	.070686	.94248	.6	10.1788	11.3097	.9	37.3928	21.6770
.4	.12566	1.2566	.7	10.7521	11.6239	7.0	38.4845	21.9911
.5	.19635	1.5708	.8	11.3411	11.9381	.1	39.5919	22.3053
.6	.28274	1.8850	.9	11.9459	12.2522	.2	40.7150	22.6195
.7	.38485	2.1991	4.0	12.5664	12.5664	.3	41.8539	22.9336
.8	.50265	2.5133	.1	13.2025	12.8805	.4	43.0084	23.2478
.9	.63617	2.8274	.2	13.8544	13.1947	.5	44.1786	23.5619
1.0	.7854	3.1416	.3	14.5220	13.5088	.6	45.3646	23.8761
.1	.9503	3.4558	.4	15.2053	13.8230	.7	46.5663	24.1903
.2	1.1310	3.7699	.5	15.9043	14.1372	.8	47.7836	24.5044
.3	1.3273	4.0841	.6	16.6190	14.4513	.9	49.0167	24.8186
.4	1.5394	4.3982	.7	17.3494	14.7655	8.0	50.2655	25.1327
.5	1.7671	4.7124	.8	18.0956	15.0796	.1	51.5300	25.4469
.6	2.0106	5.0265	.9	18.8574	15.3938	.2	52.8102	25.7611
.7	2.2698	5.3407	5.0	19.6350	15.7080	.3	54.1061	26.0752
.8	2.5447	5.6549	.1	20.4282	16.0221	.4	55.4177	26.3894
.9	2.8353	5.9690	.2	21.2372	16.3363	.5	56.7450	26.7035
2.0	3.1416	6.2832	.3	22.0618	16.6504	.6	58.0880	27.0177
.1	3.4636	6.5973	.4	22.9022	16.9646	.7	59.4468	27.3319
.2	3.8013	6.9115	.5	23.7583	17.2788	.8	60.8212	27.6460
.3	4.1548	7.2257	.6	24.6301	17.5929	.9	62.2114	27.9602
.4	4.5239	7.5398	.7	25.5176	17.9071	9.0	63.6173	28.2743
.5	4.9087	7.8540	.8	26.4208	18.2212	.1	65.0388	28.5885
.6	5.3093	8.1681	.9	27.3397	18.5354	.2	66.4761	28.9027
.7	5.7256	8.4823	6.0	28.2743	18.8496	.3	67.9291	29.2168
.8	6.1575	8.7965	.1	29.2247	19.1637	.4	69.3978	29.5310
.9	6.6052	9.1106	.2	30.1907	19.4779	.5	70.8822	29.8451
3.0	7.0686	9.4248	.3	31.1725	19.7920	.6	72.3823	30.1593
.1	7.5477	9.7389	.4	32.1699	20.1062	.7	73.8981	30.4734
.2	8.0425	10.0531	.5	33.1831	20.4204	.8	75.4296	30.7876
.3	8.5530	10.3673	.6	34.2119	20.7345	.9	76.9769	31.1018

Table 4—Continued.—*Circumferences and areas of circles.*

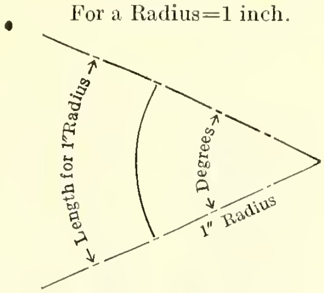
Diameter.	Circumference.	Area.
1/64	.04909	.00019
1/32	.09818	.00077
3/64	.14726	.00173
1/16	.19635	.00307
3/32	.29452	.00690
1/8	.39270	.01227
5/32	.49087	.01917
3/16	.58905	.02761
7/32	.68722	.03758
1/4	.78540	.04909
9/32	.88357	.06213
5/16	.98175	.07670
11/32	1.0799	.09281
3/8	1.1781	.11045
13/32	1.2763	.11962
7/16	1.3744	.15033
15/32	1.4726	.17257
1/2	1.5708	.19635
17/32	1.6690	.22166
9/16	1.7671	.24850
19/32	1.8653	.27688
5/8	1.9635	.30680
21/32	2.0617	.33824
11/16	2.1598	.37122
23/32	2.2580	.40574
3/4	2.3562	.44179
25/32	2.4544	.47937
13/16	2.5525	.51849
27/32	2.6507	.55914
7/8	2.7489	.60132
29/32	2.8471	.64504
15/16	2.9452	.69029
31/32	3.0434	.73708
1.	3.1416	.7854
1/16	3.3379	.8866
1/8	3.5343	.9940
3/16	3.7306	1.1075
1/4	3.9270	1.2272
5/16	4.1233	1.3530
3/8	4.3197	1.4849
7/16	4.5160	1.6230
1/2	4.7124	1.7671
9/16	4.9087	1.9175
5/8	5.1051	2.0739
11/16	5.3014	2.2365
3/4	5.4978	2.4053
13/16	5.6941	2.5802
7/8	5.8905	2.7612
15/16	6.0868	2.9483

Diameter.	Circumference.	Area.
2.	6.2832	3.1416
1/16	6.4795	3.3410
1/8	6.6759	3.5466
3/16	6.8722	3.7583
1/4	7.0686	3.9761
5/16	7.2649	4.2000
3/8	7.4613	4.4301
7/16	7.6576	4.6664
1/2	7.8540	4.9087
9/16	8.0503	5.1572
5/8	8.2467	5.4119
11/16	8.4430	5.6727
3/4	8.6394	5.9396
13/16	8.8357	6.2126
7/8	9.0321	6.4918
15/16	9.2284	6.7771
3.	9.4248	7.0686
1/16	9.6211	7.3662
1/8	9.8175	7.6699
3/16	10.014	7.9798
1/4	10.210	8.2958
5/16	10.407	8.6179
3/8	10.603	8.9462
7/16	10.799	9.2806
1/2	10.996	9.6211
9/16	11.192	9.9678
5/8	11.388	10.321
11/16	11.585	10.680
3/4	11.781	11.045
13/16	11.977	11.416
7/8	12.174	11.793
15/16	12.370	12.177
4.	12.566	12.566
1/16	12.763	12.962
1/8	12.959	13.364
3/16	13.155	13.772
1/4	13.352	14.186
5/16	13.548	14.607
3/8	13.744	15.033
7/16	13.941	15.466
1/2	14.137	15.904
9/16	14.334	16.349
5/8	14.530	16.800
11/16	14.726	17.257
3/4	14.923	17.721
13/16	15.119	18.190
7/8	15.315	18.665
15/16	15.512	19.147

Diameter.	Circumference.	Area.
5.	15.708	19.635
1/16	15.904	20.129
1/8	16.101	20.629
3/16	16.297	21.135
1/4	16.493	21.648
5/16	16.690	22.166
3/8	16.886	22.691
7/16	17.082	23.221
1/2	17.279	23.758
9/16	17.475	24.301
5/8	17.671	24.850
11/16	17.868	25.406
3/4	18.064	25.967
13/16	18.261	26.535
7/8	18.457	27.109
15/16	18.653	27.688
6.	18.850	28.274
1/8	19.242	29.465
1/4	19.635	30.680
3/8	20.028	31.919
1/2	20.420	33.183
5/8	20.813	34.472
3/4	21.206	35.785
7/8	21.598	37.122
7.	21.991	38.485
1/8	22.384	39.871
1/4	22.776	41.282
3/8	23.169	42.718
1/2	23.562	44.179
5/8	23.955	45.664
3/4	24.347	47.173
7/8	24.740	48.707
8.	25.133	50.265
1/8	25.525	51.849
1/4	25.918	53.456
3/8	26.311	55.088
1/2	26.704	56.745
5/8	27.096	58.426
3/4	27.489	60.132
7/8	27.882	61.862
9.	28.274	63.617
1/8	28.667	65.397
1/4	29.060	67.201
3/8	29.452	69.029
1/2	29.845	70.882
5/8	30.238	72.760
3/4	30.631	74.662
7/8	31.023	76.589
10.	31.416	78.540

TABLE 5.—*Circular arcs and segments.*

For a Radius=1 inch.



Degree.	Length of arc.	Height of arc.	Length of chord.	Area of segment.	Degree.	Length of arc.	Height of arc.	Length of chord.	Area of segment.
1	0.0175	0.0000	0.0175	0.00000	51	0.8901	0.0974	0.8610	0.05649
2	0.0349	0.0002	0.0349	0.00000	52	0.9076	0.1012	0.8767	0.05978
3	0.0524	0.0003	0.0524	0.00001	53	0.9250	0.1051	0.8924	0.06319
4	0.0698	0.0006	0.0698	0.00003	54	0.9425	0.1090	0.9080	0.06673
5	0.0873	0.0010	0.0872	0.00006	55	0.9599	0.1130	0.9235	0.07039
6	0.1047	0.0014	0.1047	0.00010	56	0.9774	0.1171	0.9389	0.07417
7	0.1222	0.0019	0.1221	0.00015	57	0.9948	0.1212	0.9543	0.07808
8	0.1396	0.0024	0.1395	0.00023	58	1.0123	0.1254	0.9696	0.08212
9	0.1571	0.0031	0.1569	0.00032	59	1.0297	0.1296	0.9848	0.08629
10	0.1745	0.0038	0.1743	0.00044	60	1.0472	0.1340	1.0000	0.09059
11	0.1920	0.0046	0.1917	0.00059	61	1.0647	0.1384	1.0151	0.09502
12	0.2094	0.0055	0.2091	0.00076	62	1.0821	0.1428	1.0301	0.09958
13	0.2269	0.0064	0.2264	0.00097	63	1.0996	0.1474	1.0450	0.10428
14	0.2443	0.0075	0.2437	0.00121	64	1.1170	0.1520	1.0598	0.10911
15	0.2618	0.0086	0.2611	0.00149	65	1.1345	0.1566	1.0746	0.11408
16	0.2793	0.0097	0.2783	0.00181	66	1.1519	0.1613	1.0893	0.11919
17	0.2967	0.0110	0.2956	0.00217	67	1.1694	0.1661	1.1039	0.12443
18	0.3142	0.0123	0.3129	0.00257	68	1.1868	0.1710	1.1184	0.12982
19	0.3316	0.0137	0.3301	0.00302	69	1.2043	0.1759	1.1328	0.13535
20	0.3491	0.0152	0.3473	0.00352	70	1.2217	0.1808	1.1472	0.14102
21	0.3665	0.0167	0.3645	0.00408	71	1.2392	0.1859	1.1614	0.14683
22	0.3840	0.0184	0.3816	0.00468	72	1.2566	0.1910	1.1756	0.15279
23	0.4014	0.0201	0.3987	0.00535	73	1.2741	0.1961	1.1896	0.15889
24	0.4189	0.0219	0.4158	0.00607	74	1.2915	0.2014	1.2036	0.16514
25	0.4363	0.0237	0.4329	0.00686	75	1.3090	0.2066	1.2175	0.17154
26	0.4538	0.0256	0.4499	0.00771	76	1.3265	0.2120	1.2313	0.17808
27	0.4712	0.0276	0.4669	0.00862	77	1.3439	0.2174	1.2450	0.18477
28	0.4887	0.0297	0.4838	0.00961	78	1.3614	0.2229	1.2586	0.19160
29	0.5061	0.0319	0.5008	0.01067	79	1.3788	0.2284	1.2722	0.19859
30	0.5236	0.0341	0.5176	0.01180	80	1.3963	0.2340	1.2856	0.20573
31	0.5411	0.0364	0.5345	0.01301	81	1.4137	0.2396	1.2989	0.21301
32	0.5585	0.0387	0.5512	0.01429	82	1.4312	0.2453	1.3121	0.22045
33	0.5760	0.0412	0.5680	0.01566	83	1.4486	0.2510	1.3252	0.22804
34	0.5934	0.0437	0.5847	0.01711	84	1.4661	0.2569	1.3383	0.23578
35	0.6109	0.0463	0.6014	0.01864	85	1.4835	0.2627	1.3512	0.24367
36	0.6283	0.0489	0.6180	0.02027	86	1.5010	0.2686	1.3640	0.25171
37	0.6458	0.0517	0.6346	0.02198	87	1.5184	0.2746	1.3767	0.25990
38	0.6632	0.0545	0.6511	0.02378	88	1.5359	0.2807	1.3893	0.26825
39	0.6807	0.0574	0.6676	0.02568	89	1.5533	0.2867	1.4018	0.27675
40	0.6981	0.0603	0.6840	0.02767	90	1.5708	0.2929	1.4142	0.28854
41	0.7156	0.0633	0.7004	0.02976	91	1.5882	0.2991	1.4265	0.29420
42	0.7330	0.0664	0.7167	0.03195	92	1.6057	0.3053	1.4387	0.30316
43	0.7505	0.0696	0.7330	0.03425	93	1.6232	0.3116	1.4507	0.31226
44	0.7679	0.0728	0.7492	0.03664	94	1.6406	0.3180	1.4627	0.32152
45	0.7854	0.0761	0.7654	0.03915	95	1.6581	0.3244	1.4746	0.33093
46	0.8029	0.0795	0.7815	0.04176	96	1.6755	0.3309	1.4863	0.34050
47	0.8203	0.0829	0.7975	0.04448	97	1.6930	0.3374	1.4979	0.35021
48	0.8378	0.0865	0.8135	0.04731	98	1.7104	0.3439	1.5094	0.36008
49	0.8552	0.0900	0.8294	0.05025	99	1.7279	0.3506	1.5208	0.37009
50	0.8728	0.0937	0.8452	0.05331	100	1.7453	0.3572	1.5321	0.38026

TABLE 5.—*Circular arcs and segments*—Continued.

Degree.	Length of arc.	Height of arc.	Length of chord.	Area of segment.	Degree.	Length of arc.	Height of arc.	Length of chord.	Area of segment.
101	1.7628	0.3639	1.5432	0.39058	141	2.4609	0.6662	1.8853	0.91580
102	1.7802	0.3707	1.5543	0.40104	142	2.4784	0.6744	1.8910	0.93135
103	1.7977	0.3775	1.5652	0.41166	143	2.4958	0.6827	1.8966	0.94700
104	1.8151	0.3843	1.5760	0.42242	144	2.5133	0.6910	1.9021	0.96274
105	1.8326	0.3912	1.5867	0.43333	145	2.5307	0.6993	1.9074	0.97858
106	1.8500	0.3982	1.5973	0.44439	146	2.5482	0.7076	1.9126	0.99449
107	1.8675	0.4052	1.6077	0.45560	147	2.5656	0.7160	1.9176	1.01050
108	1.8850	0.4122	1.6180	0.46695	148	2.5831	0.7244	1.9225	1.02658
109	1.9024	0.4193	1.6282	0.47844	149	2.6005	0.7328	1.9273	1.04275
110	1.9199	0.4264	1.6383	0.49008	150	2.6180	0.7412	1.9319	1.05900
111	1.9373	0.4336	1.6483	0.50187	151	2.6354	0.7496	1.9363	1.07532
112	1.9548	0.4408	1.6581	0.51379	152	2.6529	0.7581	1.9406	1.09171
113	1.9722	0.4481	1.6678	0.52586	153	2.6704	0.7666	1.9447	1.10818
114	1.9897	0.4554	1.6773	0.53807	154	2.6878	0.7750	1.9487	1.12472
115	2.0071	0.4627	1.6868	0.55041	155	2.7052	0.7836	1.9526	1.14132
116	2.0246	0.4701	1.6961	0.56289	156	2.7227	0.7921	1.9563	1.15799
117	2.0420	0.4775	1.7053	0.57551	157	2.7402	0.8006	1.9598	1.17472
118	2.0595	0.4850	1.7143	0.58827	158	2.7576	0.8092	1.9633	1.19151
119	2.0769	0.4925	1.7233	0.60116	159	2.7751	0.8178	1.9665	1.20835
120	2.0944	0.5000	1.7321	0.61418	160	2.7925	0.8264	1.9696	1.22525
121	2.1118	0.5076	1.7407	0.62734	161	2.8100	0.8350	1.9726	1.24221
122	2.1293	0.5152	1.7492	0.64063	162	2.8274	0.8436	1.9754	1.25921
123	2.1468	0.5228	1.7576	0.65404	163	2.8449	0.8522	1.9780	1.27626
124	2.1642	0.5305	1.7659	0.66759	164	2.8623	0.8608	1.9805	1.29335
125	2.1817	0.5383	1.7740	0.68125	165	2.8798	0.8695	1.9829	1.31049
126	2.1991	0.5460	1.7820	0.69505	166	2.8972	0.8781	1.9851	1.32766
127	2.2166	0.5538	1.7899	0.70897	167	2.9147	0.8868	1.9871	1.34487
128	2.2340	0.5616	1.7976	0.72301	168	2.9322	0.8955	1.9890	1.36212
129	2.2515	0.5695	1.8052	0.73716	169	2.9496	0.9042	1.9908	1.37940
130	2.2690	0.5774	1.8126	0.75144	170	2.9671	0.9128	1.9924	1.39671
131	2.2864	0.5853	1.8199	0.76584	171	2.9845	0.9215	1.9938	1.41404
132	2.3038	0.5933	1.8271	0.78034	172	3.0020	0.9302	1.9951	1.43140
133	2.3213	0.6013	1.8341	0.79497	173	3.0194	0.9390	1.9963	1.44878
134	2.3387	0.6093	1.8410	0.80970	174	3.0369	0.9477	1.9973	1.46617
135	2.3562	0.6173	1.8478	0.82454	175	3.0543	0.9564	1.9981	1.48359
136	2.3736	0.6254	1.8544	0.83940	176	3.0718	0.9651	1.9988	1.50101
137	2.3911	0.6335	1.8608	0.85455	177	3.0892	0.9738	1.9993	1.51845
138	2.4086	0.6416	1.8672	0.86971	178	3.1067	0.9825	1.9997	1.53589
139	2.4260	0.6498	1.8733	0.88497	179	3.1241	0.9913	1.9999	1.55334
140	2.4435	0.6580	1.8794	0.90034	180	3.1416	1.0000	2.0000	1.57080

Min.	Length.	Min.	Length.	Min.	Length.	Min.	Length.
1	0.0003	17	0.0050	32	0.0093	48	0.0140
2	0.0006	18	0.0052	33	0.0096	49	0.0143
3	0.0009	19	0.0055	34	0.0199	50	0.0145
4	0.0012	20	0.0058	35	0.0102	51	0.0148
5	0.0015	21	0.0061	36	0.0105	52	0.0151
6	0.0017	22	0.0064	37	0.0108	53	0.0154
7	0.0020	23	0.0067	38	0.0111	54	0.0157
8	0.0023	24	0.0070	39	0.0113	55	0.0160
9	0.0026	25	0.0073	40	0.0116	56	0.0163
10	0.0029	26	0.0076	41	0.0119	57	0.0166
11	0.0032	26	0.0076	42	0.0122	58	0.0169
12	0.0035	27	0.0079	43	0.0125	59	0.0172
13	0.0038	28	0.0081	44	0.0128	60	0.0175
14	0.0041	29	0.0084	45	0.0131		
15	0.0044	30	0.0087	46	0.0134		
16	0.0047	31	0.0090	47	0.0137		

TABLE 6.[1]—*Exponential function.*

x	$\log_{10}(e^x)$	e^x	e^{-x}	x	$\log_{10}(e^x)$	e^x	e^{-x}
0.00	0.00000	1.0000	1.000000	0.55	0.23886	1.7333	0.576950
.01	.00434	.0101	0.990050	.56	.24320	.7507	.571209
.02	.00869	.0202	.980199	.57	.24755	.7683	.565525
.03	.01303	.0305	.970446	.58	.25189	.7860	.559898
.04	.01737	.0408	.960789	.59	.25623	.8040	.554327
0.05	0.02171	1.0513	0.951229	0.60	0.26058	1.8221	0.548812
.06	.02606	.0618	.941765	.61	.26492	.8404	.543351
.07	.03040	.0725	.932394	.62	.26926	.8589	.537944
.08	.03474	.0833	.923116	.63	.27361	.8776	.532592
.09	.03909	.0942	.913931	.64	.27795	.8965	.527292
0.10	0.04343	1.1052	0.904837	0.65	0.28229	1.9155	0.522046
.11	.04777	.1163	.895834	.66	.28663	.9348	.516851
.12	.05212	.1275	.886920	.67	.29098	.9542	.511709
.13	.05646	.1388	.878095	.68	.29532	.9739	.506617
.14	.06080	.1503	.869358	.69	.29966	.9937	.501576
0.15	0.06514	1.1618	0.860708	0.70	0.30401	2.0138	0.496585
.16	.06949	.1735	.852144	.71	.30835	.0340	.491644
.17	.07383	.1853	.843665	.72	.31269	.0544	.486752
.18	.07817	.1972	.835270	.73	.31703	.0751	.481909
.19	.08252	.2092	.826959	.74	.32138	.0959	.477114
0.20	0.08686	1.2214	0.818731	0.75	0.32572	2.1170	0.472367
.21	.09120	.2337	.810584	.76	.33006	.1383	.467666
.22	.09554	.2461	.802519	.77	.33441	.1598	.463013
.23	.09989	.2586	.794534	.78	.33875	.1815	.458406
.24	.10423	.2712	.786628	.79	.34309	.2034	.453845
0.25	0.10857	1.2840	0.778801	0.80	0.34744	2.2255	0.449329
.26	.11292	.2969	.771052	.81	.35178	.2479	.444858
.27	.11726	.3100	.763379	.82	.35612	.2705	.440432
.28	.12160	.3231	.755784	.83	.36046	.2933	.436049
.29	.12595	.3364	.748264	.84	.36481	.3164	.431711
0.30	0.13029	1.3499	0.740818	0.85	0.36915	2.3396	0.427415
.31	.13463	.3634	.733447	.86	.37349	.3632	.423162
.32	.13897	.3771	.726149	.87	.37784	.3869	.418952
.33	.14332	.3910	.718924	.88	.38218	.4109	.414783
.34	.14766	.4049	.711770	.89	.38652	.4351	.410656
0.35	0.15200	1.4191	0.704688	0.90	0.39087	2.4596	0.406570
.36	.15635	.4333	.697676	.91	.39521	.4843	.402524
.37	.16069	.4477	.690734	.92	.39955	.5093	.398519
.38	.16503	.4623	.683861	.93	.40389	.5345	.394554
.39	.16937	.4770	.677057	.94	.40824	.5600	.390628
0.40	0.17372	1.4918	0.670320	0.95	0.41258	2.5857	0.386741
.41	.17806	.5068	.663650	.96	.41692	.6117	.382893
.42	.18240	.5220	.657047	.97	.42127	.6379	.379083
.43	.18675	.5373	.650509	.98	.42561	.6645	.375311
.44	.19109	.5527	.644036	.99	.42995	.6912	.371577
0.45	0.19543	1.5683	0.637628	1.00	0.43429	2.7183	0.367879
.46	.19978	.5841	.631284	.01	.43864	.7456	.364219
.47	.20412	.6000	.625002	.02	.44298	.7732	.360595
.48	.20846	.6161	.618783	.03	.44732	.8011	.357007
.49	.21280	.6323	.612626	.04	.45167	.8292	.353455
0.50	0.21715	1.6487	0.606531	1.05	0.45601	2.8577	0.349938
.51	.22149	.6653	.600496	.06	.46035	.8864	.346456
.52	.22583	.6820	.594521	.07	.46470	.9154	.343009
.53	.23018	.6989	.588605	.08	.46904	.9447	.339596
.54	.23452	.7160	.582748	.09	.47338	.9743	.336216

Table 6.[1]—*Exponential function*—Continued.

x	$\log_{10}(e^x)$	e^x	e^{-x}	x	$\log_{10}(e^x)$	e^x	e^{-x}
1.10	0.47772	3.0042	0.332871	1.65	0.71659	5.2070	0.192050
.11	.48207	.0344	.329559	.66	.72093	.2593	.190139
.12	.48641	.0649	.326280	.67	.72527	.3122	.188247
.13	.49075	.0957	.323033	.68	.72961	.3656	.186374
.14	.49510	.1268	.319819	.69	.73396	.4195	.184520
1.15	0.49944	3.1582	0.316637	1.70	0.73830	5.4739	0.182684
.16	.50378	.1899	.313486	.71	.74264	.5290	.180866
.17	.50812	.2220	.310367	.72	.74699	.5845	.179066
.18	.51247	.2544	.307279	.73	.75133	.6407	.177284
.19	.51681	.2871	.304221	.74	.75567	.6973	.175520
1.20	0.52115	3.3201	0.301194	1.75	0.76002	5.7546	0.173774
.21	.52550	.3535	.298197	.76	.76436	.8124	.172045
.22	.52984	.3872	.295230	.77	.76870	.8709	.170333
.23	.53418	.4212	.292293	.78	.77304	.9299	.168638
.24	.53853	.4556	.289384	.79	.77739	.9895	.166960
1.25	0.54287	3.4903	0.286505	1.80	0.78173	6.0496	0.165299
.26	.54721	.5254	.283654	.81	.78607	.1104	.163654
.27	.55155	.5609	.280832	.82	.79042	.1719	.162026
.28	.55590	.5966	.278037	.83	.79476	.2339	.160414
.29	.56024	.6328	.275271	.84	.79910	.2965	.158817
1.30	0.56458	3.6693	0.272532	1.85	0.80344	6.3598	0.157237
.31	.56893	.7062	.269820	.86	.80779	.4237	.155673
.32	.57327	.7434	.267135	.87	.81213	.4883	.154124
.33	.57761	.7810	.264477	.88	.81647	.5535	.152590
.34	.58195	.8190	.261846	.89	.82082	.6194	.151072
1.35	0.58630	3.8574	0.259240	1.90	0.82516	6.6859	0.149569
.36	.59064	.8962	.256661	.91	.82950	.7531	.148080
.37	.59498	.9354	.254107	.92	.83385	.8210	.146607
.38	.59933	.9749	.251579	.93	.83819	.8895	.145148
.39	.60367	4.0149	.249075	.94	.84253	.9588	.143704
1.40	0.60801	4.0552	0.246597	1.95	0.84687	7.0287	0.142274
.41	.61236	.0960	.244143	.96	.85122	.0993	.140858
.42	.61670	.1371	.241714	.97	.85556	.1707	.139457
.43	.62104	.1787	.239309	.98	.85990	.2427	.138069
.44	.62538	.2207	.236928	.99	.86425	.3155	.136695
1.45	0.62973	4.2631	0.234570	2.00	0.86859	7.3891	0.135335
.46	.63407	.3060	.232236	.01	.87293	.4633	.133989
.47	.63841	.3492	.229925	.02	.87727	.5383	.132655
.48	.64276	.3929	.227638	.03	.88162	.6141	.131336
.49	.64710	.4371	.225373	.04	.88596	.6906	.130029
1.50	0.65144	4.4817	0.223130	2.05	0.89030	7.7679	0.128735
.51	.65578	.5267	.220910	.06	.89465	.8460	.127454
.52	.66013	.5722	.218712	.07	.89899	.9248	.126186
.53	.66447	.6182	.216536	.08	.90333	8.0045	.124930
.54	.66881	.6646	.214381	.09	.90768	.0849	.123687
1.55	0.67316	4.7115	0.212248	2.10	0.91202	8.1662	0.122456
.56	.67750	.7588	.210136	.11	.91636	.2482	.121238
.57	.68184	.8066	.208045	.12	.92070	.3311	.120032
.58	.68619	.8550	.205975	.13	.92505	.4149	.118837
.59	.69053	.9037	.203926	.14	.92939	.4994	.117655
1.60	0.69487	4.9530	0.201897	2.15	0.93373	8.5849	0.116484
.61	.69921	5.0028	.199888	.16	.93808	.6711	.115325
.62	.70356	.0531	.197899	.17	.94242	.7583	.114178
.63	.70790	.1039	.195930	.18	.94676	.8463	.113042
.64	.71224	.1552	.193980	.19	.95110	.9352	.111917

TABLE 6.[1]—*Exponential function*—Continued.

x	$\log_{10}(e^x)$	e^x	e^{-x}	x	$\log_{10}(e^x)$	e^x	e^{-x}
2. 20	0. 95545	9. 0250	0. 110803	2. 75	1. 19431	15. 643	0. 063928
. 21	. 95979	. 1157	. 109701	. 76	. 19865	. 800	. 063292
. 22	. 96413	. 2073	. 108609	. 77	. 20300	. 959	. 062662
. 23	. 96848	. 2999	. 107528	. 78	. 20734	16. 119	. 062039
. 24	. 97282	. 3933	. 106459	. 79	. 21168	. 281	. 061421
2. 25	0. 97716	9. 4877	0. 105399	2. 80	1. 21602	16. 445	0. 060810
. 26	. 98151	. 5831	. 104350	. 81	. 22037	. 610	. 060205
. 27	. 98585	. 6794	. 103312	. 82	. 22471	. 777	. 059606
. 28	. 99019	. 7767	. 102284	. 83	. 22905	. 945	. 059013
. 29	. 99453	. 8749	. 101266	. 84	. 23340	17. 116	. 058426
2. 30	0. 99888	9. 9742	0. 100259	2. 85	1. 23774	17. 288	0. 057844
. 31	1. 00322	10. 074	. 099261	. 86	. 24208	. 462	. 057269
. 32	. 00756	. 176	. 098274	. 87	. 24643	. 637	. 056699
. 33	. 01191	. 278	. 097296	. 88	. 25077	. 814	. 056135
. 34	. 01625	. 381	. 096328	. 89	. 25511	. 993	. 055576
2. 35	1. 02059	10. 486	0. 095369	2. 90	1. 25945	18. 174	0. 055023
. 36	. 02493	. 591	. 094420	. 91	. 26380	. 357	. 054476
. 37	. 02928	. 697	. 093481	. 92	. 26814	. 541	. 053934
. 38	. 03362	. 805	. 092551	. 93	. 27248	. 728	. 053397
. 39	. 03796	. 913	. 091630	. 94	. 27683	. 916	. 052866
2. 40	1. 04231	11. 023	0. 090718	2. 95	1. 28117	19. 106	0. 052340
. 41	. 04665	. 134	. 089815	. 96	. 28551	. 298	. 051819
. 42	. 05099	. 246	. 088922	. 97	. 28985	. 492	. 051303
. 43	. 05534	. 359	. 088037	. 98	. 29420	. 688	. 050793
. 44	. 05968	. 473	. 087161	. 99	. 29854	. 886	. 050287
2. 45	1. 06402	11. 588	0. 086294	3. 00	1. 30288	20. 086	0. 049787
. 46	. 06836	. 705	. 085435	. 01	. 30723	. 287	. 049292
. 47	. 07271	. 822	. 084585	. 02	. 31157	. 491	. 048801
. 48	. 07705	. 941	. 083743	. 03	. 31591	. 697	. 048316
. 49	. 08139	12. 061	. 082910	. 04	. 32026	. 905	. 047835
2. 50	1. 08574	12. 182	0. 082085	3. 05	1. 32460	21. 115	0. 047359
. 51	. 09008	. 305	. 081268	. 06	. 32894	. 328	. 046888
. 52	. 09442	. 429	. 080460	. 07	. 33328	. 542	. 046421
. 53	. 09877	. 554	. 079659	. 08	. 33763	. 758	. 045959
. 54	. 10311	. 680	. 078866	. 09	. 34197	. 977	. 045502
2. 55	1. 10745	12. 807	0. 078082	3. 10	1. 34631	22. 198	0. 045049
. 56	. 11179	. 936	. 077305	. 11	. 35066	. 421	. 044601
. 57	. 11614	13. 066	. 076536	. 12	. 35500	. 646	. 044157
. 58	. 12048	. 197	. 075774	. 13	. 35934	. 874	. 043718
. 59	. 12482	. 330	. 075020	. 14	. 36368	23. 104	. 043283
2. 60	1. 12917	13. 464	0. 074274	3. 15	1. 36803	23. 336	0. 042852
. 61	. 13351	. 599	. 073535	. 16	. 37237	. 571	. 042426
. 62	. 13785	. 736	. 072803	. 17	. 37671	. 807	. 042004
. 63	. 14219	. 874	. 072078	. 18	. 38106	24. 047	. 041586
. 64	. 14654	14. 013	. 071361	. 19	. 38540	. 288	. 041172
2. 65	1. 15088	14. 154	0. 070651	3. 20	1. 38974	24. 533	0. 040762
. 66	. 15522	. 296	. 069948	. 21	. 39409	. 779	. 040357
. 67	. 15957	. 440	. 069252	. 22	. 39843	25. 028	0. 39955
. 68	. 16391	. 585	. 068563	. 23	. 40277	. 280	. 039557
. 69	. 16825	. 732	. 067881	. 24	. 40711	. 534	. 039164
2. 70	1. 17260	14. 880	0. 067206	3. 25	1. 41146	25. 790	0. 038774
. 71	. 17694	15. 029	. 066537	. 26	. 41580	26. 050	. 038388
. 72	. 18128	. 180	. 065875	. 27	. 42014	. 311	. 038006
. 73	. 18562	. 333	. 065219	. 28	. 42449	. 576	. 037628
. 74	. 18997	. 487	. 064570	. 29	. 42883	. 843	. 037254

TABLE 6.[1]—*Exponential function*—Continued.

x	$\log_{10}(\epsilon^x)$	ϵ^x	ϵ^{-x}	x	$\log_{10}(\epsilon^x)$	ϵ^x	ϵ^{-x}
3.30	1.43317	27.113	0.036883	3.85	1.67203	46.993	0.021280
.31	.43751	.385	.036516	.86	.67638	47.465	.021068
.32	.44186	.660	.036153	.87	.68072	.942	.020858
.33	.44620	.938	.035793	.88	.68506	48.424	.020651
.34	.45054	28.219	.035437	.89	.68941	.911	.020445
3.35	1.45489	28.503	0.035084	3.90	1.69375	49.402	0.020242
.36	.45923	.789	.034735	.91	.69809	.899	.020041
.37	.46357	29.079	.034390	.92	.70243	50.400	.019841
.38	.46792	.371	.034047	.93	.70678	.907	.019644
.39	.47226	.666	.033709	.94	.71112	51.419	.019448
3.40	1.47660	29.964	0.033373	3.95	1.71546	51.935	0.019255
.41	.48094	30.265	.033041	.96	.71981	52.457	.019063
.42	.48529	.569	.032712	.97	.72415	.985	.018873
.43	.48963	.877	.032387	.98	.72849	53.517	.018686
.44	.49397	31.187	.032065	.99	.73283	54.055	.018500
3.45	1.49832	31.500	0.031746	4.00	1.73718	54.598	0.018316
.46	.50266	.817	.031430	.01	.74152	55.147	.018133
.47	.50700	32.137	.031117	.02	.74586	.701	.017953
.48	.51134	.460	.030807	.03	.75021	56.261	.017774
.49	.51569	.786	.030501	.04	.75455	.826	.017597
3.50	1.52003	33.115	0.030197	4.05	1.75889	57.397	0.017422
.51	.52437	.448	.029897	.06	.76324	.974	.017249
.52	.52872	.784	.029599	.07	.76758	58.557	.017077
.53	.53306	34.124	.029305	.08	.77192	59.145	.016907
.54	.53740	.467	.029013	.09	.77626	.740	.016739
3.55	1.54175	34.813	0.028725	4.10	1.78061	60.340	0.016573
.56	.54609	35.163	.028439	.11	.78495	.947	.016408
.57	.55043	.517	.028156	.12	.78929	61.559	.016245
.58	.55477	.874	.027876	.13	.79364	62.178	.016083
.59	.55912	36.234	.027598	.14	.79798	.803	.015923
3.60	1.56346	36.598	0.027324	4.15	1.80232	63.434	0.015764
.61	.56780	.966	.027052	.16	.80667	64.072	.015608
.62	.57215	37.338	.026783	.17	.81101	.715	.015452
.63	.57649	.713	.026516	.18	.81535	65.366	.015299
.64	.58083	38.092	.026252	.19	.81969	66.023	.015146
3.65	1.58517	38.475	0.025991	4.20	1.82404	66.686	0.014996
.66	.58952	.861	.025733	.21	.82838	67.357	.014846
.67	.59386	39.252	.025476	.22	.83272	68.033	.014699
.68	.59820	.646	.025223	.23	.83707	.717	.014552
.69	.60255	40.045	.024972	.24	.84141	69.408	.014408
3.70	1.60689	40.447	0.024724	4.25	1.84575	70.105	0.014264
.71	.61123	.854	.024478	.26	.85009	.810	.014122
.72	.61558	41.264	.024234	.27	.85444	71.522	.013982
.73	.61992	.679	.023993	.28	.85878	72.240	.013843
.74	.62426	42.098	.023754	.29	.86312	.966	.013705
3.75	1.62860	42.521	0.023518	4.30	1.86747	73.700	0.013569
.76	.63295	.948	.023284	.31	.87181	74.440	.013434
.77	.63729	43.380	.023052	.32	.87615	75.189	.013300
.78	.64163	.816	.022823	.33	.88050	.944	.013168
.79	.64598	44.256	.022596	.34	.88484	76.708	.013037
3.80	1.65032	44.701	0.022371	4.35	1.88918	77.478	0.012907
.81	.65466	45.150	.022148	.36	.89352	78.257	.012778
.82	.65900	.604	.021928	.37	.89787	79.044	.012651
.83	.66335	46.063	.021710	.38	.90221	79.838	.012525
.84	.66769	.525	.021494	.39	.90655	80.640	.012401

TABLE 6.[1]—*Exponential function*—Continued.

x	$\log_{10}(e^x)$	e^x	e^{-x}	x	$\log_{10}(e^x)$	e^x	e^{-x}
4. 40	1. 91090	81. 451	0. 012277	4. 95	2. 14976	141. 17	0. 007083
. 41	. 91524	82. 269	. 012155	. 96	. 15410	142. 59	. 007013
. 42	. 91958	83. 096	. 012034	. 97	. 15844	144. 03	. 006943
. 43	. 92392	. 931	. 011914	. 98	. 16279	145. 47	. 006874
. 44	. 92827	84. 775	. 011796	. 99	. 16713	146. 94	. 006806
4. 45	1. 93261	85. 627	0. 011679	5. 00	2. 17147	148. 41	0. 006738
. 46	. 93695	86. 488	. 011562	. 01	. 17582	149. 90	. 006671
. 47	. 94130	87. 357	. 011447	. 02	. 18016	151. 41	. 006605
. 48	. 94564	88. 235	. 011333	. 03	. 18450	152. 93	. 006539
. 49	. 94998	89. 121	. 011221	. 04	. 18884	154. 47	. 006474
4. 50	1. 95433	90. 017	0. 011109	5. 05	2. 19319	156. 02	0. 006409
. 51	. 95867	. 922	. 010998	. 06	. 19753	157. 59	. 006346
. 52	. 96301	91. 836	. 010889	. 07	. 20187	159. 17	. 006282
. 53	. 96735	92. 759	. 010781	. 08	. 20622	160. 77	. 006220
. 54	. 97170	93. 691	. 010673	. 09	. 21056	162. 39	. 006158
4. 55	1. 97604	94. 632	0. 010567	5. 10	2. 21490	164. 02	0. 006097
. 56	. 98038	95. 583	. 010462	. 11	. 21924	165. 67	. 006036
. 57	. 98473	96. 544	. 010358	. 12	. 22359	167. 34	. 005976
. 58	. 98907	97. 514	. 010255	. 13	. 22793	169. 02	. 005917
. 59	. 99341	98. 494	. 010153	. 14	. 23227	170. 72	. 005858
4. 60	1. 99775	99. 484	0. 010052	5. 15	2. 23662	172. 43	0. 005799
. 61	2. 00210	100. 48	. 009952	. 16	. 24096	174. 16	. 005742
. 62	. 00644	101. 49	. 009853	. 17	. 24530	175. 91	. 005685
. 63	. 01078	102. 51	. 009755	. 18	. 24965	177. 68	. 005628
. 64	. 01513	103. 54	. 009658	. 19	. 25399	179. 47	. 005572
4. 65	2. 01947	104. 58	0. 009562	5. 20	2. 25833	181. 27	0. 005517
. 66	. 02381	105. 64	. 009466	. 21	. 26267	183. 09	. 005462
. 67	. 02816	106. 70	. 009372	. 22	. 26702	184. 93	. 005407
. 68	. 03250	107. 77	. 009279	. 23	. 27136	186. 79	. 005354
. 69	. 03684	108. 85	. 009187	. 24	. 27570	188. 67	. 005300
4. 70	2. 04118	109. 95	0. 009095	5. 25	2. 28005	190. 57	0. 005248
. 71	. 04553	111. 05	. 009005	. 26	. 28439	192. 48	. 005195
. 72	. 04987	112. 17	. 008915	. 27	. 28873	194. 42	. 005144
. 73	. 05421	113. 30	. 008826	. 28	. 29307	196. 37	. 005092
. 74	. 05856	114. 43	. 008739	. 29	. 29742	198. 34	. 005042
4. 75	2. 06290	115. 58	0. 008652	5. 30	2. 30176	200. 34	0. 004992
. 76	. 06724	116. 75	. 008566	. 31	. 30610	202. 35	. 004942
. 77	. 07158	117. 92	. 008480	. 32	. 31045	204. 38	. 004893
. 78	. 07593	119. 10	. 008396	. 33	. 31479	206. 44	. 004844
. 79	. 08027	120. 30	. 008312	. 34	. 31913	208. 51	. 004796
4. 80	2. 08461	121. 51	0. 008230	5. 35	2. 32348	210. 61	0. 004748
. 81	. 08896	122. 73	. 008148	. 36	. 32782	212. 72	. 004701
. 82	. 09330	123. 97	. 008067	. 37	. 33216	214. 86	. 004654
. 83	. 09764	125. 21	. 007987	. 38	. 33650	217. 02	. 004608
. 84	. 10199	126. 47	. 007907	. 39	. 34085	219. 20	. 004562
4. 85	2. 10633	127. 74	0. 007828	5. 40	2. 34519	221. 41	0. 004517
. 86	. 11067	129. 02	. 007750	. 41	. 34953	223. 63	. 004472
. 87	. 11501	130. 32	. 007673	. 42	. 35388	225. 88	. 004427
. 88	. 11936	131. 63	. 007597	. 43	. 35822	228. 15	. 004383
. 89	. 12370	132. 95	. 007521	. 44	. 36256	230. 44	. 004339
4. 90	2. 12804	134. 29	0. 007447	5. 45	2. 36690	232. 76	0. 004296
. 91	. 13239	135. 64	. 007372	. 46	. 37125	235. 10	. 004254
. 92	. 13673	137. 00	. 007299	. 47	. 37559	237. 46	. 004211
. 93	. 14107	138. 38	. 007227	. 48	. 37993	239. 85	. 004169
. 94	. 14541	139. 77	. 007155	. 49	. 38428	242. 26	. 004128

TABLE 6.[1]—*Exponential function*—Continued.

x	$\log_{10}(e^x)$	e^x	e^{-x}	x	$\log_{10}(e^x)$	e^x	e^{-x}
5.50	2.38862	244.69	0.004087	7.5	3.25721	1,808.0	0.000553
				.6	.30064	1,998.2	.000500
5.0	2.17147	148.41	0.006738	.7	.34407	2,208.3	.000453
.1	.21490	164.02	.006097	.8	.38750	2,440.6	.000410
.2	.25833	181.27	.005517	.9	.43093	2,697.3	.000371
.3	.30176	200.34	.004992				
.4	.34519	221.41	.004517	8.0	3.47436	2,981.0	0.000335
				.1	.51779	3,294.5	.000304
5.5	2.38862	244.69	0.004087	.2	.56121	3,641.0	.000275
.6	.43205	270.43	.003698	.3	.60464	4,023.9	.000249
.7	.47548	298.87	.003346	.4	.64807	4,447.1	.000225
.8	.51891	330.30	.003028				
.9	.56234	365.04	.002739	8.5	3.69150	4,914.8	0.000203
				.6	.73493	5,431.7	.000184
6.0	2.60577	403.43	0.002479	.7	.77836	6,002.9	.000167
.1	.64920	445.86	.002243	.8	.82179	6,634.2	.000151
.2	.69263	492.75	.002029	.9	.86522	7,332.0	.000136
.3	.73606	544.57	.001836				
.4	.77948	601.85	.001662	9.0	3.90865	8,103.1	0.000123
				.1	.95208	8,955.3	.000112
6.5	2.82291	665.14	0.001503	.2	.99551	9,897.1	.000101
.6	.86634	735.10	.001360	.3	4.03894	10,938	.000091
.7	.90977	812.41	.001231	.4	.08237	12,088	.000083
.8	.95320	897.85	.001114				
.9	.99663	992.27	.001008	9.5	4.12580	13,360	0.000075
				.6	.16923	14,765	.000068
7.0	3.04006	1,096.6	0.000912	.7	.21266	16,318	.000061
.1	.08349	1,212.0	.000825	.8	.25609	18,034	.000055
.2	.12692	1,339.4	.000747	.9	.29952	19,930	.000050
.3	.17035	1,480.3	.000676				
.4	.21378	1,636.0	.000611	10.0	4.34294	22,026	0.000045

TABLE 6.[2]—*Exponential functions.*

Value of e^{x^2} and e^{-x^2} and their logarithms.

x	e^{x^2}	$\log e^{x^2}$	e^{-x^2}	$\log e^{-x^2}$
0.1	1.0101	0.00434	0.99005	1.99566
2	1.0408	01737	96079	98263
3	1.0942	03909	91393	96091
4	1.1735	06949	85214	93051
5	1.2840	10857	77880	89143
0.6	1.4333	0.15635	0.69768	1.84365
7	1.6323	21280	61263	78720
8	1.8965	27795	52729	72205
9	2.2479	35178	44486	64822
1.0	2.7183	43429	36788	56571
1.1	3.3535	0.52550	0.29820	1.47450
2	4.2207	62538	23693	37462
3	5.4195	73396	18452	26604
4	7.0993	85122	14086	14878
5	9.4877	97716	10540	02284
1.6	1.2936×10	1.11179	0.77305×10^{-1}	2.88821
7	1.7993 "	25511	55576 "	74489
8	2.5534 "	40711	39164 "	59289
9	3.6966 "	56780	27052 "	43220
2.0	5.4598 "	73718	18316 "	26282
2.1	8.2269 "	1.91524	0.12155 "	2.08476
2	1.2647×10^{2}	2.10199	79071×10^{-2}	3.89801
3	1.9834 "	29472	50418 "	70258
4	3.1735 "	50154	31511 "	49846
5	5.1801 "	71434	19305 "	28566
2.6	8.6264 "	2.93583	0.11592 "	3.06417
7	1.4656×10^{3}	3.16601	68233×10^{-3}	4.83399
8	2.5402 "	40487	39367 "	59513
9	4.4918 "	65242	22263 "	34758
3.0	8.1031 "	90865	12341 "	09135
3.1	1.4913×10^{4}	4.17357	0.67055×10^{-4}	5.82643
2	2.8001 "	44718	35713 "	55282
3	5.3637 "	72947	18644 "	27053
4	1.0482×10^{5}	5.02044	95402×10^{-5}	6.97956
5	2.0898 "	32011	47851 "	67989
3.6	4.2507 "	5.62846	0.23526 "	6.37154
7	8.8205 "	94549	11337 "	05451
8	1.8673×10^{6}	6.27121	53553×10^{-6}	7.72879
9	4.0329 "	60562	24796 "	39438
4.0	8.8861 "	94871	11254 "	05129
4.1	1.9975×10^{7}	7.30049	0.50062×10^{-7}	8.69951
2	4.5809 "	66095	21830 "	33905
3	1.0718×10^{8}	8.03010	93303×10^{-8}	9.96990
4	2.5582 "	40794	39089 "	59206
5	6.2296 "	79446	16052 "	20554
4.6	1.5476×10^{9}	9.18967	0.64614×10^{-9}	10.81033
7	3.9225 "	59357	25494 "	40643
8	1.0142×10^{10}	10.00614	98595×10^{-10}	11.99386
9	2.6755 "	42741	37376 "	57259
5.0	7.2005 "	85736	13888 "	14264

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TABLE 6.[3]—*Exponential functions.*

Values of $e^{\frac{\pi}{4}x}$ and $e^{-\frac{\pi}{4}x}$ and their logarithms.

x	$e^{\frac{\pi}{4}x}$	$\log e^{\frac{\pi}{4}x}$	$e^{-\frac{\pi}{4}x}$	$\log e^{-\frac{\pi}{4}x}$
1	2.1933	0.34109	0.45594	1.65891
2	4.8105	.68219	.20788	.31781
3	1.0551×10	1.02328	.94780×10⁻¹	2.97672
4	2.3141 "	.36438	.43214 "	.63562
5	5.0754 "	.70547	.19703 "	.29453
6	1.1132×10²	2.04656	0.89833×10⁻²	3.95344
7	2.4415 "	.38766	.40958 "	.61234
8	5.3549 "	.72875	.18674 "	.27125
9	1.1745×10³	3.06985	.85144×10⁻³	4.93015
10	2.5760 "	.41094	.38820 "	.58906
11	5.6498 "	3.75203	0.17700 "	4.24797
12	1.2392×10⁴	4.09313	.80700×10⁻⁴	5.90687
13	2.7178 "	.43422	.36794 "	.56578
14	5.9610 "	.77532	.16776 "	.22468
15	1.3074×10⁵	5.11641	.76487×10⁻⁵	6.88359
16	2.8675 "	5.45751	0.34873 "	6.54249
17	6.2893 "	.79860	.15900 "	.20140
18	1.3794×10⁶	6.13969	.72495×10⁻⁶	7.86031
19	3.0254 "	.48079	.33053 "	.51921
20	6.6356 "	.82188	.15070 "	.17812

TABLE 6.[4]—*Exponential functions.*

Values of $e^{\frac{\sqrt{\pi}}{4}x}$ and $e^{-\frac{\sqrt{\pi}}{4}x}$ and their logarithms.

x	$e^{\frac{\sqrt{\pi}}{4}x}$	$\log e^{\frac{\sqrt{\pi}}{4}x}$	$e^{-\frac{\sqrt{\ }}{4}x}$	$\log e^{-\frac{\sqrt{\pi}}{4}x}$
1	1.5576	0.19244	0.64203	1.80756
2	2.4260	.38488	.41221	.61512
3	3.7786	.57733	.26465	.42267
4	5.8853	.76977	.16992	.23023
5	9.1666	.96221	.10909	.03779
6	14.277	1.15465	0.070041	2.84535
7	22.238	.34709	.044968	.65291
8	34.636	.53953	.028871	.46047
9	53.948	.73198	.018536	.26802
10	84.027	.92442	.011901	.07558
11	130.88	2.11686	0.0076408	3.88314
12	203.85	.30930	.0049057	.69070
13	317.50	.50174	.0031496	.49826
14	494.52	.69418	.0020222	.30582
15	770.24	.88663	.0012983	.11337
16	1,199.7	3.07907	0.00083355	4.92093
17	1,868.6	.27151	.00053517	.72849
18	2,910.4	.46395	.00034360	.53605
19	4,533.1	.65639	.00022060	.34361
20	7,060.5	.84883	.00014163	.15117

TABLE 6.[5]—*Exponential functions.*

Value of e^x and e^{-x} and their logarithms.

x	e^x	$\log e^x$	e^{-x}	x	e^x	$\log e^x$	e^{-x}
$\frac{1}{64}$	1.0157	0.00679	0.98450	$\frac{1}{3}$	1.3956	0.14476	0.71653
$\frac{1}{32}$	.0317	.01357	.96923	$\frac{1}{2}$	.6487	.21715	.60653
$\frac{1}{16}$	.0645	.02714	.93941	$\frac{3}{4}$	2.1170	.32572	.47237
$\frac{1}{10}$	.1052	.04343	.90484	1	.7183	.43429	.36788
$\frac{1}{9}$	.1175	.04825	.89484	$\frac{5}{4}$	3.4903	.54287	.28650
$\frac{1}{8}$	1.1331	0.05429	0.88250	$\frac{3}{2}$	4.4817	0.65144	0.22313
$\frac{1}{7}$	.1536	.06204	.86688	$\frac{7}{4}$	5.7546	.76002	.17377
$\frac{1}{6}$	.1814	.07238	.84648	2	7.3891	.86859	.13534
$\frac{1}{5}$	.2214	.08686	.81873	$\frac{9}{4}$	9.4877	.97716	.10540
$\frac{1}{4}$	.2840	.10857	.77880	$\frac{5}{2}$	12.1825	1.08574	.08208

TABLE 6.[6]—*Least squares.*

Values of $P = \frac{2}{\sqrt{\pi}} \int_0^{hx} e^{-(hx)^2} d(hx)$.

This table gives the value of P, the probability of an observational error having a value positive or negative equal to or less than x when h is the measure of precision, $P = \frac{2}{\sqrt{\pi}} \int_0^{hx} e^{-(hx)^2} d(hx)$.

hx	0	1	2	3	4	5	6	7	8	9
0.0	....	.01128	.02256	.03384	.04511	.05637	.06762	.07886	.09008	.10128
.1	.11246	.12362	.13476	.14587	.15695	.16800	.17901	.18999	.20094	.21184
.2	.22270	.23352	.24430	.25502	.26570	.27633	.28690	.29742	.30788	.31828
.3	.32863	.33891	.34913	.35928	.36936	.37938	.38933	.39921	.40901	.41874
.4	.42839	.43797	.44747	.45689	.46623	.47548	.48466	.49375	.50275	.51167
0.5	.52050	.52924	.53790	.54646	.55494	.56332	.57162	.57982	.58792	.59594
.6	.60386	.61168	.61941	.62705	.63459	.64203	.64938	.65663	.66378	.67084
.7	.67780	.68467	.69143	.69810	.70468	.71116	.71754	.72382	.73001	.73610
.8	.74210	.74800	.75381	.75952	.76514	.77067	.77610	.78144	.78669	.79184
.9	.79691	.80188	.80677	.81156	.81627	.82089	.82542	.82987	.83423	.83851
1.0	.84270	.84681	.85084	.85478	.85865	.86244	.86614	.86977	.87333	.87680
.1	.88021	.88353	.88679	.88997	.89308	.89612	.89910	.90200	.90484	.90761
.2	.91031	.91296	.91553	.91805	.92051	.92290	.92524	.92751	.92973	.93190
.3	.93401	.93606	.93807	.94002	.94191	.94376	.94556	.94731	.94902	.95067
.4	.95229	.95385	.95538	.95686	.95830	.95970	.96105	.96237	.96365	.96490
1.5	.96611	.96728	.96841	.96952	.97059	.97162	.97263	.97360	.97455	.97546
.6	.97635	.97721	.97804	.97884	.97962	.98038	.98110	.98181	.98249	.98315
.7	.98379	.98441	.98500	.98558	.98613	.98667	.98719	.98769	.98817	.98864
.8	.98909	.98952	.98994	.99035	.99074	.99111	.99147	.99182	.99216	.99248
.9	.99279	.99309	.99338	.99366	.99392	.99418	.99443	.99466	.99489	.99511
2.0	.99532	.99552	.99572	.99591	.99609	.99626	.99642	.99658	.99673	.99688
.1	.99702	.99715	.99728	.99741	.99753	.99764	.99775	.99785	.99795	.99805
.2	.99814	.99822	.99831	.99839	.99846	.99854	.99861	.99867	.99874	.99880
.3	.99886	.99891	.99897	.99902	.99906	.99911	.99915	.99920	.99924	.99928
.4	.99931	.99935	.99938	.99941	.99944	.99947	.99950	.99952	.99955	.99957
2.5	.99959	.99961	.99963	.99965	.99967	.99969	.99971	.99972	.99974	.99975
.6	.99976	.99978	.99979	.99980	.99981	.99982	.99983	.99984	.99985	.99986
.7	.99987	.99987	.99988	.99989	.99989	.99990	.99991	.99991	.99992	.99992
.8	.99992	.99993	.99993	.99994	.99994	.99994	.99995	.99995	.99995	.99996
.9	.99996	.99996	.99996	.99997	.99997	.99997	.99997	.99997	.99997	.99998
3.0	.99998	.99999	.99999	1.00000						

TABLE 7.—*Logarithms of numbers from 1 to 100.*

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0.000000	21	1.322219	41	1.612784	61	1.785330	81	1.908485
2	0.301030	22	1.342423	42	1.623249	62	1.792392	82	1.913814
3	0.477121	23	1.361728	43	1.633468	63	1.799341	83	1.919078
4	0.602060	24	1.380211	44	1.643453	64	1.806180	84	1.924279
5	0.698970	25	1.397940	45	1.653213	65	1.812913	85	1.929419
6	0.778151	26	1.414973	46	1.662758	66	1.819544	86	1.934498
7	0.845098	27	1.431364	47	1.672098	67	1.826075	87	1.939519
8	0.903090	28	1.447158	48	1.681241	68	1.832509	88	1.944483
9	0.954243	29	1.462398	49	1.690196	69	1.838849	89	1.949390
10	1.000000	30	1.477121	50	1.698970	70	1.845098	90	1.954243
11	1.041393	31	1.491362	51	1.707570	71	1.851258	91	1.959041
12	1.079181	32	1.505150	52	1.716003	72	1.857332	92	1.963788
13	1.113943	33	1.518514	53	1.724276	73	1.863323	93	1.968483
14	1.146128	34	1.531479	54	1.732394	74	1.869232	94	1.973128
15	1.176091	35	1.544068	55	1.740363	75	1.875061	95	1.977724
16	1.204120	36	1.556303	56	1.748188	76	1.880814	96	1.982271
17	1.230449	37	1.568202	57	1.755875	77	1.886491	97	1.986772
18	1.255273	38	1.579784	58	1.763428	78	1.892095	98	1.991226
19	1.278754	39	1.591065	59	1.770852	79	1.897627	99	1.995635
20	1.301030	40	1.602060	60	1.778151	80	1.903090	100	2.000000

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N.	0	1	2	3	4	5	6	7	8	9	Diff.
100	000000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
1	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
2	8600	9026	9451	9876							
					0300	0724	1147	1570	1993	2415	424
3	012837	3259	3680	4100	4521	4940	5360	5779	6197	6616	420
4	7033	7451	7868	8284	8700	9116	9532	9947			
									0361	0775	416
5	021189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
6	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
7	9384	9789									
			0195	0600	1004	1408	1812	2216	2619	3021	404
8	033424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
9	7426	7825	8223	8620	9017	9414	9811				
	04							0207	0602	0998	397

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
434	43.4	86.8	130.2	173.6	217.0	260.4	303.8	347.2	390.6
433	43.3	86.6	129.9	173.2	216.5	259.8	303.1	346.4	389.7
432	43.2	86.4	129.6	172.8	216.0	259.2	302.4	345.6	388.8
431	43.1	86.2	129.3	172.4	215.5	258.6	301.7	344.8	387.9
430	43.0	86.0	129.0	172.0	215.0	258.0	301.0	344.0	387.0
429	42.9	85.8	128.7	171.6	214.5	257.4	300.3	343.2	386.1
428	42.8	85.6	128.4	171.2	214.0	256.8	299.6	342.4	385.2
427	42.7	85.4	128.1	170.8	213.5	256.2	298.9	341.6	384.3
426	42.6	85.2	127.8	170.4	213.0	255.6	298.2	340.8	383.4
425	42.5	85.0	127.5	170.0	212.5	255.0	297.5	340.0	382.5
424	42.4	84.8	127.2	169.6	212.0	254.4	296.8	339.2	381.6
423	42.3	84.6	126.9	169.2	211.5	253.8	296.1	338.4	380.7
422	42.2	84.4	126.6	168.8	211.0	253.2	295.4	337.6	379.8
421	42.1	84.2	126.3	168.4	210.5	252.6	294.7	336.8	378.9
420	42.0	84.0	126.0	168.0	210.0	252.0	294.0	336.0	378.0
419	41.9	83.8	125.7	167.6	209.5	251.4	293.3	335.2	377.1
418	41.8	83.6	125.4	167.2	209.0	250.8	292.6	334.4	376.2
417	41.7	83.4	125.1	166.8	208.5	250.2	291.9	333.6	375.3
416	41.6	83.2	124.8	166.4	208.0	249.6	291.2	332.8	374.4
415	41.5	83.0	124.5	166.0	207.5	249.0	290.5	332.0	373.5
414	41.4	82.8	124.2	165.6	207.0	248.4	289.8	331.2	372.6
413	41.3	82.6	123.9	165.2	206.5	247.8	289.1	330.4	371.7
412	41.2	82.4	123.6	164.8	206.0	247.2	288.4	329.6	370.8
411	41.1	82.2	123.3	164.4	205.5	246.6	287.7	328.8	369.9
410	41.0	82.0	123.0	164.0	205.0	246.0	287.0	328.0	369.0
409	40.9	81.8	122.7	163.6	204.5	245.4	286.3	327.2	368.1
408	40.8	81.6	122.4	163.2	204.0	244.8	285.6	326.4	367.2
407	40.7	81.4	122.1	162.8	203.5	244.2	284.9	325.6	366.3
406	40.6	81.2	121.8	162.4	203.0	243.6	284.2	324.8	365.4
405	40.5	81.0	121.5	162.0	202.5	243.0	283.5	324.0	364.5
404	40.4	80.8	121.2	161.6	202.0	242.4	282.8	323.2	363.6
403	40.3	80.6	120.9	161.2	201.5	241.8	282.1	322.4	362.7
402	40.2	80.4	120.6	160.8	201.0	241.2	281.4	321.6	361.8
401	40.1	80.2	120.3	160.4	200.5	240.6	280.7	320.8	360.9
400	40.0	80.0	120.0	160.0	200.0	240.0	280.0	320.0	360.0
399	39.9	79.8	119.7	159.6	199.5	239.4	279.3	319.2	359.1
398	39.8	79.6	119.4	159.2	199.0	238.8	278.6	318.4	358.2
397	39.7	79.4	119.1	158.8	198.5	238.2	277.9	317.6	357.3
396	39.6	79.2	118.8	158.4	198.0	237.6	277.2	316.8	356.4
395	39.5	79.0	118.5	158.0	197.5	237.0	276.5	316.0	355.5

No. 110 L. 041.] [No. 119 L. 078.

N.	0	1	2	3	4	5	6	7	8	9	Diff
110	041393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
1	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
2	9218	9606	9993	0380	0766	1153	1538	1924	2309	2694	386
3	053078	3463	3846	4230	4613	4996	5378	5760	6142	6524	383
4	6905	7286	7666	8046	8426	8805	9185	9563	9942	0320	379
5	060698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
6	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
7	8186	8557	8928	9298	9668	0038	0407	0776	1145	1514	370
8	071882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
9	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
395	39.5	79.0	118.5	158.0	197.5	237.0	276.5	316.0	355.5
394	39.4	78.8	118.2	157.6	197.0	236.4	275.8	315.2	354.6
393	39.3	78.6	117.9	157.2	196.5	235.8	275.1	314.4	353.7
392	39.2	78.4	117.6	156.8	196.0	235.2	274.4	313.6	352.8
391	39.1	78.2	117.3	156.4	195.5	234.6	273.7	312.8	351.9
390	39.0	78.0	117.0	156.0	195.0	234.0	273.0	312.0	351.0
389	38.9	77.8	116.7	155.6	194.5	233.4	272.3	311.2	350.1
388	38.8	77.6	116.4	155.2	194.0	232.8	271.6	310.4	349.2
387	38.7	77.4	116.1	154.8	193.5	232.2	270.9	309.6	348.3
386	38.6	77.2	115.8	154.4	193.0	231.6	270.2	308.8	347.4
385	38.5	77.0	115.5	154.0	192.5	231.0	269.5	308.0	346.5
384	38.4	76.8	115.2	153.6	192.0	230.4	268.8	307.2	345.6
383	38.3	76.6	114.9	153.2	191.5	229.8	268.1	306.4	344.7
382	38.2	76.4	114.6	152.8	191.0	229.2	267.4	305.6	343.8
381	38.1	76.2	114.3	152.4	190.5	228.6	266.7	304.8	342.9
380	38.0	76.0	114.0	152.0	190.0	228.0	266.0	304.0	342.0
379	37.9	75.8	113.7	151.6	189.5	227.4	265.3	303.2	341.1
378	37.8	75.6	113.4	151.2	189.0	226.8	264.6	302.4	340.2
377	37.7	75.4	113.1	150.8	188.5	226.2	263.9	301.6	339.3
376	37.6	75.2	112.8	150.4	188.0	225.6	263.2	300.8	338.4
375	37.5	75.0	112.5	150.0	187.5	225.0	262.5	300.0	337.5
374	37.4	74.8	112.2	149.6	187.0	224.4	261.8	299.2	336.6
373	37.3	74.6	111.9	149.2	186.5	223.8	261.1	298.4	335.7
372	37.2	74.4	111.6	148.8	186.0	223.2	260.4	297.6	334.8
371	37.1	74.2	111.3	148.4	185.5	222.6	259.7	296.8	333.9
370	37.0	74.0	111.0	148.0	185.0	222.0	259.0	296.0	333.0
369	36.9	73.8	110.7	147.6	184.5	221.4	258.3	295.2	332.1
368	36.8	73.6	110.4	147.2	184.0	220.8	257.6	294.4	331.2
367	36.7	73.4	110.1	146.8	183.5	220.2	256.9	293.6	330.3
366	36.6	73.2	109.8	146.4	183.0	219.6	256.2	292.8	329.4
365	36.5	73.0	109.5	146.0	182.5	219.0	255.5	292.0	328.5
364	36.4	72.8	109.2	145.6	182.0	218.4	254.8	291.2	327.6
363	36.3	72.6	108.9	145.2	181.5	217.8	254.1	290.4	326.7
362	36.2	72.4	108.6	144.8	181.0	217.2	253.4	289.6	325.8
361	36.1	72.2	108.3	144.4	180.5	216.6	252.7	288.8	324.9
360	36.0	72.0	108.0	144.0	180.0	216.0	252.0	288.0	324.0
359	35.9	71.8	107.7	143.6	179.5	215.4	251.3	287.2	323.1
358	35.8	71.6	107.4	143.2	179.0	214.8	250.6	286.4	322.2
357	35.7	71.4	107.1	142.8	178.5	214.2	249.9	285.6	321.3
356	35.6	71.2	106.8	142.4	178.0	213.6	249.2	284.8	320.4

No. 120 L. 079.] [No. 134 L. 130.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
120	079181	9543	9904								
				0266	0626	0987	1347	1707	2067	2426	460
1	082785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
2	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
3	9905										
		0258	0611	0963	1315	1667	2018	2370	2721	3071	352
4	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
5	6910	7257	7604	7951	8298	8644	8990	9335	9681		
										0026	346
6	100371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
7	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
8	7210	7549	7888	8227	8565	8903	9241	9579	9916		
										0253	338
9	110590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	3943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
1	7271	7603	7934	8265	8595	8926	9256	9586	9915		
										0245	330
2	120574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
3	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
4	7105	7429	7753	8076	8399	8722	9045	9368	9690		
	13									0012	323

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
355	35.5	71.0	106.5	142.0	177.5	213.0	248.5	284.0	319.5
354	35.4	70.8	106.2	141.6	177.0	212.4	247.8	283.2	318.6
353	35.3	70.6	105.9	141.2	176.5	211.8	247.1	282.4	317.7
352	35.2	70.4	105.6	140.8	176.0	211.2	246.4	281.6	316.8
351	35.1	70.2	105.3	140.4	175.5	210.6	245.7	280.8	315.9
350	35.0	70.0	105.0	140.0	175.0	210.0	245.0	280.0	315.0
349	34.9	69.8	104.7	139.6	174.5	209.4	244.3	279.2	314.1
348	34.8	69.6	104.4	139.2	174.0	208.8	243.6	278.4	313.2
347	34.7	69.4	104.1	138.8	173.5	208.2	242.9	277.6	312.3
346	34.6	69.2	103.8	138.4	173.0	207.6	242.2	276.8	311.4
345	34.5	69.0	103.5	138.0	172.5	207.0	241.5	276.0	310.5
344	34.4	68.8	103.2	137.6	172.0	206.4	240.8	275.2	309.6
343	34.3	68.6	102.9	137.2	171.5	205.8	240.1	274.4	308.7
342	34.2	68.4	102.6	136.8	171.0	205.2	239.4	273.6	307.8
341	34.1	68.2	102.3	136.4	170.5	204.6	238.7	272.8	306.9
340	34.0	68.0	102.0	136.0	170.0	204.0	238.0	272.0	306.0
339	33.9	67.8	101.7	135.6	169.5	203.4	237.3	271.2	305.1
338	33.8	67.6	101.4	135.2	169.0	202.8	236.6	270.4	304.2
337	33.7	67.4	101.1	134.8	168.5	202.2	235.9	269.6	303.3
336	33.6	67.2	100.8	134.4	168.0	201.6	235.2	268.8	302.4
335	33.5	67.0	100.5	134.0	167.5	201.0	234.5	268.0	301.5
334	33.4	66.8	100.2	133.6	167.0	200.4	233.8	267.2	300.6
333	33.3	66.6	99.9	133.2	166.5	199.8	233.1	266.4	299.7
332	33.2	66.4	99.6	132.8	166.0	199.2	232.4	265.6	298.8
331	33.1	66.2	99.3	132.4	165.5	198.6	231.7	264.8	297.9
330	33.0	66.0	99.0	132.0	165.0	198.0	231.0	264.0	297.0
329	32.9	65.8	98.7	131.6	164.5	197.4	230.3	263.2	296.1
328	32.8	65.6	98.4	131.2	164.0	196.8	229.6	262.4	295.2
327	32.7	65.4	98.1	130.8	163.5	196.2	228.9	261.6	294.3
326	32.6	65.2	97.8	130.4	163.0	195.6	228.2	260.8	293.4
325	32.5	65.0	97.5	130.0	162.5	195.0	227.5	260.0	292.5
324	32.4	64.8	97.2	129.6	162.0	194.4	226.8	259.2	291.6
323	32.3	64.6	96.9	129.2	161.5	193.8	226.1	258.4	290.7
322	32.2	64.4	96.6	128.8	161.0	193.2	225.4	257.6	289.8

No. 135 L. 130.] [No. 149 L. 175.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
135	130334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
6	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
7	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
8	9879										
		0194	0508	0822	1136	1450	1763	2076	2389	2702	314
9	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	6128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
1	9219	9527	9835								
				0142	0449	0756	1063	1370	1676	1982	307
2	152288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
3	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
4	8362	8664	8965	9266	9567	9868					
							0168	0469	0769	1068	301
5	161368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
6	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
7	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
8	170262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
9	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
321	32. 1	64. 2	96. 3	128. 4	160. 5	192. 6	224. 7	256. 8	288. 9
320	32. 0	64. 0	96. 0	128. 0	160. 0	192. 0	224. 0	256. 0	288. 0
319	31. 9	63. 8	95. 7	127. 6	159. 5	191. 4	223. 3	255. 2	287. 1
318	31. 8	63. 6	95. 4	127. 2	159. 0	190. 8	222. 6	254. 4	286. 2
317	31. 7	63. 4	95. 1	126. 8	158. 5	190. 2	221. 9	253. 6	285. 3
316	31. 6	63. 2	94. 8	126. 4	158. 0	189. 6	221. 2	252. 8	284. 4
315	31. 5	63. 0	94. 5	126. 0	157. 5	189. 0	220. 5	252. 0	283. 5
314	31. 4	62. 8	94. 2	125. 6	157. 0	188. 4	219. 8	251. 2	282. 6
313	31. 3	62. 6	93. 9	125. 2	156. 5	187. 8	219. 1	250. 4	281. 7
312	31. 2	62. 4	93. 6	124. 8	156. 0	187. 2	218. 4	249. 6	280. 8
311	31. 1	62. 2	93. 3	124. 4	155. 5	186. 6	217. 7	248. 8	279. 9
310	31. 0	62. 0	93. 0	124. 0	155. 0	186. 0	217. 0	248. 0	279. 0
309	30. 9	61. 8	92. 7	123. 6	154. 5	185. 4	216. 3	247. 2	278. 1
308	30. 8	61. 6	92. 4	123. 2	154. 0	184. 8	215. 6	246. 4	277. 2
307	30. 7	61. 4	92. 1	122. 8	153. 5	184. 2	214. 9	245. 6	276. 3
306	30. 6	61. 2	91. 8	122. 4	153. 0	183. 6	214. 2	244. 8	275. 4
305	30. 5	61. 0	91. 5	122. 0	152. 5	183. 0	213. 5	244. 0	274. 5
304	30. 4	60. 8	91. 2	121. 6	152. 0	182. 4	212. 8	243. 2	273. 6
303	30. 3	60. 6	90. 9	121. 2	151. 5	181. 8	212. 1	242. 4	272. 7
302	30. 2	60. 4	90. 6	120. 8	151. 0	181. 2	211. 4	241. 6	271. 8
301	30. 1	60. 2	90. 3	120. 4	150. 5	180. 6	210. 7	240. 8	270. 9
300	30. 0	60. 0	90. 0	120. 0	150. 0	180. 0	210. 0	240. 0	270. 0
299	29. 9	59. 8	89. 7	119. 6	149. 5	179. 4	209. 3	239. 2	269. 1
298	29. 8	59. 6	89. 4	119. 2	149. 0	178. 8	208. 6	238. 4	268. 2
297	29. 7	59. 4	89. 1	118. 8	148. 5	178. 2	207. 9	237. 6	267. 3
296	29. 6	59. 2	88. 8	118. 4	148. 0	177. 6	207. 2	236. 8	266. 4
295	29. 5	59. 0	88. 5	118. 0	147. 5	177. 0	206. 5	236. 0	265. 5
294	29. 4	58. 8	88. 2	117. 6	147. 0	176. 4	205. 8	235. 2	264. 6
293	29. 3	58. 6	87. 9	117. 2	146. 5	175. 8	205. 1	234. 4	263. 7
292	29. 2	58. 4	87. 6	116. 8	146. 0	175. 2	204. 4	233. 6	262. 8
291	29. 1	58. 2	87. 3	116. 4	145. 5	174. 6	203. 7	232. 8	261. 9
290	29. 0	58. 0	87. 0	116. 0	145. 0	174. 0	203. 0	232. 0	261. 0
289	28. 9	57. 8	86. 7	115. 6	144. 5	173. 4	202. 3	231. 2	260. 1
288	28. 8	57. 6	86. 4	115. 2	144. 0	172. 8	201. 6	230. 4	259. 2
287	28. 7	57. 4	86. 1	114. 8	143. 5	172. 2	200. 9	229. 6	258. 3
286	28. 6	57. 2	85. 8	114. 4	143. 0	171. 6	200. 2	228. 8	257. 4

No. 150 L. 176.] [No. 169 L. 230.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
150	176091	6381	6670	6959	7248	7536	7825	8113	8401	8689	289
1	8977	9264	9552	9839	0126	0413	0699	0986	1272	1558	287
2	181844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
3	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
4	7521	7803	8084	8366	8647	8928	9209	9490	9771	0051	281
5	190332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
6	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
7	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
8	8657	8932	9206	9481	9755	0029	0303	0577	0850	1124	274
9	201397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
160	4120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
1	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
2	9515	9783	0051	0319	0586	0853	1121	1388	1654	1921	267
3	212188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
4	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
5	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
6	220108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
7	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
8	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
9	7887	8144	8400	8657	8913	9170	9426	9682	9938	0193	256
	23										

Table 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
285	28. 5	57. 0	85. 5	114. 0	142. 5	171. 0	199. 5	228. 0	256. 5
284	28. 4	56. 8	85. 2	113. 6	142. 0	170. 4	198. 8	227. 2	255. 6
283	28. 3	56. 6	84. 9	113. 2	141. 5	169. 8	198. 1	226. 4	254. 7
282	28. 2	56. 4	84. 6	112. 8	141. 0	169. 2	197. 4	225. 6	253. 8
281	28. 1	56. 2	84. 3	112. 4	140. 5	168. 6	196. 7	224. 8	252. 9
280	28. 0	56. 0	84. 0	112. 0	140. 0	168. 0	196. 0	224. 0	252. 0
279	27. 9	55. 8	83. 7	111. 6	139. 5	167. 4	195. 3	223. 2	251. 1
278	27. 8	55. 6	83. 4	111. 2	139. 0	166. 8	194. 6	222. 4	250. 2
277	27. 7	55. 4	83. 1	110. 8	138. 5	166. 2	193. 9	221. 6	249. 3
276	27. 6	55. 2	82. 8	110. 4	138. 0	165. 6	193. 2	220. 8	248. 4
275	27. 5	55. 0	82. 5	110. 0	137. 5	165. 0	192. 5	220. 0	247. 5
274	27. 4	54. 8	82. 2	109. 6	137. 0	164. 4	191. 8	219. 2	246. 6
273	27. 3	54. 6	81. 9	109. 2	136. 5	163. 8	191. 1	218. 4	245. 7
272	27. 2	54. 4	81. 6	108. 8	136. 0	163. 2	190. 4	217. 6	244. 8
271	27. 1	54. 2	81. 3	108. 4	135. 5	162. 6	189. 7	216. 8	243. 9
270	27. 0	54. 0	81. 0	108. 0	135. 0	162. 0	189. 0	216. 0	243. 0
269	26. 9	53. 8	80. 7	107. 6	134. 5	161. 4	188. 3	215. 2	242. 1
268	26. 8	53. 6	80. 4	107. 2	134. 0	160. 8	187. 6	214. 4	241. 2
267	26. 7	53. 4	80. 1	106. 8	133. 5	160. 2	186. 9	213. 6	240. 3
266	26. 6	53. 2	79. 8	106. 4	133. 0	159. 6	186. 2	212. 8	239. 4
265	26. 5	53. 0	79. 5	106. 0	132. 5	159. 0	185. 5	212. 0	238. 5
264	26. 4	52. 8	79. 2	105. 6	132. 0	158. 4	184. 8	211. 2	237. 6
263	26. 3	52. 6	78. 9	105. 2	131. 5	157. 8	184. 1	210. 4	236. 7
262	26. 2	52. 4	78. 6	104. 8	131. 0	157. 2	183. 4	209. 6	235. 8
261	26. 1	52. 2	78. 3	104. 4	130. 5	156. 6	182. 7	208. 8	234. 9
260	26. 0	52. 0	78. 0	104. 0	130. 0	156. 0	182. 0	208. 0	234. 0
259	25. 9	51. 8	77. 7	103. 6	129. 5	155. 4	181. 3	207. 2	233. 1
258	25. 8	51. 6	77. 4	103. 2	129. 0	154. 8	180. 6	206. 4	232. 2
257	25. 7	51. 4	77. 1	102. 8	128. 5	154. 2	179. 9	205. 6	231. 3
256	25. 6	51. 2	76. 8	102. 4	128. 0	153. 6	179. 2	204. 8	230. 4
255	25. 5	51. 0	76. 5	102. 0	127. 5	153. 0	178. 5	204. 0	229. 5

No. 170 L. 230.] [No. 189 L. 278.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
170	230449	0704	0960	1215	1470	1724	1979	2234	2488	2742	255
1	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
2	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
3	8046	8297	8548	8799	9049	9299	9550	9800	0050	0300	250
4	240549	0799	1048	1297	1546	1795	2044	2293	2541	2790	249
5	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
6	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
7	7973	8219	8464	8709	8954	9198	9443	9687	9932	0176	245
8	250420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
9	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	5273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
1	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
2	260071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
3	2451	2688	2925	3162	3399	3636	3873	4109	4346	4583	237
4	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
5	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
6	9513	9746	9980	0213	0446	0679	0912	1144	1377	1609	233
7	271842	2074	2306	2538	2770	3001	3233	3464	3696	3927	232
8	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
9	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
255	25.5	51.0	76.5	102.0	127.5	153.0	178.5	204.0	229.5
254	25.4	50.8	76.2	101.6	127.0	152.4	177.8	203.2	228.6
253	25.3	50.6	75.9	101.2	126.5	151.8	177.1	202.4	227.7
252	25.2	50.4	75.6	100.8	126.0	151.2	176.4	201.6	226.8
251	25.1	50.2	75.3	100.4	125.5	150.6	175.7	200.8	225.9
250	25.0	50.0	75.0	100.0	125.0	150.0	175.0	200.0	225.0
249	24.9	49.8	74.7	99.6	124.5	149.4	174.3	199.2	224.1
248	24.8	49.6	74.4	99.2	124.0	148.8	173.6	198.4	223.2
247	24.7	49.4	74.1	98.8	123.5	148.2	172.9	197.6	222.3
246	24.6	49.2	73.8	98.4	123.0	147.6	172.2	196.8	221.4
245	24.5	49.0	73.5	98.0	122.5	147.0	171.5	196.0	220.5
244	24.4	48.8	73.2	97.6	122.0	146.4	170.8	195.2	219.6
243	24.3	48.6	72.9	97.2	121.5	145.8	170.1	194.4	218.7
242	24.2	48.4	72.6	96.8	121.0	145.2	169.4	193.6	217.8
241	24.1	48.2	72.3	66.4	120.5	144.6	168.7	192.8	216.9
240	24.0	48.0	72.0	96.0	120.0	144.0	168.0	192.0	216.0
239	23.9	47.8	71.7	95.6	119.5	143.4	167.3	191.2	215.1
238	23.8	47.6	71.4	95.2	119.0	142.8	166.6	190.4	214.2
237	23.7	47.4	71.1	94.8	118.5	142.2	165.9	189.6	213.3
236	23.6	47.2	70.8	94.4	118.0	141.6	165.2	188.8	212.4
235	23.5	47.0	70.5	94.0	117.5	141.0	164.5	188.0	211.5
234	23.4	46.8	70.2	93.6	117.0	140.4	163.8	187.2	210.6
233	23.3	46.6	69.9	93.2	116.5	139.8	163.1	186.4	209.7
232	23.2	46.4	69.6	92.8	116.0	139.2	162.4	185.6	208.8
231	23.1	46.2	69.3	92.4	115.5	138.6	161.7	184.8	207.9
230	23.0	46.0	69.0	92.0	115.0	138.0	161.0	184.0	207.0
229	22.9	45.8	68.7	91.6	114.5	137.4	160.3	183.2	206.1
228	22.8	45.6	68.4	91.2	114.0	136.8	159.6	182.4	205.2
227	22.7	45.4	68.1	90.8	113.5	136.2	158.9	181.6	204.3
226	22.6	45.2	67.8	90.4	113.0	135.6	158.2	180.8	203.4

No. 190 L. 278.] [No. 214 L. 332.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
190	278754	8982	9211	9439	9667	9895	0123	0351	0578	0806	228
1	281033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
2	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
3	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
4	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
5	290035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
6	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
7	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
8	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
9	8853	9071	9289	9507	9725	9943	0161	0378	0595	0813	218
200	301030	1247	1464	1681	1898	2114	2331	2547	2764	2980	217
1	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
2	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
3	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
4	9630	9843	0056	0268	0481	0693	0906	1118	1330	1542	212
5	311754	1966	2177	2389	2600	2812	3023	3234	3445	3656	211
6	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
7	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
8	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
9	320146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	2219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
1	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
2	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
3	8380	8583	8787	8991	9194	9398	9601	9805	0008	0211	203
4	330414	0617	0819	1022	1225	1427	1630	1832	2034	2236	202

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
225	22.5	45.0	67.5	90.0	112.5	135.0	157.5	180.0	202.5
224	22.4	44.8	67.2	89.6	112.0	134.4	156.8	179.2	201.6
223	22.3	44.6	66.9	89.2	111.5	133.8	156.1	178.4	200.7
222	22.2	44.4	66.6	88.8	111.0	133.2	155.4	177.6	199.8
221	22.1	44.2	66.3	88.4	110.5	132.6	154.7	176.8	198.9
220	22.0	44.0	66.0	88.0	110.0	132.0	154.0	176.0	198.0
219	21.9	43.8	65.7	87.6	109.5	131.4	153.3	175.2	197.1
218	21.8	43.6	65.4	87.2	109.0	130.8	152.6	174.4	196.2
217	21.7	43.4	65.1	86.8	108.5	130.2	151.9	173.6	195.3
216	21.6	43.2	64.8	86.4	108.0	129.6	151.2	172.8	194.4
215	21.5	43.0	64.5	86.0	107.5	129.0	150.5	172.0	193.5
214	21.4	42.8	64.2	85.6	107.0	128.4	149.8	171.2	192.6
213	21.3	42.6	63.9	85.2	106.5	127.8	149.1	170.4	191.7
212	21.2	42.4	63.6	84.8	106.0	127.2	148.4	169.6	190.8
211	21.1	42.2	63.3	84.4	105.5	126.6	147.7	168.8	189.9
210	21.0	42.0	63.0	84.0	105.0	126.0	147.0	168.0	189.0
209	20.9	41.8	62.7	83.6	104.5	125.4	146.3	167.2	188.1
208	20.8	41.6	62.4	83.2	104.0	124.8	145.6	166.4	187.2
207	20.7	41.4	62.1	82.8	103.5	124.2	144.9	165.6	186.3
206	20.6	41.2	61.8	82.4	103.0	123.6	144.2	164.8	185.4
205	20.5	41.0	61.5	82.0	102.5	123.0	143.5	164.0	184.5
204	20.4	40.8	61.2	81.6	102.0	122.4	142.8	163.2	183.6
203	20.3	40.6	60.9	81.2	101.5	121.8	142.1	162.4	182.7
202	20.2	40.4	60.6	80.8	101.0	121.2	141.4	161.6	181.8

No. 215 L. 332.] [No. 239 L. 380

N.	0	1	2	3	4	5	6	7	8	9	Diff.
215	332438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
6	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
7	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
8	8456	8656	8855	9054	9253	9451	9650	9849	0047	0246	199
9	340444	0642	0841	1039	1237	1435	1632	1830	2028	2225	198
220	2423	2620	2817	3014	3212	3409	3606	3802	3999	4196	197
1	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
2	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
3	8305	8500	8694	8889	9083	9278	9472	9666	9860	0054	194
4	350248	0442	0636	0829	1023	1216	1410	1603	1796	1989	193
5	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
6	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
7	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
8	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
9	9835	0025	0215	0404	0593	0783	0972	1161	1350	1539	189
230	361728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
1	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
2	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
3	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
4	9216	9401	9587	9772	9958	0143	0328	0513	0698	0883	185
5	371068	1253	1437	1622	1806	1991	2175	2360	2544	2728	184
6	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
7	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
8	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
9	8398	8580	8761	8943	9124	9306	9487	9668	9849	0030	181
	38										

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
202	20.2	40.4	60.6	80.8	101.0	121.2	141.4	161.6	181.8
201	20.1	40.2	60.3	80.4	100.5	120.6	140.7	160.8	180.9
200	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0	180.0
199	19.9	39.8	59.7	79.6	99.5	119.4	139.3	159.2	179.1
198	19.8	39.6	59.4	79.2	99.0	118.8	138.6	158.4	178.2
197	19.7	39.4	59.1	78.8	98.5	118.2	137.9	157.6	177.3
196	19.6	39.2	58.8	78.4	98.0	117.6	137.2	156.8	176.4
195	19.5	39.0	58.5	78.0	97.5	117.0	136.5	156.0	175.5
194	19.4	38.8	58.2	77.6	97.0	116.4	135.8	155.2	174.6
193	19.3	38.6	57.9	77.2	96.5	115.8	135.1	154.4	173.7
192	19.2	38.4	57.6	76.8	96.0	115.2	134.4	153.6	172.8
191	19.1	38.2	57.3	76.4	95.5	114.6	133.7	152.8	171.9
190	19.0	38.0	57.0	76.0	95.0	114.0	133.0	152.0	171.0
189	18.9	37.8	56.7	75.6	94.5	113.4	132.3	151.2	170.1
188	18.8	37.6	56.4	75.2	94.0	112.8	131.6	150.4	169.2
187	18.7	37.4	56.1	74.8	93.5	112.2	130.9	149.6	168.3
186	18.6	37.2	55.8	74.4	93.0	111.6	130.2	148.8	167.4
185	18.5	37.0	55.5	74.0	92.5	111.0	129.5	148.0	166.5
184	18.4	36.8	55.2	73.6	92.0	110.4	128.8	147.2	165.6
183	18.3	36.6	54.9	73.2	91.5	109.8	128.1	146.4	164.7
182	18.2	36.4	54.6	72.8	91.0	109.2	127.4	145.6	163.8
181	18.1	36.2	54.3	72.4	90.5	108.6	126.7	144.8	162.9
180	18.0	36.0	54.0	72.0	90.0	108.0	126.0	144.0	162.0
179	17.9	35.8	53.7	71.6	89.5	107.4	125.3	143.2	161.1

No. 240 L. 380.] [No. 269 L. 431.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
240	380211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
1	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
2	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
3	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
4	7390	7568	7746	7924	8101	8279	8456	8634	8811	8989	178
5	9166	9343	9520	9698	9875	0051	0228	0405	0582	0759	177
6	390935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
7	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
8	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
9	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	7940	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
1	9674	9847	0020	0192	0365	0538	0711	0883	1056	1228	173
2	401401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
3	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
4	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
5	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
6	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
7	9933	0102	0271	0440	0609	0777	0946	1114	1283	1451	169
8	411620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
9	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	4973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
1	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
2	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
3	9956	0121	0286	0451	0616	0781	0945	1110	1275	1439	165
4	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
5	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
6	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
7	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
8	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
9	9752	9914									
	43		0075	0236	0398	0559	0720	0881	1042	1203	161

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
178	17. 8	35. 6	53. 4	71. 2	89. 0	106. 8	124. 6	142. 4	160. 2
177	17. 7	35. 4	53. 1	70. 8	88. 5	106. 2	123. 9	141. 6	159. 3
176	17. 6	35. 2	52. 8	70. 4	88. 0	105. 6	123. 2	140. 8	158. 4
175	17. 5	35. 0	52. 5	70. 0	87. 5	105. 0	122. 5	140. 0	157. 5
174	17. 4	34. 8	52. 2	69. 6	87. 0	104. 4	121. 8	139. 2	156. 6
173	17. 3	34. 6	51. 9	69. 2	86. 5	103. 8	121. 1	138. 4	155. 7
172	17. 2	34. 4	51. 6	68. 8	86. 0	103. 2	120. 4	137. 6	154. 8
171	17. 1	34. 2	51. 3	68. 4	85. 5	102. 6	119. 7	136. 8	153. 9
170	17. 0	34. 0	51. 0	68. 0	85. 0	102. 0	119. 0	136. 0	153. 0
169	16. 9	33. 8	50. 7	67. 6	84. 5	101. 4	118. 3	135. 2	152. 1
168	16. 8	33. 6	50. 4	67. 2	84. 0	100. 8	117. 6	134. 4	151. 2
167	16. 7	33. 4	50. 1	66. 8	83. 5	100. 2	116. 9	133. 6	150. 3
166	16. 6	33. 2	49. 8	66. 4	83. 0	99. 6	116. 2	132. 8	149. 4
165	16. 5	33. 0	49. 5	66. 0	82. 5	99. 0	115. 5	132. 0	148. 5
164	16. 4	32. 8	49. 2	65. 6	82. 0	98. 4	114. 8	131. 2	147. 6
163	16. 3	32. 6	48. 9	65. 2	81. 5	97. 8	114. 1	130. 4	146. 7
162	16. 2	32. 4	48. 6	64. 8	81. 0	97. 2	113. 4	129. 6	145. 8
161	16. 1	32. 2	48. 3	64. 4	80. 5	96. 6	112. 7	128. 8	144. 9

No. 270 L. 431.] [No. 299 L. 476.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
270	431364	1525	1685	1846	2007	2167	2328	2488	2649	2809	161
1	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
2	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
3	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
4	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
5	9333	9491	9648	9806	9964	0122	0279	0437	0594	0752	158
6	440909	1066	1224	1381	1538	1695	1852	2009	2166	2323	157
7	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
8	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
9	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155
280	7158	7313	7468	7623	7778	7933	8088	8242	8397	8552	155
1	8706	8861	9015	9170	9324	9478	9633	9787	9941	0095	154
2	450249	0403	0557	0711	0865	1018	1172	1326	1479	1633	154
3	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
4	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
5	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
6	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
7	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
8	9392	9543	9694	9845	9995	0146	0296	0447	0597	0748	151
9	460898	1048	1198	1348	1499	1649	1799	1948	2098	2248	150
290	2398	2548	2697	2847	2997	3146	3296	3445	3594	3744	150
1	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
2	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
3	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
4	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
5	9822	9969	0116	0263	0410	0557	0704	0851	0998	1145	147
6	471292	1438	1585	1732	1878	2025	2171	2318	2464	2610	146
7	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
8	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
9	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
161	16.1	32.2	48.3	64.4	80.5	96.6	112.7	128.8	144.9
160	16.0	32.0	48.0	64.0	80.0	96.0	112.0	128.0	144.0
159	15.9	31.8	47.7	63.6	79.5	95.4	111.3	127.2	143.1
158	15.8	31.6	47.4	63.2	79.0	94.8	110.6	126.4	142.2
157	15.7	31.4	47.1	62.8	78.5	94.2	109.9	125.6	141.3
156	15.6	31.2	46.8	62.4	78.0	93.6	109.2	124.8	140.4
155	15.5	31.0	46.5	62.0	77.5	93.0	108.5	124.0	139.5
154	15.4	30.8	46.2	61.6	77.0	92.4	107.8	123.2	138.6
153	15.3	30.6	45.9	61.2	76.5	91.8	107.1	122.4	137.7
152	15.2	30.4	45.6	60.8	76.0	91.2	106.4	121.6	136.8
151	15.1	30.2	45.3	60.4	75.5	90.6	105.7	120.8	135.9
150	15.0	30.0	45.0	60.0	75.0	90.0	105.0	120.0	135.0
149	14.9	29.8	44.7	59.6	74.5	89.4	104.3	119.2	134.1
148	14.8	29.6	44.4	59.2	74.0	88.8	103.6	118.4	133.2
147	14.7	29.4	44.1	58.8	73.5	88.2	102.9	117.6	132.3
146	14.6	29.2	43.8	58.4	73.0	87.6	102.2	116.8	131.4
145	14.5	29.0	43.5	58.0	72.5	87.0	101.5	116.0	130.5
144	14.4	28.8	43.2	57.6	72.0	86.4	100.8	115.2	129.6
143	14.3	28.6	42.9	57.2	71.5	85.8	100.1	114.4	128.7
142	14.2	28.4	42.6	56.8	71.0	85.2	99.4	113.6	127.8
141	14.1	28.2	42.3	56.4	70.5	84.6	98.7	112.8	126.9
140	14.0	28.0	42.0	56.0	70.0	84.0	98.0	112.0	126.0

No. 300 L. 477.] [No. 339 L. 531.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
300	477121	7266	7411	7555	7700	7844	7989	8133	8278	8422	145
1	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
2	480007	0151	0294	0438	0582	0725	0869	1012	1156	1299	144
3	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
4	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
5	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
6	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
7	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
8	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
9	9958	0099	0239	0380	0520	0661	0801	0941	1081	1222	140
310	491362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
1	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
2	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
3	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
4	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
5	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
6	9687	9824	9962	0099	0236	0374	0511	0648	0785	0922	137
7	501059	1196	1333	1470	1607	1744	1880	2017	2154	2291	137
8	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
9	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	5150	5286	5421	5557	5693	5828	5964	6099	6234	6370	136
1	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
2	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
3	9203	9337	9471	9606	9740	9874	0009	0143	0277	0411	134
4	510545	0679	0813	0947	1081	1215	1349	1482	1616	1750	134
5	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
6	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
7	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
8	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
9	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	8514	8646	8777	8909	9040	9171	9303	9434	9566	9697	131
1	9828	9959	0090	0221	0353	0484	0615	0745	0876	1007	131
2	521138	1269	1400	1530	1661	1792	1922	2053	2183	2314	131
3	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
4	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
5	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
6	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
7	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
8	8917	9045	9174	9302	9430	9559	9687	9815	9943	0072	128
9	530200	0328	0456	0584	0712	0840	0968	1096	1223	1351	128

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
139	13.9	27.8	41.7	55.6	69.5	83.4	97.3	111.2	125.1
138	13.8	27.6	41.4	55.2	69.0	82.8	96.6	110.4	124.2
137	13.7	27.4	41.1	54.8	68.5	82.2	95.9	109.6	123.3
136	13.6	27.2	40.8	54.4	68.0	81.6	95.2	108.8	122.4
135	13.5	27.0	40.5	54.0	67.5	81.0	94.5	108.0	121.5
134	13.4	26.8	40.2	53.6	67.0	80.4	93.8	107.2	120.6
133	13.3	26.6	39.9	53.2	66.5	79.8	93.1	106.4	119.7
132	13.2	26.4	39.6	52.8	66.0	79.2	92.4	105.6	118.8
131	13.1	26.2	39.3	52.4	65.5	78.6	91.7	104.8	117.9
130	13.0	26.0	39.0	52.0	65.0	78.0	91.0	104.0	117.0
129	12.9	25.8	38.7	51.6	64.5	77.4	90.3	103.2	116.1
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2
127	12.7	25.4	38.1	50.8	63.5	76.2	88.9	101.6	114.3

No. 340 L. 531.] [No. 379 L. 579.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
340	531479	1607	1734	1862	1990	2117	2245	2372	2500	2627	128
1	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
2	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
3	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
4	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
5	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
6	9076	9202	9327	9452	9578	9703	9829	9954	0079	0204	125
7	540329	0455	0580	0705	0830	0955	1080	1205	1330	1454	125
8	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
9	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	4068	4192	4316	4440	4564	4688	4812	4936	5060	5183	124
1	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
2	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
3	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
4	9003	9126	9249	9371	9494	9616	9739	9861	9984	0106	123
5	550228	0351	0473	0595	0717	0840	0962	1084	1206	1328	122
6	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
7	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
8	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
9	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	6303	6423	6544	6664	6785	6905	7026	7146	7267	7387	120
1	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
2	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
3	9907	0026	0146	0265	0385	0504	0624	0743	0863	0982	119
4	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
5	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
6	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
7	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
8	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
9	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	8202	8319	8436	8554	8671	8788	8905	9023	9140	9257	117
1	9374	9491	9608	9725	9842	9959	0076	0193	0309	0426	117
2	570543	0660	0776	0893	1010	1126	1243	1359	1476	1592	117
3	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
4	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
5	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
6	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
7	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
8	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
9	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2
127	12.7	25.4	38.1	50.8	63.5	76.2	88.9	101.6	114.3
126	12.6	25.2	37.8	50.4	63.0	75.6	88.2	100.8	113.4
125	12.5	25.0	37.5	50.0	62.5	75.0	87.5	100.0	112.5
124	12.4	24.8	37.2	49.6	62.0	74.4	86.8	99.2	111.6
123	12.3	24.6	36.9	49.2	61.5	73.8	86.1	98.4	110.7
122	12.2	24.4	36.6	48.8	61.0	73.2	85.4	97.6	109.8
121	12.1	24.2	36.3	48.4	60.5	72.6	84.7	96.8	108.9
120	12.0	24.0	36.0	48.0	60.0	72.0	84.0	96.0	108.0
119	11.9	23.8	35.7	47.6	59.5	71.4	83.3	95.2	107.1

No. 380 L. 579.] [No. 414 L. 617.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
380	579784	9898	0012	0126	0241	0355	0469	0583	0697	0811	114
1	580925	1039	1153	1267	1381	1495	1608	1722	1836	1950	
2	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	
3	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	
4	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
5	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	
6	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	
7	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
8	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	
9	9950	0061	0173	0284	0396	0507	0619	0730	0842	0953	
390	591065	1176	1287	1399	1510	1621	1732	1843	1955	2066	
1	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
2	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	
3	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	
4	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	
5	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
6	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	
7	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	
8	9883	9992	0101	0210	0319	0428	0537	0646	0755	0864	109
9	600973	1082	1191	1299	1408	1517	1625	1734	1843	1951	
400	2060	2169	2277	2386	2494	2603	2711	2819	2928	3036	
1	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
2	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	
3	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	
4	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	
5	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
6	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	
7	9594	9701	9808	9914	0021	0128	0234	0341	0447	0554	
8	610660	0767	0873	0979	1086	1192	1298	1405	1511	1617	
9	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	2784	2890	2996	3102	3207	3313	3419	3525	3630	3736	
1	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	
2	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	
3	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
4	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
118	11. 8	23. 6	35. 4	47. 2	59. 0	70. 8	82. 6	94. 4	106. 2
117	11. 7	23. 4	35. 1	46. 8	58. 5	70. 2	81. 9	93. 6	105. 3
116	11. 6	23. 2	34. 8	46. 4	58. 0	69. 6	81. 2	92. 8	104. 4
115	11. 5	23. 0	34. 5	46. 0	57. 5	69. 0	80. 5	92. 0	103. 5
114	11. 4	22. 8	34. 2	45. 6	57. 0	68. 4	79. 8	91. 2	102. 6
113	11. 3	22. 6	33. 9	45. 2	56. 5	67. 8	79. 1	90. 4	101. 7
112	11. 2	22. 4	33. 6	44. 8	56. 0	67. 2	78. 4	89. 6	100. 8
111	11. 1	22. 2	33. 3	44. 4	55. 5	66. 6	77. 7	88. 8	99. 9
110	11. 0	22. 0	33. 0	44. 0	55. 0	66. 0	77. 0	88. 0	99. 0
109	10. 9	21. 8	32. 7	43. 6	54. 5	65. 4	76. 3	87. 2	98. 1
108	10. 8	21. 6	32. 4	43. 2	54. 0	64. 8	75. 6	86. 4	97. 2
107	10. 7	21. 4	32. 1	42. 8	53. 5	64. 2	74. 9	85. 6	96. 3
106	10. 6	21. 2	31. 8	42. 4	53. 0	63. 6	74. 2	84. 8	95. 4
105	10. 5	21. 0	31. 5	42. 0	52. 5	63. 0	73. 5	84. 0	94. 5
104	10. 4	20. 8	31. 2	41. 6	52. 0	62. 4	72. 8	83. 2	93. 6

No. 415 L. 618.] [No. 459 L. 662.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
415	618048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
6	9093	9198	9302	9406	9511	9615	9719	9824	9928	0032	
7	620136	0240	0344	0448	0552	0656	0760	0864	0968	1072	104
8	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	
9	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	
420	3249	3353	3456	3559	3663	3766	3869	3973	4076	4179	
1	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
2	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	
3	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	
4	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	
5	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
6	9410	9512	9613	9715	9817	9919	0021	0123	0224	0326	
7	630428	0530	0631	0733	0835	0936	1038	1139	1241	1342	
8	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	
9	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	
430	3468	3569	3670	3771	3872	3973	4074	4175	4276	4376	101
1	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	
2	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	
3	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	
4	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	100
5	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	
6	9486	9586	9686	9785	9885	9984	0084	0183	0283	0382	
7	640481	0581	0680	0779	0879	0978	1077	1177	1276	1375	
8	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	
9	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	3453	3551	3650	3749	3847	3946	4044	4143	4242	4340	
1	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	
2	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	
3	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
4	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	
5	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	
6	9335	9432	9530	9627	9724	9821	9919	0016	0113	0210	
7	650308	0405	0502	0599	0696	0793	0890	0987	1084	1181	
8	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
9	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	
450	3213	3309	3405	3502	3598	3695	3791	3888	3984	4080	
1	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	
2	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
3	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	
4	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	
5	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	
6	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	
7	9916	0011	0106	0201	0296	0391	0486	0581	0676	0771	95
8	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	
9	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
105	10. 5	21. 0	31. 5	42. 0	52. 5	63. 0	73. 5	84. 0	94. 5
104	10. 4	20. 8	31. 2	41. 6	52. 0	62. 4	72. 8	83. 2	93. 6
103	10. 3	20. 6	30. 9	41. 2	51. 5	61. 8	72. 1	82. 4	92. 7
102	10. 2	20. 4	30. 6	40. 8	51. 0	61. 2	71. 4	81. 6	91. 8
101	10. 1	20. 2	30. 3	40. 4	50. 5	60. 6	70. 7	80. 8	90. 9
100	10. 0	20. 0	30. 0	40. 0	50. 0	60. 0	70. 0	80. 0	90. 0
99	9. 9	19. 8	29. 7	39. 6	49. 5	59. 4	69. 3	79. 2	89. 1

No. 460 L. 662.] [No. 499 L. 698.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
460	662758	2852	2947	3041	3135	3230	3324	3418	3512	3607	
1	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	
2	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
3	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	
4	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	
5	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	
6	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	
7	9317	9410	9503	9596	9689	9782	9875	9967	0060	0153	93
8	670246	0339	0431	0524	0617	0710	0802	0895	0988	1080	
9	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	
470	2098	2190	2283	2375	2467	2560	2652	2744	2836	2929	
1	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	
2	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
3	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	
4	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	
5	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	
6	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	
7	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
8	9428	9519	9610	9700	9791	9882	9973	0063	0154	0245	
9	680336	0426	0517	0607	0698	0789	0879	0970	1060	1151	
480	1241	1332	1422	1513	1603	1693	1784	1874	1964	2055	
1	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	
2	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
3	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	
4	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	
5	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	
6	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	
7	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	
8	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
9	9309	9398	9486	9575	9664	9753	9841	9930	0019	0107	
490	690196	0285	0373	0462	0550	0639	0728	0816	0905	0993	
1	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	
2	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	
3	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
4	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	
5	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	
6	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	
7	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	
8	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
9	8100	8188	8275	8362	8449	8535	8622	8709	8796	8883	

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
98	9.8	19.6	29.4	39.2	49.0	58.8	68.6	78.4	88.2
97	9.7	19.4	29.1	38.8	48.5	58.2	67.9	77.6	87.3
96	9.6	19.2	28.8	38.4	48.0	57.6	67.2	76.8	86.4
95	9.5	19.0	28.5	38.0	47.5	57.0	66.5	76.0	85.5
94	9.4	18.8	28.2	37.6	47.0	56.4	65.8	75.2	84.6
93	9.3	18.6	27.9	37.2	46.5	55.8	65.1	74.4	83.7
92	9.2	18.4	27.6	36.8	46.0	55.2	64.4	73.6	82.8
91	9.1	18.2	27.3	36.4	45.5	54.6	63.7	72.8	81.9
90	9.0	18.0	27.0	36.0	45.0	54.0	63.0	72.0	81.0
89	8.9	17.8	26.7	35.6	44.5	53.4	62.3	71.2	80.1
88	8.8	17.6	26.4	35.2	44.0	52.8	61.6	70.4	79.2
87	8.7	17.4	26.1	34.8	43.5	52.2	60.9	69.6	78.3
86	8.6	17.2	25.8	34.4	43.0	51.6	60.2	68.8	77.4

No. 500 L. 698.] [No. 544 L. 736.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
500	698970	9057	9144	9231	9317	9404	9491	9578	9664	9751	
1	9838	9924	0011	0098	0184	0271	0358	0444	0531	0617	
2	700704	0790	0877	0963	1050	1136	1222	1309	1395	1482	
3	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	
4	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	
5	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
6	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	
7	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	
8	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	
9	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	
510	7570	7655	7740	7826	7911	7996	8081	8166	8251	8336	
1	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
2	9270	9355	9440	9524	9609	9694	9779	9863	9948	0033	
3	710117	0202	0287	0371	0456	0540	0625	0710	0794	0879	
4	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	
5	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	
6	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	
7	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
8	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	
9	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	
520	6003	6087	6170	6254	6337	6421	6504	6588	6671	6754	
1	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	
2	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
3	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	
4	9331	9414	9497	9580	9663	9745	9828	9911	9994	0077	
5	720159	0242	0325	0407	0490	0573	0655	0738	0821	0903	
6	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	
7	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	
8	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	
9	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	4276	4358	4440	4522	4604	4685	4767	4849	4931	5013	
1	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	
2	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	
3	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	
4	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	
5	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	
6	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
7	9974	0055	0136	0217	0298	0378	0459	0540	0621	0702	
8	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	
9	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	
540	2394	2474	2555	2635	2715	2796	2876	2956	3037	3117	
1	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	
2	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
3	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	
4	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
87	8. 7	17. 4	26. 1	34. 8	43. 5	52. 2	60. 9	69. 6	78. 3
86	8. 6	17. 2	25. 8	34. 4	43. 0	51. 6	60. 2	68. 8	77. 4
85	8. 5	17. 0	25. 5	34. 0	42. 5	51. 0	59. 5	68. 0	76. 5
84	8. 4	16. 8	25. 2	33. 6	42. 0	50. 4	58. 8	67. 2	75. 6

No. 545 L. 736.] [No. 584 L. 767.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
545	736397	6476	6556	6635	6715	6795	6874	6954	7034	7113	
6	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	
7	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	
8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	
9	9572	9651	9731	9810	9889	9968					
							0047	0126	0205	0284	79
550	740363	0442	0521	0600	0678	0757	0836	0915	0994	1073	
1	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	
2	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	
3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	
4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	
5	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	
6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	
8	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	
9	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	
560	8188	8266	8343	8421	8498	8576	8653	8731	8808	8885	
1	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	
2	9736	9814	9891	9968							
					0045	0123	0200	0277	0354	0431	
3	750508	0586	0663	0740	0817	0894	0971	1048	1125	1202	
4	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
5	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	
6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	
7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	
8	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	
9	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	
570	5875	5951	6027	6103	6180	6256	6332	6408	6484	6560	
1	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	
3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	
4	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	
5	9668	9743	9819	9894	9970						
						0045	0121	0196	0272	0347	
6	760422	0498	0573	0649	0724	0799	0875	0950	1025	1101	
7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	
8	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
9	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	
580	3428	3503	3578	3653	3727	3802	3877	3952	4027	4101	
1	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	
2	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	
3	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	
4	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
83	8.3	16.6	24.9	33.2	41.5	49.8	58.1	66.4	74.7
82	8.2	16.4	24.6	32.8	41.0	49.2	57.4	65.6	73.8
81	8.1	16.2	24.3	32.4	40.5	48.6	56.7	64.8	72.9
80	8.0	16.0	24.0	32.0	40.0	48.0	56.0	64.0	72.0
79	7.9	15.8	23.7	31.6	39.5	47.4	55.3	63.2	71.1
78	7.8	15.6	23.4	31.2	39.0	46.8	54.6	62.4	70.2
77	7.7	15.4	23.1	30.8	38.5	46.2	53.9	61.6	69.3
76	7.6	15.2	22.8	30.4	38.0	45.6	53.2	60.8	68.4
75	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5
74	7.4	14.8	22.2	29.6	37.0	44.4	51.8	59.2	66.6

No. 585 L. 767.] [No. 629 L. 799.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
585	767156	7230	7304	7379	7453	7527	7601	7675	7749	7823	
6	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
7	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	
8	9377	9451	9525	9599	9673	9746	9820	9894	9968	0042	
9	770115	0189	0263	0336	0410	0484	0557	0631	0705	0778	
590	0852	0926	0999	1073	1146	1220	1293	1367	1440	1514	
1	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	
2	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	
3	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	
4	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
5	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	
6	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	
7	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	
8	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	
9	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	
600	8151	8224	8296	8368	8441	8513	8585	8658	8730	8802	
1	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	
2	9596	9669	9741	9813	9885	9957	0029	0101	0173	0245	
3	780317	0389	0461	0533	0605	0677	0749	0821	0893	0965	72
4	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	
5	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	
6	2473	2544	2616	2683	2759	2831	2902	2974	3046	3117	
7	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	
8	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	
9	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	
610	5330	5401	5472	5543	5615	5686	5757	5828	5899	5970	
1	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
2	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	
3	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	
4	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	
5	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	
6	9581	9651	9722	9792	9863	9933	0004	0074	0144	0215	
7	790285	0356	0426	0496	0567	0637	0707	0778	0848	0918	
8	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	
9	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	
620	2392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
1	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	
2	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	
3	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	
4	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	
5	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	
6	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	
7	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	
8	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	
9	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
75	7. 5	15. 0	22. 5	30. 0	37. 5	45. 0	52. 5	60. 0	67. 5
74	7. 4	14. 8	22. 2	29. 6	37. 0	44. 4	51. 8	59. 2	66. 6
73	7. 3	14. 6	21. 9	29. 2	36. 5	43. 8	51. 1	58. 4	65. 7
72	7. 2	14. 4	21. 6	28. 8	36. 0	43. 2	50. 4	57. 6	64. 8
71	7. 1	14. 2	21. 3	28. 4	35. 5	42. 6	49. 7	56. 8	63. 9
70	7. 0	14. 0	21. 0	28. 0	35. 0	42. 0	49. 0	56. 0	63. 0
69	6. 9	13. 8	20. 7	27. 6	34. 5	41. 4	48. 3	55. 2	62. 1

No. 630 L. 799.] [No. 674 L. 829.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
630	799341	9409	9478	9547	9616	9685	9754	9823	9892	9961	
1	800029	0098	0167	0236	0305	0373	0442	0511	0580	0648	
2	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	
3	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	
4	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	
5	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	
6	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	
7	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	
8	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
9	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	
640	806180	6248	6316	6384	6451	6519	6587	6655	6723	6790	
1	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	
2	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	
3	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	
4	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	
5	9560	9627	9694	9762	9829	9896	9964	0031	0098	0165	
6	810233	0300	0367	0434	0501	0569	0636	0703	0770	0837	
7	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
8	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	
9	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	
650	2913	2980	3047	3114	3181	3247	3314	3381	3448	3514	
1	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	
2	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	
3	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	
4	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	
5	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	
6	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	
7	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	
8	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
9	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	
660	9544	9610	9676	9741	9807	9873	9939	0004	0070	0136	
1	820201	0267	0333	0399	0464	0530	0595	0661	0727	0792	
2	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	
3	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	
4	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	
5	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	
6	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	
7	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	
8	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
9	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	
670	6075	6140	6204	6269	6334	6399	6464	6528	6593	6658	
1	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	
2	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	
3	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	
4	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
68	6. 8	13. 6	20. 4	27. 2	34. 0	40. 8	47. 6	54. 4	61. 2
67	6. 7	13. 4	20. 1	26. 8	33. 5	40. 2	46. 9	53. 6	60. 3
66	6. 6	13. 2	19. 8	26. 4	33. 0	39. 6	46. 2	52. 8	59. 4
65	6. 5	13. 0	19. 5	26. 0	32. 5	39. 0	45. 5	52. 0	58. 5
64	6. 4	12. 8	19. 2	25. 6	32. 0	38. 4	44. 8	51. 2	57. 6

No. 675 L. 829.] [No. 719 L. 857.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
675	829304	9368	9432	9497	9561	9625	9690	9754	9818	9382	
6	9947	0011	0075	0139	0204	0268	0332	0396	0460	0525	
7	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	
8	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
9	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	
680	2509	2573	2637	2700	2764	2828	2892	2956	3020	3083	
1	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	
2	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	
3	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	
4	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	
5	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	
6	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	
7	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	
8	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
9	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	
690	8849	8912	8975	9038	9101	9164	9227	9289	9352	9415	
1	9478	9541	9604	9667	9729	9792	9855	9918	9981	0043	
2	840106	0169	0232	0294	0357	0420	0482	0545	0608	0671	
3	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	
4	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	
5	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	
6	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	
7	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	
8	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	
9	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	
700	5098	5160	5222	5284	5346	5408	5470	5532	5594	5656	62
1	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	
2	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	
3	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	
4	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	
5	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	
6	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	
7	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	
8	850033	0095	0156	0217	0279	0340	0401	0462	0524	0585	
9	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	
710	1258	1320	1381	1442	1503	1564	1625	1686	1747	1809	
1	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
2	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	
3	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	
4	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	
5	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	
6	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	
7	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	
8	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	
9	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
65	6. 5	13. 0	19. 5	26. 0	32. 5	39. 0	45. 5	52. 0	58. 5
64	6. 4	12. 8	19. 2	25. 6	32. 0	38. 4	44. 8	51. 2	57. 6
63	6. 3	12. 6	18. 9	25. 2	31. 5	37. 8	44. 1	50. 4	56. 7
62	6. 2	12. 4	18. 6	24. 8	31. 0	37. 2	43. 4	49. 6	55. 8
61	6. 1	12. 2	18. 3	24. 4	30. 5	36. 6	42. 7	48. 8	54. 9
60	6. 0	12. 0	18. 0	24. 0	30. 0	36. 0	42. 0	48. 0	54. 0

No. 720 L. 857.] [No. 764 L. 883.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
720	857332	7393	7453	7513	7574	7634	7694	7755	7815	7875	
1	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	
2	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	
3	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
4	9739	9799	9859	9918	9978	0038	0098	0158	0218	0278	
5	860338	0398	0458	0518	0578	0637	0697	0757	0817	0877	
6	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	
7	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	
8	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	
9	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	
730	3323	3382	3442	3501	3561	3620	3680	3739	3799	3858	
1	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	
2	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	
3	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	
4	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	
5	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	
6	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
7	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	
8	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	
9	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	
740	9232	9290	9349	9408	9466	9525	9584	9642	9701	9760	
1	9818	9877	9935	9994	0053	0111	0170	0228	0287	0345	
2	870404	0462	0521	0579	0638	0696	0755	0813	0872	0930	
3	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	
4	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	
5	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	
6	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	
7	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	
8	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
9	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	
750	5061	5119	5177	5235	5293	5351	5409	5466	5524	5582	
1	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	
2	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	
3	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	
4	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	
5	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	
6	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	
7	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	
8	9669	9726	9784	9841	9898	9956	0013	0070	0127	0185	
9	880242	0299	0356	0413	0471	0528	0585	0642	0699	0756	
760	0814	0871	0928	0985	1042	1099	1156	1213	1271	1328	
1	1385	1442	1499	1556	1613	1670	1727	1784	1741	1898	
2	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
3	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	
4	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
59	5.9	11.8	17.7	23.6	29.5	35.4	41.3	47.2	53.1
58	5.8	11.6	17.4	23.2	29.0	34.8	40.6	46.4	52.2
57	5.7	11.4	17.1	22.8	28.5	34.2	39.9	45.6	51.3
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4

No. 765 L. 883.] [No. 809 L. 908.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
765	883661	3718	3775	3832	3888	3945	4002	4059	4115	4172	
6	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	
7	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	
8	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	
9	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	
770	6491	6547	6604	6660	6716	6773	6829	6885	6942	6998	
1	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	
2	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	
3	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	
4	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	
5	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
6	9862	9918	9974	0030	0086	0141	0197	0253	0309	0365	
7	890421	0477	0533	0589	0645	0700	0756	0812	0868	0924	
8	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	
9	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	
780	2095	2150	2206	2262	2317	2373	2429	2484	2540	2595	
1	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	
2	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	
3	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	
4	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	
5	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	
6	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	
7	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	
8	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	
9	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	
790	7627	7682	7737	7792	7847	7902	7957	8012	8067	8122	55
1	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	
2	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	
3	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	
4	9821	9875	9930	9985	0039	0094	0149	0203	0258	0312	
5	900367	0422	0476	0531	0586	0640	0695	0749	0804	0859	
6	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	
7	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	
8	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	
9	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	
800	3090	3144	3199	3253	3307	3361	3416	3470	3524	3578	
1	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	
2	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	
3	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	
4	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
5	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	
6	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	
7	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	
8	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	
9	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
57	5. 7	11. 4	17. 1	22. 8	28. 5	34. 2	39. 9	45. 6	51. 3
56	5. 6	11. 2	16. 8	22. 4	28. 0	33. 6	39. 2	44. 8	50. 4
55	5. 5	11. 0	16. 5	22. 0	27. 5	33. 0	38. 5	44. 0	49. 5
54	5. 4	10. 8	16. 2	21. 6	27. 0	32. 4	37. 8	43. 2	48. 6

No. 810 L. 908.] [No. 854 L. 931.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
810	908485	8539	8592	8646	8699	8753	8807	8860	8914	8967	
1	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	
2	9556	9610	9663	9716	9770	9823	9877	9930	9984	0037	
3	910091	0144	0197	0251	0304	0358	0411	0464	0518	0571	
4	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	
5	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	
6	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	
7	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	
8	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	
9	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
820	3814	3867	3920	3973	4026	4079	4132	4184	4237	4290	
1	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	
2	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	
3	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	
4	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	
5	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	
6	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	
7	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	
8	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	
9	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	
830	9078	9130	9183	9235	9287	9340	9392	9444	9496	9549	
1	9601	9653	9706	9758	9810	9862	9914	9967	0019	0071	
2	920123	0176	0228	0280	0332	0384	0436	0489	0541	0593	
3	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
4	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	
5	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	
6	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	
7	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	
8	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	
9	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	
840	4279	4331	4383	4434	4486	4538	4589	4641	4693	4744	
1	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	
2	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	
3	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	
4	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	
5	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	
6	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	
7	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	
8	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	
9	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	
850	9419	9470	9521	9572	9623	9674	9725	9776	9827	9879	
1	9930	9981	0032	0083	0134	0185	0236	0287	0338	0389	51
2	930440	0491	0542	0592	0643	0694	0745	0796	0847	0898	
3	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	
4	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
53	5.3	10.6	15.9	21.2	26.5	31.8	37.1	42.4	47.7
52	5.2	10.4	15.6	20.8	26.0	31.2	36.4	41.6	46.8
51	5.1	10.2	15.3	20.4	25.5	30.6	35.7	40.8	45.9
50	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0

No. 855 L. 931.] [No. 899 L. 954.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
855	931966	2017	2068	2118	2169	2220	2271	2322	2372	2423	
6	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	
7	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	
8	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	
9	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	
860	4498	4549	4599	4650	4700	4751	4801	4852	4902	4953	
1	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	
2	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	
3	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	
4	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	
5	7016	7066	7116	7167	7217	7267	7317	7367	7418	7468	
6	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	
7	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
8	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	
9	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	
870	9519	9569	9619	9669	9719	9769	9819	9869	9918	9968	
1	940018	0068	0118	0168	0218	0267	0317	0367	0417	0467	
2	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	
3	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	
4	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	
5	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	
6	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	
7	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	
8	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	
9	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	
880	4483	4532	4581	4631	4680	4729	4779	4828	4877	4927	
1	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	
2	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	
3	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	
4	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	
5	6943	6992	7041	7090	7139	7189	7238	7287	7336	7385	
6	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
7	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	
8	8413	8462	8511	8560	8608	8657	8706	8755	8804	8853	
9	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	
890	9390	9439	9488	9536	9585	9634	9683	9731	9780	9829	
1	9878	9926	9975	0024	0073	0121	0170	0219	0267	0316	
2	950365	0414	0462	0511	0560	0608	0657	0706	0754	0303	
3	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	
4	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	
5	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	
6	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	
7	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	
8	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	
9	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
51	5. 1	10. 2	15. 3	20. 4	25. 5	30. 6	35. 7	40. 8	45. 9
50	5. 0	10. 0	15. 0	20. 0	25. 0	30. 0	35. 0	40. 0	45. 0
49	4. 9	9. 8	14. 7	19. 6	24. 5	29. 4	34. 3	39. 2	44. 1
48	4. 8	9. 6	14. 4	19. 2	24. 0	28. 8	33. 6	38. 4	43. 2

No. 900 L. 954.] [No. 944 L. 975.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
900	954243	4291	4339	4387	4435	4484	4532	4580	4628	4677	
1	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	
2	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	
3	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	
4	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	
5	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
6	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	
7	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	
8	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	
9	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	
910	9041	9089	9137	9185	9232	9280	9328	9375	9423	9471	
1	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	
2	9995	0042	0090	0138	0185	0233	0280	0328	0376	0423	
3	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	
4	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	
5	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	
6	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	
7	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	
8	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	
9	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	
920	3788	3835	3882	3929	3977	4024	4071	4118	4165	4212	
1	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	
2	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	
3	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	
4	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
5	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	
6	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	
7	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	
8	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	
9	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	
930	8483	8530	8576	8623	8670	8716	8763	8810	8856	8903	
1	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	
2	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	
3	9882	9928	9975	0021	0068	0114	0161	0207	0254	0300	
4	970347	0393	0440	0486	0533	0579	0626	0672	0719	0765	
5	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	
6	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	
7	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	
8	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	
9	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	
940	3128	3174	3220	3266	3313	3359	3405	3451	3497	3543	
1	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	
2	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	
3	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	
4	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
47	4.7	9.4	14.1	18.8	23.5	28.2	32.9	37.6	42.3
46	4.6	9.2	13.8	18.4	23.0	27.6	32.2	36.8	41.4

No. 945 L. 975.] [No. 989 L. 995.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
945	975432	5478	5524	5570	5616	5662	5707	5753	5799	5845	
6	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	
7	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	
8	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	
9	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	
950	7724	7769	7815	7861	7906	7952	7998	8043	8089	8135	
1	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	
2	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	
3	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	
4	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	
5	980003	0049	0094	0140	0185	0231	0276	0322	0367	0412	
6	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	
7	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	
8	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	
9	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	
960	2271	2316	2362	2407	2452	2497	2543	2588	2633	2678	
1	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	
2	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	
3	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	
4	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	
5	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
6	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	
7	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	
8	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	
9	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	
970	6772	6817	6861	6906	6951	6996	7040	7085	7130	7175	
1	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	
2	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	
3	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	
4	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	
5	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	
6	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	
7	9895	9939	9983	0028	0072	0117	0161	0206	0250	0294	
8	990339	0383	0428	0472	0516	0561	0605	0650	0694	0738	
9	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	
980	1226	1270	1315	1359	1403	1448	1492	1536	1580	1625	
1	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	
2	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	
3	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	
4	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	
5	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	
6	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	
7	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
8	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	
9	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	

TABLE 7.—*Logarithms of numbers from 1 to 100*—Continued.

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
46	4. 6	9. 2	13. 8	18. 4	23. 0	27. 6	32. 2	36. 8	41. 4
45	4. 5	9. 0	13. 5	18. 0	22. 5	27. 0	31. 5	36. 0	40. 5
44	4. 4	8. 8	13. 2	17. 6	22. 0	26. 4	30. 8	35. 2	39. 6
43	4. 3	8. 6	12. 9	17. 2	21. 5	25. 8	30. 1	34. 4	38. 7

No. 990 L. 995.] [No. 999 L. 999.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
990	995635	5679	5723	5767	5811	5854	5898	5942	5986	6030	
1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	
3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	
4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	
5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	
6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	
7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	
8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	
9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	
											43

TABLE 8.—HYPERBOLIC LOGARITHMS.

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
1. 01	. 0099	1. 35	. 3001	1. 69	. 5247	2. 03	. 7080	2. 37	. 8629
1. 02	. 0198	1. 36	. 3075	1. 70	. 5306	2. 04	. 7129	2. 38	. 8671
1. 03	. 0296	1. 37	. 3148	1. 71	. 5365	2. 05	. 7178	2. 39	. 8713
1. 04	. 0392	1. 38	. 3221	1. 72	. 5423	2. 06	. 7227	2. 40	. 8755
1. 05	. 0488	1. 39	. 3293	1. 73	. 5481	2. 07	. 7275	2. 41	. 8796
1. 06	. 0583	1. 40	. 3365	1. 74	. 5539	2. 08	. 7324	2. 42	. 8838
1. 07	. 0677	1. 41	. 3436	1. 75	. 5596	2. 09	. 7372	2. 43	. 8879
1. 08	. 0770	1. 42	. 3507	1. 76	. 5653	2. 10	. 7419	2. 44	. 8920
1. 09	. 0862	1. 43	. 3577	1. 77	. 5710	2. 11	. 7467	2. 45	. 8961
1. 10	. 0953	1. 44	. 3646	1. 78	. 5766	2. 12	. 7514	2. 46	. 9002
1. 11	. 1044	1. 45	. 3716	1. 79	. 5822	2. 13	. 7561	2. 47	. 9042
1. 12	. 1133	1. 46	. 3784	1. 80	. 5878	2. 14	. 7608	2. 48	. 9083
1. 13	. 1222	1. 47	. 3853	1. 81	. 5933	2. 15	. 7655	2. 49	. 9123
1. 14	. 1310	1. 48	. 3920	1. 82	. 5988	2. 16	. 7701	2. 50	. 9163
1. 15	. 1398	1. 49	. 3988	1. 83	. 6043	2. 17	. 7747	2. 51	. 9203
1. 16	. 1484	1. 50	. 4055	1. 84	. 6098	2. 18	. 7793	2. 52	. 9243
1. 17	. 1570	1. 51	. 4121	1. 85	. 6152	2. 19	. 7839	2. 53	. 9282
1. 18	. 1655	1. 52	. 4187	1. 86	. 6206	2. 20	. 7885	2. 54	. 9322
1. 19	. 1740	1. 53	. 4253	1. 87	. 6259	2. 21	. 7930	2. 55	. 9361
1. 20	. 1823	1. 54	. 4318	1. 88	. 6313	2. 22	. 7975	2. 56	. 9400
1. 21	. 1906	1. 55	. 4383	1. 89	. 6366	2. 23	. 8020	2. 57	. 9439
1. 22	. 1988	1. 56	. 4447	1. 90	. 6419	2. 24	. 8065	2. 58	. 9478
1. 23	. 2070	1. 57	. 4511	1. 91	. 6471	2. 25	. 8109	2. 59	. 9517
1. 24	. 2151	1. 58	. 4574	1. 92	. 6523	2. 26	. 8154	2. 60	. 9555
1. 25	. 2231	1. 59	. 4637	1. 93	. 6575	2. 27	. 8198	2. 61	. 9594
1. 26	. 2311	1. 60	. 4700	1. 94	. 6627	2. 28	. 8242	2. 62	. 9632
1. 27	. 2390	1. 61	. 4762	1. 95	. 6678	2. 29	. 8286	2. 63	. 9670
1. 28	. 2469	1. 62	. 4824	1. 96	. 6729	2. 30	. 8329	2. 64	. 9708
1. 29	. 2546	1. 63	. 4886	1. 97	. 6780	2. 31	. 8372	2. 65	. 9746
1. 30	. 2624	1. 64	. 4947	1. 98	. 6831	2. 32	. 8416	2. 66	. 9783
1. 31	. 2700	1. 65	. 5008	1. 99	. 6881	2. 33	. 8458	2. 67	. 9821
1. 32	. 2776	1. 66	. 5068	2. 00	. 6931	2. 34	. 8502	2. 68	. 9858
1. 33	. 2852	1. 67	. 5128	2. 01	. 6981	2. 35	. 8544	2. 69	. 9895
1. 34	. 2927	1. 68	. 5188	2. 02	. 7031	2. 36	. 8587	2. 70	. 9933

TABLE 8.—HYPERBOLIC LOGARITHMS—Continued.

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
2.71	.9969	3.34	1.2060	3.97	1.3788	4.60	1.5261	5.23	1.6544
2.72	1.0006	3.35	1.2090	3.98	1.3813	4.61	1.5282	5.24	1.6563
2.73	1.0043	3.36	1.2119	3.99	1.3838	4.62	1.5304	5.25	1.6582
2.74	1.0080	3.37	1.2149	4.00	1.3863	4.63	1.5326	5.26	1.6601
2.75	1.0116	3.38	1.2179	4.01	1.3888	4.64	1.5347	5.27	1.6620
2.76	1.0152	3.39	1.2208	4.02	1.3913	4.65	1.5369	5.28	1.6639
2.77	1.0188	3.40	1.2238	4.03	1.3938	4.66	1.5390	5.29	1.6658
2.78	1.0225	3.41	1.2267	4.04	1.3962	4.67	1.5412	5.30	1.6677
2.79	1.0260	3.42	1.2296	4.05	1.3987	4.68	1.5433	5.31	1.6696
2.80	1.0296	3.43	1.2326	4.06	1.4012	4.69	1.5454	5.32	1.6715
2.81	1.0332	3.44	1.2355	4.07	1.4036	4.70	1.5476	5.33	1.6734
2.82	1.0367	3.45	1.2384	4.08	1.4061	4.71	1.5497	5.34	1.6752
2.83	1.0403	3.46	1.2413	4.09	1.4085	4.72	1.5518	5.35	1.6771
2.84	1.0438	3.47	1.2442	4.10	1.4110	4.73	1.5539	5.36	1.6790
2.85	1.0473	3.48	1.2470	4.11	1.4134	4.74	1.5560	5.37	1.6808
2.86	1.0508	3.49	1.2499	4.12	1.4159	4.75	1.5581	5.38	1.6827
2.87	1.0543	3.50	1.2528	4.13	1.4183	4.76	1.5602	5.39	1.6845
2.88	1.0578	3.51	1.2556	4.14	1.4207	4.77	1.5623	5.40	1.6864
2.89	1.0613	3.52	1.2585	4.15	1.4231	4.78	1.5644	5.41	1.6882
2.90	1.0647	3.53	1.2613	4.16	1.4255	4.79	1.5665	5.42	1.6901
2.91	1.0682	3.54	1.2641	4.17	1.4279	4.80	1.5686	5.43	1.6919
2.92	1.0716	3.55	1.2669	4.18	1.4303	4.81	1.5707	5.44	1.6938
2.93	1.0750	3.56	1.2698	4.19	1.4327	4.82	1.5728	5.45	1.6956
2.94	1.0784	3.57	1.2726	4.20	1.4351	4.83	1.5748	5.46	1.6974
2.95	1.0818	3.58	1.2754	4.21	1.4375	4.84	1.5769	5.47	1.6993
2.96	1.0852	3.59	1.2782	4.22	1.4398	4.85	1.5790	5.48	1.7011
2.97	1.0886	3.60	1.2809	4.23	1.4422	4.86	1.5810	5.49	1.7029
2.98	1.0919	3.61	1.2837	4.24	1.4446	4.87	1.5831	5.50	1.7047
2.99	1.0953	3.62	1.2865	4.25	1.4469	4.88	1.5851	5.51	1.7066
3.00	1.0986	3.63	1.2892	4.26	1.4493	4.89	1.5872	5.52	1.7084
3.01	1.1019	3.64	1.2920	4.27	1.4516	4.90	1.5892	5.53	1.7102
3.02	1.1056	3.65	1.2947	4.28	1.4540	4.91	1.5913	5.54	1.7120
3.03	1.1081	3.66	1.2975	4.29	1.4563	4.92	1.5933	5.55	1.7138
3.04	1.1113	3.67	1.3002	4.30	1.4586	4.93	1.5953	5.56	1.7156
3.05	1.1154	3.68	1.3029	4.31	1.4609	4.94	1.5974	5.57	1.7174
3.06	1.1187	3.69	1.3056	4.32	1.4633	4.95	1.5994	5.58	1.7192
3.07	1.1219	3.70	1.3083	4.33	1.4656	4.96	1.6014	5.59	1.7210
3.08	1.1246	3.71	1.3110	4.34	1.4679	4.97	1.6034	5.60	1.7228
3.09	1.1284	3.72	1.3137	4.35	1.4702	4.98	1.6054	5.61	1.7246
3.10	1.1312	3.73	1.3164	4.36	1.4725	4.99	1.6074	5.62	1.7263
3.11	1.1349	3.74	1.3191	4.37	1.4748	5.00	1.6094	5.63	1.7281
3.12	1.1378	3.75	1.3218	4.38	1.4770	5.01	1.6114	5.64	1.7299
3.13	1.1410	3.76	1.3244	4.39	1.4793	5.02	1.6134	5.65	1.7317
3.14	1.1442	3.77	1.3271	4.40	1.4816	5.03	1.6154	5.66	1.7334
3.15	1.1474	3.78	1.3297	4.41	1.4839	5.04	1.6174	5.67	1.7352
3.16	1.1506	3.79	1.3324	4.42	1.4861	5.05	1.6194	5.68	1.7370
3.17	1.1537	3.80	1.3350	4.43	1.4884	5.06	1.6214	5.69	1.7387
3.18	1.1569	3.81	1.3376	4.44	1.4907	5.07	1.6233	5.70	1.7405
3.19	1.1600	3.82	1.3403	4.45	1.4929	5.08	1.6253	5.71	1.7422
3.20	1.1632	3.83	1.3429	4.46	1.4951	5.09	1.6273	5.72	1.7440
3.21	1.1663	3.84	1.3455	4.47	1.4974	5.10	1.6292	5.73	1.7457
3.22	1.1694	3.85	1.3481	4.48	1.4996	5.11	1.6312	5.74	1.7475
3.23	1.1725	3.86	1.3507	4.49	1.5019	5.12	1.6332	5.75	1.7492
3.24	1.1756	3.87	1.3533	4.50	1.5041	5.13	1.6351	5.76	1.7509
3.25	1.1787	3.88	1.3558	4.51	1.5063	5.14	1.6371	5.77	1.7527
3.26	1.1817	3.89	1.3584	4.52	1.5085	5.15	1.6390	5.78	1.7544
3.27	1.1848	3.90	1.3610	4.53	1.5107	5.16	1.6409	5.79	1.7561
3.28	1.1878	3.91	1.3635	4.54	1.5129	5.17	1.6429	5.80	1.7579
3.29	1.1909	3.92	1.3661	4.55	1.5151	5.18	1.6448	5.81	1.7596
3.30	1.1939	3.93	1.3686	4.56	1.5173	5.19	1.6467	5.82	1.7613
3.31	1.1969	3.94	1.3712	4.57	1.5195	5.20	1.6487	5.83	1.7630
3.32	1.1999	3.95	1.3737	4.58	1.5217	5.21	1.6506	5.84	1.7647
3.33	1.2030	3.96	1.3762	4.59	1.5239	5.22	1.6525	5.85	1.7664

TABLE 8.—HYPERBOLIC LOGARITHMS—Continued.

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
5.86	1.7681	6.49	1.8703	7.12	1.9629	7.75	2.0477	8.56	2.1471
5.87	1.7699	6.50	1.8718	7.13	1.9643	7.76	2.0490	8.58	2.1494
5.88	1.7716	6.51	1.8733	7.14	1.9657	7.77	2.0503	8.60	2.1518
5.89	1.7733	6.52	1.8749	7.15	1.9671	7.78	2.0516	8.62	2.1541
5.90	1.7750	6.53	1.8764	7.16	1.9685	7.79	2.0528	8.64	2.1564
5.91	1.7766	6.54	1.8779	7.17	1.9699	7.80	2.0541	8.66	2.1587
5.92	1.7783	6.55	1.8795	7.18	1.9713	7.81	2.0554	8.68	2.1610
5.93	1.7800	6.56	1.8810	7.19	1.9727	7.82	2.0567	8.70	2.1633
5.94	1.7817	6.57	1.8825	7.20	1.9741	7.83	2.0580	8.72	2.1656
5.95	1.7834	6.58	1.8840	7.21	1.9754	7.84	2.0592	8.74	2.1679
5.96	1.7851	6.59	1.8856	7.22	1.9769	7.85	2.0605	8.76	2.1702
5.97	1.7867	6.60	1.8871	7.23	1.9782	7.86	2.0618	8.78	2.1725
5.98	1.7884	6.61	1.8886	7.24	1.9796	7.87	2.0631	8.80	2.1748
5.99	1.7901	6.62	1.8901	7.25	1.9810	7.88	2.0643	8.82	2.1770
6.00	1.7918	6.63	1.8916	7.26	1.9824	7.89	2.0656	8.84	2.1793
6.01	1.7934	6.64	1.8931	7.27	1.9838	7.90	2.0669	8.86	2.1815
6.02	1.7951	6.65	1.8946	7.28	1.9851	7.91	2.0681	8.88	2.1838
6.03	1.7967	6.66	1.8961	7.29	1.9865	7.92	2.0694	8.90	2.1861
6.04	1.7984	6.67	1.8976	7.30	1.9879	7.93	2.0707	8.92	2.1883
6.05	1.8001	6.68	1.8991	7.31	1.9892	7.94	2.0719	8.94	2.1905
6.06	1.8017	6.69	1.9006	7.32	1.9906	7.95	2.0732	8.96	2.1928
6.07	1.8034	6.70	1.9021	7.33	1.9920	7.96	2.0744	8.98	2.1950
6.08	1.8050	6.71	1.9036	7.34	1.9933	7.97	2.0757	9.00	2.1972
6.09	1.8066	6.72	1.9051	7.35	1.9947	7.98	2.0769	9.02	2.1994
6.10	1.8083	6.73	1.9066	7.36	1.9961	7.99	2.0782	9.04	2.2017
6.11	1.8099	6.74	1.9081	7.37	1.9974	8.00	2.0794	9.06	2.2039
6.12	1.8116	6.75	1.9095	7.38	1.9988	8.01	2.0807	9.08	2.2061
6.13	1.8132	6.76	1.9110	7.39	2.0001	8.02	2.0819	9.10	2.2083
6.14	1.8148	6.77	1.9125	7.40	2.0015	8.03	2.0832	9.12	2.2105
6.15	1.8165	6.78	1.9140	7.41	2.0028	8.04	2.0844	9.14	2.2127
6.16	1.8181	6.79	1.9155	7.42	2.0041	8.05	2.0857	9.16	2.2148
6.17	1.8197	6.80	1.9169	7.43	2.0055	8.06	2.0869	9.18	2.2170
6.18	1.8213	6.81	1.9184	7.44	2.0069	8.07	2.0882	9.20	2.2192
6.19	1.8229	6.82	1.9199	7.45	2.0082	8.08	2.0894	9.22	2.2214
6.20	1.8245	6.83	1.9213	7.46	2.0096	8.09	2.0906	9.24	2.2235
6.21	1.8262	6.84	1.9228	7.47	2.0108	8.10	2.0919	9.26	2.2257
6.22	1.8278	6.85	1.9242	7.48	2.0122	8.11	2.0931	9.28	2.2279
6.23	1.8294	6.86	1.9257	7.49	2.0136	8.12	2.0943	9.30	2.2300
6.24	1.8310	6.87	1.9272	7.50	2.0149	8.13	2.0956	9.32	2.2322
6.25	1.8326	6.88	1.9286	7.51	2.0162	8.14	2.0968	9.34	2.2343
6.26	1.8342	6.89	1.9301	7.52	2.0176	8.15	2.0980	9.36	2.2364
6.27	1.8358	6.90	1.9315	7.53	2.0189	8.16	2.0992	9.38	2.2386
6.28	1.8374	6.91	1.9330	7.54	2.0202	8.17	2.1005	9.40	2.2407
6.29	1.8390	6.92	1.9344	7.55	2.0215	8.18	2.1017	9.42	2.2428
6.30	1.8405	6.93	1.9359	7.56	2.0229	8.19	2.1029	9.44	2.2450
6.31	1.8421	6.94	1.9373	7.57	2.0242	8.20	2.1041	9.46	2.2471
6.32	1.8437	6.95	1.9387	7.58	2.0255	8.22	2.1066	9.48	2.2492
6.33	1.8453	6.96	1.9402	7.59	2.0268	8.24	2.1090	9.50	2.2513
6.34	1.8469	6.97	1.9416	7.60	2.0281	8.26	2.1114	9.52	2.2534
6.35	1.8485	6.98	1.9430	7.61	2.0295	8.28	2.1138	9.54	2.2555
6.36	1.8500	6.99	1.9445	7.62	2.0308	8.30	2.1163	9.56	2.2576
6.37	1.8516	7.00	1.9459	7.63	2.0321	8.32	2.1187	9.58	2.2597
6.38	1.8532	7.01	1.9473	7.64	2.0334	8.34	2.1211	9.60	2.2618
6.39	1.8547	7.02	1.9488	7.65	2.0347	8.36	2.1235	9.62	2.2638
6.40	1.8563	7.03	1.9502	7.66	2.0360	8.38	2.1258	9.64	2.2659
6.41	1.8579	7.04	1.9516	7.67	2.0373	8.40	2.1282	9.66	2.2680
6.42	1.8594	7.05	1.9530	7.68	2.0386	8.42	2.1306	9.68	2.2701
6.43	1.8610	7.06	1.9544	7.69	2.0399	8.44	2.1330	9.70	2.2721
6.44	1.8625	7.07	1.9559	7.70	2.0412	8.46	2.1353	9.72	2.2742
6.45	1.8641	7.08	1.9573	7.71	2.0425	8.48	2.1377	9.74	2.2762
6.46	1.8656	7.09	1.9587	7.72	2.0438	8.50	2.1401	9.76	2.2783
6.47	1.8672	7.10	1.9601	7.73	2.0451	8.52	2.1424	9.78	2.2803
6.48	1.8687	7.11	1.9615	7.74	2.0464	8.54	2.1448	9.80	2.2824

Table 8.—HYPERBOLIC LOGARITHMS—Continued.

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
9.82	2.2844	11.25	2.4201	14.75	2.6913	23	3.1355	37	3.6109
9.84	2.2865	11.50	2.4430	15.00	2.7081	24	3.1781	38	3.6376
9.86	2.2885	11.75	2.4636	15.50	2.7408	25	3.2189	39	3.6636
9.88	2.2905	12.00	2.4849	16.00	2.7726	26	3.2581	40	3.6889
9.90	2.2925	12.25	2.5052	16.50	2.8034	27	3.2958	41	3.7136
9.92	2.2946	12.50	2.5262	17.00	2.8332	28	3.3322	42	3.7377
9.94	2.2966	12.75	2.5455	17.50	2.8621	29	3.3673	43	3.7612
9.96	2.2986	13.00	2.5649	18.00	2.8904	30	3.4012	44	3.7842
9.98	2.3006	13.25	2.5840	18.50	2.9178	31	3.4340	45	3.8067
10.00	2.3026	13.50	2.6027	19.00	2.9444	32	3.4657	46	3.8286
10.25	2.3279	13.75	2.6211	19.50	2.9703	33	3.4965	47	5.8501
10.50	2.3513	14.00	2.6391	20.00	2.9957	34	3.5263	48	3.8712
10.75	2.3749	14.25	2.6567	21	3.0445	35	3.5553	49	3.8918
11.00	2.3979	14.50	2.6740	22	3.0910	36	3.5835	50	3.9120

98257—19—No. 1——6

V.—TRIGONOMETRIC FUNCTIONS.

Natural sines and cosines.

Natural sines at intervals of 0°.1, or 6′. (For 10′ intervals see pp. 88–92.)

Deg.	°.0 =(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. diff.
											0.0000	90°	
0°	0.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	0175	89	17
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	0349	88	17
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	0523	87	17
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	0698	86	17
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	0.0872	85	17
5	0.0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	1045	84	17
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	1219	83	17
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	1392	82	17
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	1564	81	17
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	0.1736	80°	17
10°	0.1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	1908	79	17
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	2079	78	17
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	2250	77	17
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	2419	76	17
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	0.2588	75	17
15	0.2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	2756	74	17
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	2924	73	17
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3090	72	17
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3256	71	17
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	0.3420	70°	16
20°	0.3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3584	69	16
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3746	68	16
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3907	67	16
23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	4067	66	16
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	0.4226	65	16
25	0.4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	4384	64	16
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	4540	63	16
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	4695	62	16
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	4848	61	15
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	0.5000	60°	15
30°	0.5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	5150	59	15
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	5299	58	15
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	5446	57	15
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	5592	56	15
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	0.5736	55	14
35	0.5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	5878	54	14
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	6018	53	14
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	6157	52	14
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	6293	51	14
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	0.6428	50°	13
40°	0.6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	6561	49	13
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	6691	48	13
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	6820	47	13
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	6947	46	13
44	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	0.7071	45°	12
45°	0.7071												
		°.9 (54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

Natural cosines.

Natural sines and cosines—Continued.

Natural sines at intervals of 0°.1, or 6′. (For 10′ intervals see pp. 88–92.)

Deg.	°.0 =(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. diff.
											0.7071	45°	
45°	0.7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	7193	44	12
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	7314	43	12
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	7431	42	12
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	7547	41	12
49	7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	0.7660	40°	11
50°	0.7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	7771	39	11
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	7880	38	11
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	7986	37	11
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	8090	36	10
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	0.8192	35	10
55	0.8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	8290	34	10
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	8387	33	10
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	8480	32	9
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	8572	31	9
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	0.8660	30°	9
60°	0.8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	8746	29	9
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	8829	28	8
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	8910	27	8
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	8988	26	8
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	0.9063	25	7
65	0.9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	9135	24	7
66	9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	9205	23	7
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	9272	22	7
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	9336	21	6
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	0.9397	20°	6
70°	0.9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	9455	19	6
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	9511	18	6
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	9563	17	5
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	9613	16	5
74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	0.9659	15	5
75	0.9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	9703	14	4
76	9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	9744	13	4
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	9781	12	4
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	9816	11	3
79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	0.9848	10°	3
80°	0.9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	9877	9	3
81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	9903	8	3
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	9925	7	2
83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	9945	6	2
84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0.9962	5	2
85	0.9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	9976	4	1
86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	9986	3	1
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	9994	2	1
88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0.9998	1	0
89	0.9998	9999	9999	9999	9999	0000	0000	0000	0000	0000	1.0000	0°	0
90°	1.0000												
		°.9 =(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

Natural cosines.

Natural tangents and cotangents.

Natural tangents at intervals of 0°.1, or 6′. (For 10′ intervals. see pp. 88–92.)

Deg.	°.0 =(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. diff.
											0. 0000	90°	
0°	0. 0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	0175	89	17
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	0349	88	17
2	0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	0524	87	17
3	0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	0699	86	18
4	0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	0. 0875	85	18
5	0. 0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	1051	84	18
6	1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	1228	83	18
7	1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	1405	82	18
8	1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	1584	81	18
9	1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	0. 1763	80°	18
10°	0. 1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	1944	79	18
11	1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	2126	78	18
12	2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	2309	77	18
13	2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	2493	76	18
14	2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	0. 2679	75	19
15	0. 2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	2867	74	19
16	2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3057	73	19
17	3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3249	72	19
18	3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3443	71	19
19	3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	0. 3640	70°	20
20°	0. 3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3839	69	20
21	3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	4040	68	20
22	4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	4245	67	21
23	4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	4452	66	21
24	4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	0. 4663	65	21
25	0. 4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4877	64	21
26	4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	5095	63	22
27	5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	5317	62	22
28	5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	5543	61	23
29	5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	0. 5774	60°	23
30°	0. 5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	6009	59	24
31	6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	6249	58	24
32	6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	6494	57	25
33	6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	6745	56	25
34	6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	0. 7002	55	26
35	0. 7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	7265	54	26
36	7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	7536	53	27
37	7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	7813	52	28
38	7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	8098	51	28
39	8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	0. 8391	50°	29
40°	0. 8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	8693	49	30
41	8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	9004	48	31
42	9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	9325	47	32
43	9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	0. 9657	46	33
44	0. 9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	1. 0000	45°	34
45°	1. 0000												
		°.9 =(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

Natural cotangents.

Natural tangents and cotangents—Continued.

Natural tangents at intervals of 0°.1, or 6′. (For 10′ intervals, see pp. 88–92.)

Deg.	°.0 =(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. diff.
											1. 0000	45°	
45°	1. 0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	0355	44	35
46	0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	0724	43	37
47	0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	1106	42	38
48	1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	1504	41	40
49	1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	1. 1918	40°	41
50°	1. 1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	2349	39	43
51	2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	2799	38	45
52	2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	3270	37	47
53	3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	3764	36	49
54	3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	1. 4281	35	52
55	1. 4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	4826	34	55
56	4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	5399	33	57
57	5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	6003	32	60
58	6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	6643	31	64
59	1. 6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	1. 7321	30°	67
60°	1.732	1.739	1.746	1.753	1.760	1.767	1.775	1.782	1.789	1.797	1.804	29	7
61	1.804	1.811	1.819	1.827	1.834	1.842	1.849	1.857	1.865	1.873	1.881	28	8
62	1.881	1.889	1.897	1.905	1.913	1.921	1.929	1.937	1.946	1.954	1.963	27	8
63	1.963	1.971	1.980	1.988	1.997	2.006	2.014	2.023	2.032	2.041	2.050	26	9
64	2.050	2.059	2.069	2.078	2.087	2.097	2.106	2.116	2.125	2.135	2.145	25	9
65	2.145	2.154	2.164	2.174	2.184	2.194	2.204	2.215	2.225	2.236	2.246	24	10
66	2.246	2.257	2.267	2.278	2.289	2.300	2.311	2.322	2.333	2.344	2.356	23	11
67	2.356	2.367	2.379	2.391	2.402	2.414	2.426	2.438	2.450	2.463	2.475	22	12
68	2.475	2.488	2.500	2.513	2.526	2.539	2.552	2.565	2.578	2.592	2.605	21	13
69	2.605	2.619	2.633	2.646	2.660	2.675	2.689	2.703	2.718	2.733	2.747	20°	14
70°	2.747	2.762	2.778	2.793	2.808	2.824	2.840	2.856	2.872	2.888	2.904	19	16
71	2.904	2.921	2.937	2.954	2.971	2.989	3.006	3.024	3.042	3.060	3.078	18	17
72	3.078	3.096	3.115	3.133	3.152	3.172	3.191	3.211	3.230	3.251	3.271	17	19
73	3.271	3.291	3.312	3.333	3.354	3.376	3.398	3.420	3.442	3.465	3.487	16	22
74	3.487	3.511	3.534	3.558	3.582	3.606	3.630	3.655	3.681	3.706	3.732	15	24
75	3.732	3.758	3.785	3.812	3.839	3.867	3.895	3.923	3.952	3.981	4.011	14	28
76	4.011	4.041	4.071	4.102	4.134	4.165	4.198	4.230	4.264	4.297	4.331	13	32
77	4.331	4.366	4.402	4.437	4.474	4.511	4.548	4.586	4.625	4.665	4.705	12	37
78	4.705	4.745	4.787	4.829	4.872	4.915	4.959	5.005	5.050	5.097	5.145	11	44
79	5.145	5.193	5.242	5.292	5.343	5.396	5.449	5.503	5.558	5.614	5.671	10°	53
80°	5.671	5.730	5.789	5.850	5.912	5.976	6.041	6.107	6.174	6.243	6.314	9	
81	6.314	6.386	6.460	6.535	6.612	6.691	6.772	6.855	6.940	7.026	7.115	8	
82	7.115	7.207	7.300	7.396	7.495	7.596	7.700	7.806	7.916	8.028	8.144	7	
83	8.144	8.264	8.386	8.513	8.643	8.777	8.915	9.058	9.205	9.357	9.514	6	
84	9.514	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20	11.43	5	
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95	14.30	4	
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46	19.08	3	
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27	28.64	2	
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08	57.29	1	
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0	∞	0°	
90°	∞												
		°.9 =(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

Natural cotangents.

Natural secants and cosecants.

Natural secants at intervals of 0°.1, or 6′. (For 10′ intervals see pp. 88–92.)

Deg.	°.0 =(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. diff.
											1.0000	90°	
0°	1.0000	0000	0000	0000	0000	0000	0001	0001	0001	0001	0002	89	0
1	0002	0002	0002	0003	0003	0003	0004	0004	0005	0006	0006	88	0
2	0006	0007	0007	0008	0009	0010	0010	0011	0012	0013	0014	87	1
3	0014	0015	0016	0017	0018	0019	0020	0021	0022	0023	0024	86	1
4	0024	0026	0027	0028	0030	0031	0032	0034	0035	0037	1.0038	85	1
5	1.0038	0040	0041	0043	0045	0046	0048	0050	0051	0053	0055	84	2
6	0055	0057	0059	0061	0063	0065	0067	0069	0071	0073	0075	83	2
7	0075	0077	0079	0082	0084	0086	0089	0091	0093	0096	0098	82	2
8	0098	0101	0103	0106	0108	0111	0114	0116	0119	0122	0125	81	3
9	0125	0127	0130	0133	0136	0139	0142	0145	0148	0151	1.0154	80°	3
10°	1.0154	0157	0161	0164	0167	0170	0174	0177	0180	0184	0187	79	3
11	0187	0191	0194	0198	0201	0205	0209	0212	0216	0220	0223	78	4
12	0223	0227	0231	0235	0239	0243	0247	0251	0255	0259	0263	77	4
13	0263	0267	0271	0276	0280	0284	0288	0293	0297	0302	0306	76	4
14	0306	0311	0315	0320	0324	0329	0334	0338	0343	0348	1.0353	75	5
15	1.0353	0358	0363	0367	0372	0377	0382	0388	0393	0398	0403	74	5
16	0403	0408	0413	0419	0424	0429	0435	0440	0446	0451	0457	73	5
17	0457	0463	0468	0474	0480	0485	0491	0497	0503	0509	0515	72	6
18	0515	0521	0527	0533	0539	0545	0551	0557	0564	0570	0576	71	6
19	0576	0583	0589	0595	0602	0608	0615	0622	0628	0635	1.0642	70°	7
20°	1.0642	0649	0655	0662	0669	0676	0683	0690	0697	0704	0711	69	7
21	0711	0719	0726	0733	0740	0748	0755	0763	0770	0778	0785	68	7
22	0785	0793	0801	0808	0816	0824	0832	0840	0848	0856	0864	67	8
23	0864	0872	0880	0888	0896	0904	0913	0921	0929	0938	0946	66	8
24	0946	0955	0963	0972	0981	0989	0998	1007	1016	1025	1.1034	65	9
25	1.1034	1043	1052	1061	1070	1079	1089	1098	1107	1117	1126	64	9
26	1126	1136	1145	1155	1164	1174	1184	1194	1203	1213	1223	63	10
27	1223	1233	1243	1253	1264	1274	1284	1294	1305	1315	1326	62	10
28	1326	1336	1347	1357	1368	1379	1390	1401	1412	1423	1434	61	11
29	1434	1445	1456	1467	1478	1490	1501	1512	1524	1535	1.1547	60°	11
30°	1.1547	1559	1570	1582	1594	1606	1618	1630	1642	1654	1666	59	12
31	1666	1679	1691	1703	1716	1728	1741	1753	1766	1779	1792	58	13
32	1792	1805	1818	1831	1844	1857	1870	1883	1897	1910	1924	57	13
33	1924	1937	1951	1964	1978	1992	2006	2020	2034	2048	2062	56	14
34	2062	2076	2091	2105	2120	2134	2149	2163	2178	2193	1.2208	55	15
35	1.2208	2223	2238	2253	2268	2283	2299	2314	2329	2345	2361	54	15
36	2361	2376	2392	2408	2424	2440	2456	2472	2489	2505	2521	53	16
37	2521	2538	2554	2571	2588	2605	2622	2639	2656	2673	2690	52	17
38	2690	2708	2725	2742	2760	2778	2796	2813	2831	2849	2868	51	18
39	2868	2886	2904	2923	2941	2960	2978	2997	3016	3035	1.3054	50°	19
40°	1.3054	3073	3093	3112	3131	3151	3171	3190	3210	3230	3250	49	20
41	3250	3270	3291	3311	3331	3352	3373	3393	3414	3435	3456	48	21
42	3456	3478	3499	3520	3542	3563	3585	3607	3629	3651	3673	47	22
43	3673	3696	3718	3741	3763	3786	3809	3832	3855	3878	3902	46	23
44	3902	3925	3949	3972	3996	4020	4044	4069	4093	4118	1.4142	45°	24
45°	1.4142												
		°.9 =(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

Natural cosecants.

Natural secants and cosecants.—Continued.

Natural secants at intervals of 0°.1, or 6′. (For 10′ intervals see pp. 88–92.)

Deg.	°.0 =(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. diff.
											1.4142	45°	
45°	1.4142	4167	4192	4217	4242	4267	4293	4318	4344	4370	4396	44	25
46	4396	4422	4448	4474	4501	4527	4554	4581	4608	4635	4663	43	27
47	4663	4690	4718	4746	4774	4802	4830	4859	4887	4916	4945	42	28
48	4945	4974	5003	5032	5062	5092	5121	5151	5182	5212	5243	41	30
49	5243	5273	5304	5335	5366	5398	5429	5461	5493	5525	1.5557	40°	31
50°	1.5557	5590	5622	5655	5688	5721	5755	5788	5822	5856	5890	39	33
51	5890	5925	5959	5994	6029	6064	6099	6135	6171	6207	6243	38	35
52	6243	6279	6316	6353	6390	6427	6464	6502	6540	6578	6616	37	37
53	6616	6655	6694	6733	6772	6812	6852	6892	6932	6972	7013	36	40
54	7013	7054	7095	7137	7179	7221	7263	7305	7348	7391	1.7434	35	42
55	1.7434	7478	7522	7566	7610	7655	7700	7745	7791	7837	7883	34	45
56	7883	7929	7976	8023	8070	8118	8166	8214	8263	8312	8361	33	48
57	8361	8410	8460	8510	8561	8612	8663	8714	8766	8818	8871	32	51
58	8871	8924	8977	9031	9084	9139	9194	9249	9304	9360	1.9416	31	54
59	1.9416	9473	9530	9587	9645	9703	9762	9821	9880	9940	2.0000	30°	58
60°	2.000	2.006	2.012	2.018	2.025	2.031	2.037	2.043	2.050	2.056	2.063	29	6
61	2.063	2.069	2.076	2.082	2.089	2.096	2.103	2.109	2.116	2.123	2.130	28	7
62	2.130	2.137	2.144	2.151	2.158	2.166	2.173	2.180	2.188	2.195	2.203	27	7
63	2.203	2.210	2.218	2.226	2.233	2.241	2.249	2.257	2.265	2.273	2.281	26	8
64	2.281	2.289	2.298	2.306	2.314	2.323	2.331	2.340	2.349	2.357	2.366	25	8
65	2.366	2.375	2.384	2.393	2.402	2.411	2.421	2.430	2.439	2.449	2.459	24	9
66	2.459	2.468	2.478	2.488	2.498	2.508	2.518	2.528	2.538	2.549	2.559	23	10
67	2.559	2.570	2.581	2.591	2.602	2.613	2.624	2.635	2.647	2.658	2.669	22	11
68	2.669	2.681	2.693	2.705	2.716	2.729	2.741	2.753	2.765	2.778	2.790	21	12
69	2.790	2.803	2.816	2.829	2.842	2.855	2.869	2.882	2.896	2.910	2.924	20°	13
70°	2.924	2.938	2.952	2.967	2.981	2.996	3.011	3.026	3.041	3.056	3.072	19	15
71	3.072	3.087	3.103	3.119	3.135	3.152	3.168	3.185	3.202	3.219	3.236	18	16
72	3.236	3.254	3.271	3.289	3.307	3.326	3.344	3.363	3.382	3.401	3.420	17	18
73	3.420	3.440	3.460	3.480	3.500	3.521	3.542	3.563	3.584	3.606	3.628	16	21
74	3.628	3.650	3.673	3.695	3.719	3.742	3.766	3.790	3.814	3.839	3.864	15	24
75	3.864	3.889	3.915	3.941	3.967	3.994	4.021	4.049	4.077	4.105	4.134	14	27
76	4.134	4.163	4.192	4.222	4.253	4.284	4.315	4.347	4.379	4.412	4.445	13	31
77	4.445	4.479	4.514	4.549	4.584	4.620	4.657	4.694	4.732	4.771	4.810	12	36
78	4.810	4.850	4.890	4.931	4.973	5.016	5.059	5.103	5.148	5.194	5.241	11	43
79	5.241	5.288	5.337	5.386	5.436	5.487	5.540	5.593	5.647	5.702	5.759	10°	52
80°	5.759	5.816	5.875	5.935	5.996	6.059	6.123	6.188	6.255	6.323	6.392	9	
81	6.392	6.464	6.537	6.611	6.687	6.765	6.845	6.927	7.011	7.097	7.185	8	
82	7.185	7.276	7.368	7.463	7.561	7.661	7.764	7.870	7.979	8.091	8.206	7	
83	8.206	8.324	8.446	8.571	8.700	8.834	8.971	9.113	9.259	9.411	9.567	6	
84	9.567	9.728	9.895	10.07	10.25	10.43	10.63	10.83	11.03	11.25	11.47	5	
85	11.47	11.71	11.95	12.20	12.47	12.75	13.03	13.34	13.65	13.99	14.34	4	
86	14.34	14.70	15.09	15.50	15.93	16.38	16.86	17.37	17.91	18.49	19.11	3	
87	19.11	19.77	20.47	21.23	22.04	22.93	23.88	24.92	26.05	27.29	28.65	2	
88	28.65	30.16	31.84	33.71	35.81	38.20	40.93	44.08	47.75	52.09	57.30	1	
89	57.30	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0	∞	0°	
90°	∞												
		°.9 =(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

Natural cosecants.

Trigonometric functions (at intervals of 10′).

Annex—10 in columns marked *. (For 0.°1 intervals, see pp. 82–87.)

Degrees.	Radians.	Sines.		Cosines.		Tangents.		Cotangents.			
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
0° 00′	0.0000	.0000	∞	1.0000	0.0000	.0000	∞	∞	∞	1.5708	90° 00′
10	0.0029	.0029	7.4637	1.0000	.0000	.0029	7.4637	343.77	2.5363	1.5679	50
20	0.0058	.0058	.7648	1.0000	.0000	.0058	.7648	171.89	.2352	1.5650	40
30	0.0087	.0087	.9408	1.0000	.0000	.0087	.9409	114.59	.0591	1.5621	30
40	0.0116	.0116	8.0658	0.9999	.0000	.0116	8.0658	85.940	1.9342	1.5592	20
50	0.0145	.0145	.1627	.9999	.0000	.0145	.1627	68.750	.8373	1.5563	10
1° 00′	0.0175	.0175	8.2419	.9998	9.9999	.0175	8.2419	57.290	1.7581	1.5533	89° 00′
10	0.0204	.0204	.3088	.9998	.9999	.0204	.3089	49.104	.6911	1.5504	50
20	0.0233	.0233	.3668	.9997	.9999	.0233	.3669	42.964	.6331	1.5475	40
30	0.0262	.0262	.4179	.9997	.9999	.0262	.4181	38.188	.5819	1.5446	30
40	0.0291	.0291	.4637	.9996	.9998	.0291	.4638	34.368	.5362	1.5417	20
50	0.0320	.0320	.5050	.9995	.9998	.0320	.5053	31.242	.4947	1.5388	10
2° 00′	0.0349	.0349	8.5428	.9994	9.9997	.0349	8.5431	28.636	1.4569	1.5359	88° 00′
10	0.0378	.0378	.5776	.9993	.9997	.0378	.5779	26.432	.4221	1.5330	50
20	0.0407	.0407	.6097	.9992	.9996	.0407	.6101	24.542	.3899	1.5301	40
30	0.0436	.0436	.6397	.9990	.9996	.0437	.6401	22.904	.3599	1.5272	30
40	0.0465	.0465	.6677	.9989	.9995	.0466	.6682	21.470	.3318	1.5243	20
50	0.0495	.0494	.6940	.9988	.9995	.0495	.6945	20.206	.3055	1.5213	10
3° 00′	0.0524	.0523	8.7188	.9986	9.9994	.0524	8.7194	19.081	1.2806	1.5184	87° 00′
10	0.0553	.0552	.7423	.9985	.9993	.0553	.7429	18.075	.2571	1.5155	50
20	0.0582	.0581	.7645	.9983	.9993	.0582	.7652	17.169	.2348	1.5126	40
30	0.0611	.0610	.7857	.9981	.9992	.0612	.7865	16.350	.2135	1.5097	30
40	0.0640	.0640	.8059	.9980	.9991	.0641	.8067	15.605	.1933	1.5068	20
50	0.0669	.0669	.8251	.9978	.9990	.0670	.8261	14.924	.1739	1.5039	10
4° 00′	0.0698	.0698	8.8436	.9976	9.9989	.0699	8.8446	14.301	1.1554	1.5010	86° 00′
10	0.0727	.0727	.8613	.9974	.9989	.0729	.8624	13.727	.1376	1.4981	50
20	0.0756	.0756	.8783	.9971	.9988	.0758	.8795	13.197	.1205	1.4952	40
30	0.0785	.0785	.8946	.9969	.9987	.0787	.8960	12.706	.1040	1.4923	30
40	0.0814	.0814	.9104	.9967	.9986	.0816	.9118	12.251	.0882	1.4893	20
50	0.0844	.0843	.9256	.9964	.9985	.0846	.9272	11.826	.0728	1.4864	10
5° 00′	0.0873	.0872	8.9403	.9962	9.9983	.0875	8.9420	11.430	1.0580	1.4835	85° 00′
10	0.0902	.0901	.9545	.9959	.9982	.0904	.9563	11.059	.0437	1.4806	50
20	0.0931	.0929	.9682	.9957	.9981	.0934	.9701	10.712	.0299	1.4777	40
30	0.0960	.0958	.9816	.9954	.9980	.0963	.9836	10.385	.0164	1.4748	30
40	0.0989	.0987	.9945	.9951	.9979	.0992	.9966	10.078	.0034	1.4719	20
50	0.1018	.1016	9.0070	.9948	.9977	.1022	9.0093	9.7882	0.9907	1.4690	10
6° 00′	0.1047	.1045	9.0192	.9945	9.9976	.1051	9.0216	9.5144	0.9784	1.4661	84° 00′
10	0.1076	.1074	.0311	.9942	.9975	.1080	.0336	9.2553	.9664	1.4632	50
20	0.1105	.1103	.0426	.9939	.9973	.1110	.0453	9.0098	.9547	1.4603	40
30	0.1134	.1132	.0539	.9936	.9972	.1139	.0567	8.7769	.9433	1.4574	30
40	0.1164	.1161	.0648	.9932	.9971	.1169	.0678	8.5555	.9322	1.4544	20
50	0.1193	.1190	.0755	.9929	.9969	.1198	.0786	8.3450	.9214	1.4515	10
7° 00′	0.1222	.1219	9.0859	.9925	9.9968	.1228	9.0891	8.1443	0.9109	1.4486	83° 00′
10	0.1251	.1248	.0961	.9922	.9966	.1257	.0995	7.9530	.9005	1.4457	50
20	0.1280	.1276	.1060	.9918	.9964	.1287	.1096	7.7704	.8904	1.4428	40
30	0.1309	.1305	.1157	.9914	.9963	.1317	.1194	7.5958	.8806	1.4399	30
40	0.1338	.1334	.1252	.9911	.9961	.1346	.1291	7.4287	.8709	1.4370	20
50	0.1367	.1363	.1543	.9907	.9959	.1376	.1385	7.2687	.8615	1.4341	10
8° 00′	0.1396	.1392	9.1436	.9903	9.9958	.1405	9.1478	7.1154	0.8522	1.4312	82° 00′
10	0.1425	.1421	.1525	.9899	.9956	.1435	.1569	6.9682	.8431	1.4283	50
20	0.1454	.1449	.1612	.9894	.9954	.1465	.1658	6.8269	.8342	1.4254	40
30	0.1484	.1478	.1697	.9890	.9952	.1495	.1745	6.6912	.8255	1.4224	30
40	0.1513	.1507	.1781	.9886	.9950	.1524	.1831	6.5606	.8169	1.4195	20
50	0.1542	.1536	.1863	.9881	.9948	.1554	.1915	6.4348	.8085	1.4166	10
9° 00′	0.1571	.1564	9.1943	.9877	9.9946	.1584	9.1997	6.3138	0.8003	1.4137	81° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines.		Sines.		Cotangents.		Tangents.		Radians.	Degrees.

Trigonometric functions—Continued.

Annex—10 in columns marked *. (For 0.°1 intervals, see pp. 82–87.)

Degrees.	Radians.	Sines.		Cosines.		Tangents.		Cotangents.			
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
9° 00′	0.1571	.1564	9.1943	.9877	9.9946	.1584	9.1997	6.3138	0.8003	1.4137	81° 00′
10	0.1600	.1593	.2022	.9872	.9944	.1614	.2078	6.1970	.7922	1.4108	50
20	0.1629	.1622	.2100	.9868	.9942	.1644	.2158	6.0844	.7842	1.4079	40
30	0.1658	.1650	.2176	.9863	.9940	.1673	.2236	5.9758	.7764	1.4050	30
40	0.1687	.1679	.2251	.9858	.9938	.1703	.2313	5.8708	.7687	1.4021	20
50	0.1716	.1708	.2324	.9853	.9936	.1733	.2389	5.7694	.7611	1.3992	10
10° 00′	0.1745	.1736	9.2397	.9848	9.9934	.1763	9.2463	5.6713	0.7537	1.3963	80° 00′
10	0.1774	.1765	.2468	.9843	.9931	.1793	.2536	5.5764	.7464	1.3934	50
20	0.1804	.1794	.2538	.9838	.9929	.1823	.2609	5.4845	.7391	1.3904	40
30	0.1833	.1822	.2606	.9833	.9927	.1853	.2680	5.3955	.7320	1.3875	30
40	0.1862	.1851	.2674	.9827	.9924	.1883	.2750	5.3093	.7250	1.3846	20
50	0.1891	.1880	.2740	.9822	.9922	.1914	.2819	5.2257	.7181	1.3817	10
11° 00′	0.1920	.1908	9.2806	.9816	9.9919	.1944	9.2887	5.1446	0.7113	1.3788	79° 00′
10	0.1949	.1937	.2870	.9811	.9917	.1974	.2953	5.0658	.7047	1.3759	50
20	0.1978	.1965	.2934	.9805	.9914	.2004	.3020	4.9894	.6980	1.3730	40
30	0.2007	.1994	.2997	.9799	.9912	.2035	.3085	4.9152	.6915	1.3701	30
40	0.2036	.2022	.3058	.9793	.9909	.2065	.3149	4.8430	.6851	1.3672	20
50	0.2065	.2051	.3119	.9787	.9907	.2095	.3212	4.7729	.6788	1.3643	10
12° 00′	0.2094	.2079	9.3179	.9781	9.9904	.2126	9.3275	4.7046	0.6725	1.3614	78° 00′
10	0.2123	.2108	.3238	.9775	.9901	.2156	.3336	4.6382	.6664	1.3584	50
20	0.2153	.2136	.3296	.9769	.9899	.2186	.3397	4.5736	.6603	1.3555	40
30	0.2182	.2164	.3353	.9763	.9896	.2217	.3458	4.5107	.6542	1.3526	30
40	0.2211	.2193	.3410	.9757	.9893	.2247	.3517	4.4494	.6483	1.3497	20
50	0.2240	.2221	.3466	.9750	.9890	.2278	.3576	4.3897	.6424	1.3468	10
13° 00′	0.2269	.2250	9.3521	.9744	9.9887	.2309	9.3634	4.3315	0.6366	1.3439	77° 00′
10	0.2298	.2278	.3575	.9737	.9884	.2339	.3691	4.2747	.6309	1.3410	50
20	0.2327	.2306	.3629	.9730	.9881	.2370	.3748	4.2193	.6252	1.3381	40
30	0.2356	.2334	.3682	.9724	.9878	.2401	.3804	4.1653	.6196	1.3352	30
40	0.2385	.2363	.3734	.9717	.9875	.2432	.3859	4.1126	.6141	1.3323	20
50	0.2414	.2391	.3786	.9710	.9872	.2462	.3914	4.0611	.6086	1.3294	10
14° 00′	0.2443	.2419	9.3837	.9703	9.9869	.2493	9.3968	4.0108	0.6032	1.3265	76° 00′
10	0.2473	.2447	.3887	.9696	.9866	.2524	.4021	3.9617	.5979	1.3235	50
20	0.2502	.2476	.3937	.9689	.9863	.2555	.4074	3.9136	.5926	1.3206	40
30	0.2531	.2504	.3986	.9681	.9859	.2586	.4127	3.8667	.5873	1.3177	30
40	0.2560	.2532	.4035	.9674	.9856	.2617	.4178	3.8208	.5822	1.3148	20
50	0.2589	.2560	.4083	.9667	.9853	.2648	.4230	3.7760	.5770	1.3119	10
15° 00′	0.2618	.2588	9.4130	.9659	9.9849	.2679	9.4281	3.7321	0.5719	1.3090	75° 00′
10	0.2647	.2616	.4177	.9652	.9846	.2711	.4331	3.6891	.5669	1.3061	50
20	0.2676	.2644	.4223	.9644	.9843	.2742	.4381	3.6470	.5619	1.3032	40
30	0.2705	.2672	.4269	.9636	.9839	.2773	.4430	3.6059	.5570	1.3003	30
40	0.2734	.2700	.4314	.9628	.9836	.2805	.4479	3.5656	.5521	1.2974	20
50	0.2763	.2728	.4359	.9621	.9832	.2836	.4527	3.5261	.5473	1.2945	10
16° 00′	0.2793	.2756	9.4403	.9613	9.9828	.2867	9.4575	3.4874	0.5425	1.2915	74° 00′
10	0.2822	.2784	.4447	.9605	.9825	.2899	.4622	3.4495	.5378	1.2886	50
20	0.2851	.2812	.4491	.9596	.9821	.2931	.4669	3.4124	.5331	1.2857	40
30	0.2880	.2840	.4533	.9588	.9817	.2962	.4716	3.3759	.5284	1.2828	30
40	0.2909	.2868	.4576	.9580	.9814	.2994	.4762	3.3402	.5238	1.2799	20
50	0.2938	.2896	.4618	.9572	.9810	.3026	.4808	3.3052	.5192	1.2770	10
17° 00′	0.2967	.2924	9.4659	.9563	9.9806	.3057	9.4853	3.2709	0.5147	1.2741	73° 00′
10	0.2996	.2952	.4700	.9555	.9802	.3089	.4898	3.2371	.5102	1.2712	50
20	0.3025	.2979	.4741	.9546	.9798	.3121	.4943	3.2041	.5057	1.2683	40
30	0.3054	.3007	.4781	.9537	.9794	.3153	.4987	3.1716	.5013	1.2654	30
40	0.3083	.3035	.4821	.9528	.9790	.3185	.5031	3.1397	.4969	1.2625	20
50	0.3113	.3062	.4861	.9520	.9786	.3217	.5075	3.1084	.4925	1.2595	10
18° 00′	0.3142	.3090	9.4900	.9511	9.9782	.3249	9.5118	3.0777	0.4882	1.2566	72° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines.		Sines.		Cotangents.		Tangents.		Radians.	Degrees.

Trigonometric functions—Continued.

Annex −10 in columns marked *. (For 0.°1 intervals see pp. 82–87.)

Degrees.	Radians.	Sines.		Cosines.		Tangents.		Cotangents.			
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
18° 00′	0. 3142	. 3090	9. 4900	. 9511	9. 9782	. 3249	9. 5118	3. 0777	0. 4882	1. 2566	72° 00′
10	0. 3171	. 3118	. 4939	. 9502	. 9778	. 3281	. 5161	3. 0475	. 4839	1. 2537	50
20	0. 3200	. 3145	. 4977	. 9492	. 9774	. 3314	. 5203	3. 0178	. 4797	1. 2508	40
30	0. 3229	. 3173	. 5015	. 9483	. 9770	. 3346	. 5245	2. 9887	. 4755	1. 2479	30
40	0. 3258	. 3201	. 5052	. 9474	. 9765	. 3378	. 5287	2. 9600	. 4713	1. 2450	20
50	0. 3287	. 3228	. 5090	. 9465	. 9761	. 3411	. 5329	2. 9319	. 4671	1. 2421	10
19° 00′	0. 3316	. 3256	9. 5126	. 9455	9. 9757	. 3443	9. 5370	2. 9042	0. 4630	1. 2392	71° 00′
10	0. 3345	. 3283	. 5163	. 9446	. 9752	. 3476	. 5411	2. 8770	. 4589	1. 2363	50
20	0. 3374	. 3311	. 5199	. 9436	. 9748	. 3508	. 5451	2. 8502	. 4549	1. 2334	40
30	0. 3403	. 3338	. 5235	. 9426	. 9743	. 3541	. 5491	2. 8239	. 4509	1. 2305	30
40	0. 3432	. 3365	. 5270	. 9417	. 9739	. 3574	. 5531	2. 7980	. 4469	1. 2275	20
50	0. 3462	. 3393	. 5306	. 9407	. 9734	. 3607	. 5571	2. 7725	. 4429	1. 2246	10
20° 00′	0. 3491	. 3420	9. 5341	. 9397	9. 9730	. 3640	9. 5611	2. 7475	0. 4389	1. 2217	70° 00′
10	0. 3520	. 3448	. 5375	. 9387	. 9725	. 3673	. 5650	2. 7228	. 4350	1. 2188	50
20	0. 3549	. 3475	. 5409	. 9377	. 9721	. 3706	. 5689	2. 6985	. 4311	1. 2159	40
30	0. 3578	. 3502	. 5443	. 9367	. 9716	. 3739	. 5727	2. 6746	. 4273	1. 2130	30
40	0. 3607	. 3529	. 5477	. 9356	. 9711	. 3772	. 5766	2. 6511	. 4234	1. 2101	20
50	0. 3636	. 3557	. 5510	. 9346	. 9706	. 3805	. 5804	2. 6279	. 4196	1. 2072	10
21° 00′	0. 3665	. 3584	9. 5543	. 9336	9. 9702	. 3839	9. 5842	2. 6051	0. 4158	1. 2043	69° 00′
10	0. 3694	. 3611	. 5576	. 9325	. 9697	. 3872	. 5879	2. 5826	. 4121	1. 2014	50
20	0. 3723	. 3638	. 5609	. 9315	. 9692	. 3906	. 5917	2. 5605	. 4083	1. 1985	40
30	0. 3752	. 3665	. 5641	. 9304	. 9687	. 3939	. 5954	2. 5386	. 4046	1. 1956	30
40	0. 3782	. 3692	. 5673	. 9293	. 9682	. 3973	. 5991	2. 5172	. 4009	1. 1926	20
50	0. 3811	. 3719	. 5704	. 9283	. 9677	. 4006	. 6028	2. 4960	. 3972	1. 1897	10
22° 00′	0. 3840	. 3746	9. 5736	. 9272	9. 9672	. 4040	9. 6064	2. 4751	0. 3936	1. 1868	68° 00′
10	0. 3869	. 3773	. 5767	. 9261	. 9667	. 4074	. 6100	2. 4545	. 3900	1. 1839	50
20	0. 3898	. 3800	. 5798	. 9250	. 9661	. 4108	. 6136	2. 4342	. 3864	1. 1810	40
30	0. 3927	. 3827	. 5828	. 9239	. 9656	. 4142	. 6172	2. 4142	. 3828	1. 1781	30
40	0. 3956	. 3854	. 5859	. 9228	. 9651	. 4176	. 6208	2. 3945	. 3792	1. 1752	20
50	0. 3985	. 3881	. 5889	. 9216	. 9646	. 4210	. 6243	2. 3750	. 3757	1. 1723	10
23° 00′	0. 4014	. 3907	9. 5919	. 9205	9. 9640	. 4245	9. 6279	2. 3559	0. 3721	1. 1694	67° 00′
10	0. 4043	. 3934	. 5948	. 9194	. 9635	. 4279	. 6314	2. 3369	. 3686	1. 1665	50
20	0. 4072	. 3961	. 5978	. 9182	. 9629	. 4314	. 6348	2. 3183	. 3652	1. 1636	40
30	0. 4102	. 3987	. 6007	. 9171	. 9624	. 4348	. 6383	2. 2998	. 3617	1. 1606	30
40	0. 4131	. 4014	. 6036	. 9159	. 9618	. 4383	. 6417	2. 2817	. 3583	1. 1577	20
50	0. 4160	. 4041	. 6065	. 9147	. 9613	. 4417	. 6452	2. 2637	. 3548	1. 1548	10
24° 00′	0. 4189	. 4067	9. 6093	. 9135	9. 9607	. 4452	9. 6486	2. 2460	0. 3514	1. 1519	66° 00′
10	0. 4218	. 4094	. 6121	. 9124	. 9602	. 4487	. 6520	2. 2286	. 3480	1. 1490	50
20	0. 4247	. 4120	. 6149	. 9112	. 9596	. 4522	. 6553	2. 2113	. 3447	1. 1461	40
30	0. 4276	. 4147	. 6177	. 9100	. 9590	. 4557	. 6587	2. 1943	. 3413	1. 1432	30
40	0. 4305	. 4173	. 6205	. 9088	. 9584	. 4592	. 6620	2. 1775	. 3380	1. 1403	20
50	0. 4334	. 4100	. 6232	. 9075	. 9579	. 4628	. 6654	2. 1609	. 3346	1. 1374	10
25° 00′	0. 4363	. 4226	9. 6259	. 9063	9. 9573	. 4663	9. 6687	2. 1445	0. 3313	1. 1345	65° 00′
10	0. 4392	. 4253	. 6286	. 9051	. 9567	. 4699	. 6720	2. 1283	. 3280	1. 1316	50
20	0. 4422	. 4279	. 6313	. 9038	. 9561	. 4734	. 6752	2. 1123	. 3248	1. 1286	40
30	0. 4451	. 4305	. 6340	. 9026	. 9555	. 4770	. 6785	2. 0965	. 3215	1. 1257	30
40	0. 4480	. 4331	. 6366	. 9013	. 9549	. 4806	. 6817	2. 0809	. 3183	1. 1228	20
50	0. 4509	. 4358	. 6392	. 9001	. 9543	. 4841	. 6850	2. 0655	. 3150	1. 1199	10
26° 00′	0. 4538	. 4384	9. 6418	. 8988	9. 9537	. 4877	9. 6882	2. 0503	0. 3118	1. 1170	64° 00′
10	0. 4567	. 4410	. 6444	. 8975	. 9530	. 4913	. 6914	2. 0353	. 3086	1. 1141	50
20	0. 4596	. 4436	. 6470	. 8962	. 9524	. 4950	. 6946	2. 0204	. 3054	1. 1112	40
30	0. 4625	. 4462	. 6495	. 8949	. 9518	. 4986	. 6977	2. 0057	. 3023	1. 1083	30
40	0. 4654	. 4488	. 6521	. 8936	. 9512	. 5022	. 7009	1. 9912	. 2991	1. 1054	20
50	0. 4683	. 4514	. 6546	. 8923	. 9505	. 5059	. 7040	1. 9768	. 2960	1. 1025	10
27° 00′	0. 4712	. 4540	9. 6570	. 8910	9. 9499	. 5095	9. 7072	1. 9626	0. 2928	1. 0996	63° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines.		Sines.		Cotangents.		Tangents.		Radians.	Degrees.

Trigonometric functions—Continued.

Annex −10 in columns marked *. (For 0°.1 intervals see pp. 82–87.)

Degrees.	Radians.	Sines.		Cosines.		Tangents.		Cotangents.			
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
27° 00′	0.4712	.4540	9.6570	.8910	9.9499	.5095	9.7072	1.9626	0.2928	1.0996	63° 00′
10	0.4741	.4566	.6595	.8897	.9492	.5132	.7103	1.9486	.2897	1.0966	50
20	0.4771	.4592	.6620	.8884	.9486	.5169	.7134	1.9347	.2866	1.0937	40
30	0.4800	.4617	.6644	.8870	.9479	.5206	.7165	1.9210	.2835	1.0908	30
40	0.4829	.4643	.6668	.8857	.9473	.5243	.7196	1.9074	.2804	1.0879	20
50	0.4858	.4669	.6692	.8843	.9466	.5280	.7226	1.8940	.2774	1.0850	10
28° 00′	0.4887	.4695	9.6716	.8829	9.9459	.5317	9.7257	1.8807	0.2743	1.0821	62° 00′
10	0.4916	.4720	.6740	.8816	.9453	.5354	.7287	1.8676	.2713	1.0792	50
20	0.4945	.4746	.6763	.8802	.9446	.5392	.7317	1.8546	.2683	1.0763	40
30	0.4974	.4772	.6787	.8788	.9439	.5430	.7348	1.8418	.2652	1.0734	30
40	0.5003	.4797	.6810	.8774	.9432	.5467	.7378	1.8291	.2622	1.0705	20
50	0.5032	.4823	.6833	.8760	.9425	.5505	.7408	1.8165	.2592	1.0676	10
29° 00′	0.5061	.4848	9.6856	.8746	9.9418	.5543	9.7438	1.8040	0.2562	1.0647	61° 00′
10	0.5091	.4874	.6878	.8732	.9411	.5581	.7467	1.7917	.2533	1.0617	50
20	0.5120	.4899	.6901	.8718	.9404	.5619	.7497	1.7796	.2503	1.0588	40
30	0.5149	.4924	.6923	.8704	.9397	.5658	.7526	1.7675	.2474	1.0559	30
40	0.5178	.4950	.6946	.8689	.9390	.5696	.7556	1.7556	.2444	1.0530	20
50	0.5207	.4975	.6968	.8675	.9383	.5735	.7585	1.7437	.2415	1.0501	10
30° 00′	0.5236	.5000	9.6990	.8660	9.9375	.5774	9.7614	1.7321	0.2386	1.0472	60° 00′
10	0.5265	.5025	.7012	.8646	.9368	.5812	.7644	1.7205	.2356	1.0443	50
20	0.5294	.5050	.7033	.8631	.9361	.5851	.7673	1.7090	.2327	1.0414	40
30	0.5323	.5075	.7055	.8616	.9353	.5890	.7701	1.6977	.2299	1.0385	30
40	0.5352	.5100	.7076	.8601	.9346	.5930	.7730	1.6864	.2270	1.0356	20
50	0.5381	.5125	.7097	.8587	.9338	.5969	.7759	1.6753	.2241	1.0327	10
31° 00′	0.5411	.5150	9.7118	.8572	9.9331	.6009	9.7788	1.6643	0.2212	1.0297	59° 00′
10	0.5440	.5175	.7139	.8557	.9323	.6048	.7816	1.6534	.2184	1.0268	50
20	0.5469	.5200	.7160	.8542	.9315	.6088	.7845	1.6426	.2155	1.0239	40
30	0.5498	.5225	.7181	.8526	.9308	.6128	.7873	1.6319	.2127	1.0210	30
40	0.5527	.5250	.7201	.8511	.9300	.6168	.7902	1.6212	.2098	1.0181	20
50	0.5556	.5275	.7222	.8496	.9292	.6208	.7930	1.6107	.2070	1.0152	10
32° 00′	0.5585	.5299	9.7242	.8480	9.9284	.6249	9.7958	1.6003	0.2042	1.0123	58° 00′
10	0.5614	.5324	.7262	.8465	.9276	.6289	.7986	1.5900	.2014	1.0094	50
20	0.5643	.5348	.7282	.8450	.9268	.6330	.8014	1.5798	.1986	1.0065	40
30	0.5672	.5373	.7302	.8434	.9260	.6371	.8042	1.5697	.1958	1.0036	30
40	0.5701	.5398	.7322	.8418	.9252	.6412	.8070	1.5597	.1930	1.0007	20
50	0.5730	.5422	.7342	.8403	.9244	.6453	.8097	1.5497	.1903	0.9977	10
33° 00′	0.5760	.5446	9.7361	.8387	9.9236	.6494	9.8125	1.5399	0.1875	0.9948	57° 00′
10	0.5789	.5471	.7380	.8371	.9228	.6536	.8153	1.5301	.1847	0.9919	50
20	0.5818	.5495	.7400	.8355	.9219	.6577	.8180	1.5204	.1820	0.9890	40
30	0.5847	.5519	.7419	.8339	.9211	.6619	.8208	1.5108	.1792	0.9861	30
40	0.5876	.5544	.7438	.8323	.9203	.6661	.8235	1.5013	.1765	0.9832	20
50	0.5905	.5568	.7457	.8307	.9194	.6703	.8263	1.4919	.1737	0.9803	10
34° 00′	0.5934	.5592	9.7476	.8290	9.9186	.6745	9.8290	1.4826	0.1710	0.9774	56° 00′
10	0.5963	.5616	.7494	.8274	.9177	.6787	.8317	1.4733	.1683	0.9745	50
20	0.5992	.5640	.7513	.8258	.9169	.6830	.8344	1.4641	.1656	0.9716	40
30	0.6021	.5664	.7531	.8241	.9160	.6873	.8371	1.4550	.1629	0.9687	30
40	0.6050	.5688	.7550	.8225	.9151	.6916	.8398	1.4460	.1602	0.9657	20
50	0.6080	.5712	.7568	.8208	.9142	.6959	.8425	1.4370	.1575	0.9628	10
35° 00′	0.6109	.5736	9.7586	.8192	9.9134	.7002	9.8452	1.4281	0.1548	0.9599	55° 00′
10	0.6138	.5760	.7604	.8175	.9125	.7046	.8479	1.4193	.1521	0.9570	50
20	0.6167	.5783	.7622	.8158	.9116	.7089	.8506	1.4106	.1494	0.9541	40
30	0.6196	.5807	.7640	.8141	.9107	.7133	.8533	1.4019	.1467	0.9512	30
40	0.6225	.5831	.7657	.8124	.9098	.7177	.8559	1.3934	.1441	0.9483	20
50	0.6254	.5854	.7675	.8107	.9089	.7221	.8586	1.3848	.1414	0.9454	10
36° 00′	0.6283	.5878	9.7692	.8090	9.9080	.7265	9.8613	1.3764	0.1387	0.9425	54° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines.		Sines.		Cotangents.		Tangents.		Radians.	Degrees.

Trigonometric functions—Continued.

Annex –10 in columns marked *. (For 0°.1 intervals see pp. 82–87.)

Degrees.	Radians.	Sines.		Cosines.		Tangents.		Cotangents.			
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
36° 00′	0. 6283	. 5878	9. 7692	. 8090	9. 9080	. 7265	9. 8613	1. 3764	0. 1387	0. 9425	54° 00′
10	0. 6312	. 5901	. 7710	. 8073	. 9070	. 7310	. 8639	1. 3680	. 1361	0. 9396	50
20	0. 6341	. 5925	. 7727	. 8056	. 9061	. 7355	. 8666	1. 3597	. 1334	0. 9367	40
30	0. 6370	. 5948	. 7744	. 8039	. 9052	. 7400	. 8692	1. 3514	. 1308	0. 9338	30
40	0. 6400	. 5972	. 7761	. 8021	. 9042	. 7445	. 8718	1. 3432	. 1282	0. 9308	20
50	0. 6429	. 5995	. 7778	. 8004	. 9033	. 7490	. 8745	1. 3351	. 1255	0. 9279	10
37° 00′	0. 6458	. 6018	9. 7795	. 7986	9. 9023	. 7536	9. 8771	1. 3270	0. 1229	0. 9250	53° 00′
10	0. 6487	. 6041	. 7811	. 7969	. 9014	. 7581	. 8797	1. 3190	. 1203	0. 9221	50
20	0. 6516	. 6065	. 7828	. 7951	. 9004	. 7627	. 8824	1. 3111	. 1176	0. 9192	40
30	0. 6545	. 6088	. 7844	. 7934	. 8995	. 7673	. 8850	1. 3032	. 1150	0. 9163	30
40	0. 6574	. 6111	. 7861	. 7916	. 8985	. 7720	. 8876	1. 2954	. 1124	0. 9134	20
50	0. 6603	. 6134	. 7877	. 7898	. 8975	. 7766	. 8902	1. 2876	. 1098	0. 9105	10
38° 00′	0. 6632	. 6157	9. 7893	. 7880	9. 8965	. 7813	9. 8928	1. 2799	0. 1072	0. 9076	52° 00′
10	0. 6661	. 6180	. 7910	. 7862	. 8955	. 7860	. 8954	1. 2723	. 1046	0. 9047	50
20	0. 6690	. 6202	. 7926	. 7844	. 8945	. 7907	. 8980	1. 2647	. 1020	0. 9018	40
30	0. 6720	. 6225	. 7941	. 7826	. 8935	. 7954	. 9006	1. 2572	. 0994	0. 8988	30
40	0. 6749	. 6248	. 7957	. 7808	. 8925	. 8002	. 9032	1. 2497	. 0968	0. 8959	20
50	0. 6778	. 6271	. 7973	. 7790	. 8915	. 8050	. 9058	1. 2423	. 0942	0. 8930	10
39° 00′	0. 6807	. 6293	9. 7989	. 7771	9. 8905	. 8098	9. 9084	1. 2349	0. 0916	0. 8901	51° 00′
10	0. 6836	. 6316	. 8004	. 7753	. 8895	. 8146	. 9110	1. 2276	. 0890	0. 8872	50
20	0. 6865	. 6338	. 8020	. 7735	. 8884	. 8195	. 9135	1. 2203	. 0865	0. 8843	40
30	0. 6894	. 6361	. 8035	. 7716	. 8874	. 8243	. 9161	1. 2131	. 0839	0. 8814	30
40	0. 6923	. 6383	. 8050	. 7698	. 8864	. 8292	. 9187	1. 2059	. 0813	0. 8785	20
50	0. 6952	. 6406	. 8066	. 7679	. 8853	. 8342	. 9212	1. 1988	. 0788	0. 8756	10
40° 00′	0. 6981	. 6428	9. 8081	. 7660	9. 8843	. 8391	9. 9238	1. 1918	0. 0762	0. 8727	50° 00′
10	0. 7010	. 6450	. 8096	. 7642	. 8832	. 8441	. 9264	1. 1847	. 0736	0. 8698	50
20	0. 7039	. 6472	. 8111	. 7623	. 8821	. 8491	. 9289	1. 1778	. 0711	0. 8668	40
30	0. 7069	. 6494	. 8125	. 7604	. 8810	. 8541	. 9315	1. 1708	. 0685	0. 8639	30
40	0. 7098	. 6517	. 8140	. 7585	. 8800	. 8591	. 9341	1. 1640	. 0659	0. 8610	20
50	0. 7127	. 6539	. 8155	. 7566	. 8789	. 8642	. 9366	1. 1571	. 0634	0. 8581	10
41° 00′	0. 7156	. 6561	9. 8169	. 7547	9. 8778	. 8693	9. 9392	1. 1504	0. 0608	0. 8552	49° 00′
10	0. 7185	. 6583	. 8184	. 7528	. 8767	. 8744	. 9417	1. 1436	. 0583	0. 8523	50
20	0. 7214	. 6604	. 8198	. 7509	. 8756	. 8796	. 9443	1. 1369	. 0557	0. 8494	40
30	0. 7243	. 6626	. 8213	. 7490	. 8745	. 8847	. 9468	1. 1303	. 0532	0. 8465	30
40	0. 7272	. 6648	. 8227	. 7470	. 8733	. 8899	. 9494	1. 1237	. 0506	0. 8436	20
50	0. 7301	. 6670	. 8241	. 7451	. 8722	. 8952	. 9519	1. 1171	. 0481	0. 8407	10
42° 00′	0. 7330	. 6691	9. 8255	. 7431	9. 8711	. 9004	9. 9544	1. 1106	0. 0456	0. 8378	48° 00′
10	0. 7359	. 6713	. 8269	. 7412	. 8699	. 9057	. 9570	1. 1041	. 0430	0. 8348	50
20	0. 7389	. 6734	. 8283	. 7392	. 8688	. 9110	. 9595	1. 0977	. 0405	0. 8319	40
30	0. 7418	. 6756	. 8297	. 7373	. 8676	. 9163	. 9621	1. 0913	. 0379	0. 8290	30
40	0. 7447	. 6777	. 8311	. 7353	. 8665	. 9217	. 9646	1. 0850	. 0354	0. 8261	20
50	0. 7476	. 6799	. 8324	. 7333	. 8653	. 9271	. 9671	1. 0786	. 0329	0. 8232	10
43° 00′	0. 7505	. 6820	9. 8338	. 7314	9. 8641	. 9325	9. 9697	1. 0724	0. 0303	0. 8203	47° 00′
10	0. 7534	. 6841	. 8351	. 7294	. 8629	. 9380	. 9722	1. 0661	. 0278	0. 8174	50
20	0. 7563	. 6862	. 8365	. 7274	. 8618	. 9435	. 9747	1. 0599	. 0253	0. 8145	40
30	0. 7592	. 6884	. 8378	. 7254	. 8606	. 9490	. 9772	1. 0538	. 0228	0. 8116	30
40	0. 7621	. 6905	. 8391	. 7234	. 8594	. 9545	. 9798	1. 0477	. 0202	0. 8087	20
50	0. 7650	. 6926	. 8405	. 7214	. 8582	. 9601	. 9823	1. 0416	. 0177	0. 8058	10
44° 00′	0. 7679	. 6947	9. 8418	. 7193	9. 8569	. 9657	9. 9848	1. 0355	0. 0152	0. 8029	46° 00′
10	0. 7709	. 6967	. 8431	. 7173	. 8557	. 9713	. 9874	1. 0295	. 0126	0. 7999	50
20	0. 7738	. 6988	. 8444	. 7153	. 8545	. 9770	. 9899	1. 0235	. 0101	0. 7970	40
30	0. 7767	. 7009	. 8457	. 7133	. 8532	. 9827	. 9924	1. 0176	. 0076	0. 7941	30
40	0. 7796	. 7030	. 8469	. 7112	. 8520	. 9884	. 9949	1. 0117	. 0051	0. 7912	20
50	0. 7825	. 7050	. 8482	. 7092	. 8507	. 9942	. 9975	1. 0058	. 0025	0. 7883	10
45° 00′	0. 7854	. 7071	9. 8495	. 7071	9. 8495	1. 0000	0. 0000	1. 0000	0. 0000	0. 7854	45° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines.		Sines.		Cotangents.		Tangents.		Radians.	Degrees.

VI.--DERIVATIVES AND INTEGRALS.

DERIVATIVES.

$d\,ax = a\,dx$

$d\,u\,v = \left(u\frac{dv}{dx} + v\frac{du}{dx}\right)dx$

$d\frac{u}{v} = \left(\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\right)dx$

$d\,x^n = nx^{n-1}\,dx$

$d\,f\,(u) = d\frac{f\,(u)}{du}\cdot\frac{du}{dx}\cdot dx$

$d\,e^x = e^x\,dx$

$d\,e^{ax} = a\,e^{ax}\,dx$

$d\log_e x = \frac{1}{x}dx$

$d\,x^x = x^x\,(1+\log_e x)$

$d\sin x = \cos x\,dx$

$d\cos x = -\sin x\,dx$

$d\tan x = \sec^2 x\,dx$

$d\cot x = -\csc^2 x\,dx$

$d\sec x = \tan x\sec x\,dx$

$d\csc x = -\cot x.\csc x\,dx$

$d\sin^{-1} x = (1-x^2)^{-½}\,dx$

$d\cos^{-1} x = -(1-x^2)^{-½}\,dx$

$d\tan^{-1} x = (1+x^2)^{-1}\,dx$

$d\cot^{-1} x = -(1+x^2)^{-1}\,dx$

$d\sec^{-1} x = x^{-1}\,(x^2-1)^{-½}\,dx$

$d\csc^{-1} x = -x^{-1}\,(x^2-1)^{-½}\,dx$

$d\sinh x = \cosh x\,dx$

$d\cosh x = \sinh x\,dx$

$d\tanh x = \mathrm{sech}^2\,x\,dx$

$d\coth x = -\mathrm{csch}^2\,x\,dx$

$d\,\mathrm{sech}\,x = -\mathrm{sech}\,x\tanh dx$

$d\,\mathrm{csch}\,x = -\mathrm{csch}\,x.\coth x\,dx$

$d\sinh^{-1} = (x^2+1)^{-½}\,dx$

$d\cosh^{-1} x = (x^2-1)^{-½}\,dx$

$d\tanh^{-1} x = (1-x^2)^{-1}\,dx$

$d\coth^{-1} x = (1-x^2)^{-1}\,dx$

$d\,\mathrm{sech}^{-1}\,x = -x^{-1}\,(1-x^2)^{-½}\,dx$

$d\,\mathrm{csch}^{-1}\,x = -x^{-1}\,(x^2+1)^{-½}$

FUNDAMENTAL EQUATIONS.

$$\int a.f(x)dx = a\int f(x)dx;\quad \int\phi\,(y)dx = \int\frac{\phi(y)}{y'}dy,\ \text{where } y^1 = dy/dx.$$

$$\int(u+v)dx = \int u\,dx + \int v\,dx,\ \text{where } u \text{ and } v \text{ are any functions of } x.$$

$$\int u\,dv = uv - \int v\,du;\quad \int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx.$$

$$\int x^m dx = \frac{x^{m+1}}{m+1},\ \text{if } m \neq -1;\quad \int\frac{dx}{x} = \log x,\ \text{or } \log(-x).$$

$$\int e^{ax}dx = e^{ax}/a;\quad \int b^{ax}\,dx = \frac{b^{ax}}{a\log b}.$$

$$\int\sin x\,dx = -\cos x;\quad \int\cos x\,dx = \sin x.$$

$$\int\tan x\,dx = -\log\ \cos x;\quad \int\mathrm{ctn}\,x\,dx = \log\ \sin x.$$

$$\int\sec^2 x\,dx = \tan x;\quad \int\csc^2 x\,dx = -\mathrm{ctn}\,x.$$

$$\int\cosh x\,dx = \sinh x;\quad \int\sinh x\,dx = \cosh x.$$

$$\int \tanh x\, dx = \log \cosh x; \quad \int \operatorname{ctnh} x = \log \sinh x.$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right), \text{or} -\frac{1}{a}\operatorname{ctn}^{-1}\left(\frac{x}{a}\right).$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right), \text{ or } \frac{1}{2a}\log\frac{a+x}{a-x}.$$

$$\int \frac{dx}{x^2-a^2} = -\frac{1}{a}\operatorname{ctnh}^{-1}\left(\frac{x}{a}\right), \text{ or } \frac{1}{2a}\log\frac{x-a}{x+a}.$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \text{ or} -\cos^{-1}\left(\frac{x}{a}\right).$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log\left(x+\sqrt{x^2 \pm a^2}\right).$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\cos^{-1}\left(\frac{a}{x}\right).$$

$$\int \frac{dx}{x\sqrt{a^2 \pm x^2}} = -\frac{1}{a}\log\left(\frac{a+\sqrt{a^2 \pm x^2}}{x}\right).$$

$$\int \frac{dx}{x\sqrt{a+bx}} = \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bx}{-a}}, \text{ or } \frac{-2}{\sqrt{a}}\tanh^{-1}\sqrt{\frac{a+bx}{a}}.$$

In such a case as this, that one of the alternate values of the integral which makes the quantities under the radical signs positive is to be used, and each radical itself is to be considered positive. Of course an arbitrary constant may be added to the value of every integral given.

$$e^{xi} = \cos x + i \sin x; \quad e^{-xi} = \cos x - i \sin x.$$

$$\sinh x = \tfrac{1}{2}(e^x - e^{-x}); \quad \cosh x = 1(e^x + e^{-x}).$$

$$\sin xi = i \sinh x; \quad \cos xi = \cosh x.$$

$$\sin x = -i \sinh xi; \quad \cos x = \cosh xi$$

$$\log u = \log (cu) - \log c.$$

$$\log x = \log(-x) + (2k+1)\pi i; \quad \log_e x = (2.3025851).\log_{10} x.$$

$$\log(x \pm yi) = \tfrac{1}{2}\log(x^2+y^2) \pm i \tan^{-1}(y/x).$$

For acute angles and some other cases easily to be determined in each instance,

$$\sin^{-1}u = \cos^{-1}\sqrt{1-u^2} = \tan^{-1}(u/\sqrt{1-u^2}) = \csc^{-1}(1/u).$$

$$\sin^{-1} u = -\sin^{-1}\sqrt{1-u^2} + \text{a constant} = \tfrac{1}{2}\sin^{-1}(2u^2-1) + \text{ a constant}.$$

$$\tan^{-1} u = -\tan^{-1}(1/u) + \text{ a constant}.$$

I.--RATIONAL ALGEBRAIC FUNCTIONS.

A.—Expressions involving $(a+bx)$

The substitution of y or z for x, where $y=xz=a+bx$, gives

$$\int (a+bx)^m dx=\frac{1}{b}\int y^m dy.$$

$$\int x(a+bx)^m dx=\frac{1}{b^2}\int y^m (y-a)dy.$$

$$\int x^n (a+bx)^m dx=\frac{1}{b^{n+1}}\int y^m (y-a)^n dy.$$

$$\int \frac{x^n dx}{(a+bx)^m}=\frac{1}{b^{n+1}}\int \frac{(y-a)^n dy}{y^m}.$$

$$\int \frac{dx}{x^n (a+bx)^m}=-\frac{1}{a^{m+n-1}}\int \frac{(z-b)^{m+n-2}dz}{z^m}.$$

Whence

$$\int \frac{dx}{a+bx}=\frac{1}{b}\log (a+bx).$$

$$\int \frac{dx}{(a+bx)^2}=-\frac{1}{b(a+bx)}.$$

$$\int \frac{dx}{(a+bx)^3}=-\frac{1}{2\,b(a+bx)^2}.$$

$$\int \frac{xdx}{a+bx}=\frac{1}{b^2}[a+bx-a\log (a+bx)].$$

$$\int \frac{xdx}{(a+bx)^2}=\frac{1}{b^2}\left[\log(a+bx)+\frac{a}{a+bx}\right].$$

$$\int \frac{xdx}{(a+bx)^3}=\frac{1}{b^2}\left[-\frac{1}{a+bx}+\frac{a}{2(a+bx)^2}\right].$$

$$\int \frac{x^2dx}{a+bx}=\frac{1}{b^3}\left[\frac{1}{2}(a+bx)^2-2\,a(a+bx)+a^2\log (a+bx)\right].$$

$$\int \frac{x^2dx}{(a+bx)^2}=\frac{1}{b^3}\left[a+bx-2\,a\log (a+bx)-\frac{a^2}{a+bx}\right].$$

$$\int \frac{dx}{x(a+bx)}=-\frac{1}{a}\log\frac{a+bx}{x}.$$

$$\int \frac{dx}{x(a+bx)^2}=\frac{1}{a(a+bx)}-\frac{1}{a^2}\log\frac{a+bx}{x}.$$

$$\int \frac{dx}{x^2(a+bx)}=-\frac{1}{ax}+\frac{b}{a^2}\log\frac{a+bx}{x}.$$

B. Expressions involving $(a+bx^n)$

$$\int \frac{dx}{c^2+x^2}=\frac{1}{c}\tan^{-1}\frac{x}{c}=\frac{1}{c}\sin^{-1}\frac{x}{\sqrt{c^2+x^2}}.$$

$$\int \frac{dx}{c^2-x^2}=\frac{1}{2c}\log\frac{c+x}{c-x};\quad \int \frac{dx}{x^2-c^2}=\frac{1}{2c}\log\frac{x-c}{x+c}.$$

$$\int \frac{dx}{c^2-x^2}=\frac{1}{c}\tanh^{-1}\left(\frac{x}{c}\right);\quad \int \frac{dx}{x^2-c^2}=-\frac{1}{c}\text{ctnh}^{-1}\left(\frac{x}{c}\right).$$

$$\int \frac{dx}{a+bx^2}=\frac{1}{\sqrt{ab}}\tan^{-1}\left(x\sqrt{\frac{b}{a}}\right),\ [a>0,\ b>0].$$

$$\int \frac{dx}{a+bx^2}=\frac{1}{2\sqrt{-ab}}\log\frac{\sqrt{a}+x\sqrt{-b}}{\sqrt{a}-x\sqrt{-b}},\quad\text{or}\quad \frac{1}{\sqrt{-ab}}\tanh^{-1}\left(x\sqrt{\frac{-b}{a}}\right),\ [a>0,\ b<0]$$

$$\int \frac{dx}{(a+bx^2)^2}=\frac{x}{2\,a(a+bx^2)}+\frac{1}{2a}\int\frac{dx}{a+bx^2}.$$

$$\int \frac{dx}{(a+bx^2)^{m+1}}=\frac{1}{2\,ma}\,\frac{x}{(a+bx^2)^m}+\frac{2\,m-1}{2\,ma}\int\frac{dx}{(a+bx^2)^m}.$$

$$\int \frac{xdx}{a+bx^2}=\frac{1}{2\,b}\log\left(x^2+\frac{a}{b}\right).$$

$$\int \frac{xdx}{(a+bx^2)^{m+1}}=\frac{1}{2}\int\frac{dz}{(a+bz)^{m+1}},[z=x^2].$$

$$\int \frac{dx}{x(a+bx^2)}=\frac{1}{2\,a}\log\frac{x^2}{a+bx^2}.$$

$$\int \frac{x^2dx}{a+bx^2}=\frac{x}{b}-\frac{a}{b}\int\frac{dx}{a+bx^2}.$$

$$\int \frac{dx}{x^2(a+bx^2)}=-\frac{1}{ax}-\frac{b}{a}\int\frac{dx}{a+bx^2}.$$

$$\int \frac{x^2dx}{(a+bx^2)^{m+1}}=\frac{-x}{2\,mb\,(a+bx^2)^m}+\frac{1}{2\,mb}\int\frac{dx}{(a+bx^2)^m}.$$

$$\int \frac{dx}{x^2(a+bx^2)^{m+1}}=\frac{1}{a}\int\frac{dx}{x^2(a+bx^2)^m}-\frac{b}{a}\int\frac{dx}{(a+bx^2)^{m+1}}.$$

$$\int \frac{dx}{a+bx^3}=\frac{k}{3\,a}\left[\frac{1}{2}\log\frac{(k+x)^2}{k^2-kx+x^2}+\sqrt{3}\tan^{-1}\frac{2\,x-k}{k\sqrt{3}}\right],\quad [bk^3=a].$$

$$\int \frac{xdx}{a+bx^3}=\frac{1}{3\,bk}\left[\frac{1}{2}\log\frac{k^2-kx+x^2}{(k+x)^2}+\sqrt{3}\tan^{-1}\frac{2\,x-k}{k\sqrt{3}}\right],\quad [bk^3=a].$$

$$\int \frac{dx}{x(a+bx^n)}=\frac{1}{an}\log\frac{x^n}{a+bx^n}.$$

$$\int \frac{dx}{(a+bx^n)^{m+1}}=\frac{1}{a}\int\frac{dx}{(a+bx^n)^m}-\frac{b}{a}\int\frac{x^ndx}{(a+bx^n)^{m+1}}.$$

$$\int \frac{x^m dx}{(a+bx^n)^{p+1}} = \frac{1}{b}\int \frac{x^{m-n}}{(a+bx^n)^p} - \frac{a}{b}\int \frac{x^{m-n}dx}{(a+bx^n)^{p+1}}.$$

$$\int \frac{dx}{x^m(a+bx^n)^{p+1}} = \frac{1}{a}\int \frac{dx}{x^m(a+bx^n)^p} - \frac{b}{a}\int \frac{dx}{x^{m-n}(a+bx^n)^{p+1}}.$$

$$\int x^{m-1}(a+bx^n)^p dx = \begin{cases} \dfrac{1}{b(m+np)}\left[x^{m-n}(a+bx^n)^{p+1} - (m-n)a\displaystyle\int x^{m-n-}(a+bx^n)^p dx\right]. \\ \dfrac{1}{m+np}\left[x^m(a+bx^n)^p + npa\displaystyle\int x^{m-1}(a+bx^n)^{p-1}dx\right]. \\ \dfrac{1}{ma}\left[x^m(a+bx^n)^{p+1} - (m+np+n)b\displaystyle\int x^{m+n-1}(a+bx^n)^p dx\right]. \\ \dfrac{1}{an(p+1)}\left[-x^m(a+bx^n)^{p+1} + (m+np+n)\displaystyle\int x^{m-1}(a+bx^n)^{p+1}dx\right]. \end{cases}$$

C.—Expressions involving $(a+bx+cx^2)$

Let $X = a+bx+cx^2$ and $q = 4\,ac - b^2$, then

$$\int \frac{dx}{X} = \frac{2}{\sqrt{q}}\tan^{-1}\left(\frac{2\,cx+b}{\sqrt{q}}\right), \text{ when } q>0; \quad \text{or} \quad \frac{-2}{\sqrt{-q}}\cdot\tanh^{-1}\left(\frac{2cx+b}{\sqrt{-q}}\right), \text{ when } q<0.$$

$$\int \frac{dx}{X} = \frac{1}{\sqrt{-q}}\log\frac{2\,cx+b-\sqrt{-q}}{2\,cx+b+\sqrt{-q}}, \text{ when } q<0.$$

$$\int \frac{dx}{X^2} = \frac{2\,cx+b}{qX} + \frac{2\,c}{q}\int \frac{dx}{X}.$$

$$\int \frac{dx}{X^3} = \frac{2\,cx+b}{q}\left(\frac{1}{2\,X^2} + \frac{3\,c}{qX}\right) + \frac{6\,c^2}{q^2}\int \frac{dx}{X}.$$

$$\int \frac{dx}{X^{n+1}} = \frac{2\,cx+b}{nqX^n} + \frac{2(2\,n-1)c}{qn}\int \frac{dx}{X^n}.$$

$$\int \frac{xdx}{X} = \frac{1}{2c}\log X - \frac{b}{2c}\int \frac{dx}{X}.$$

$$\int \frac{xdx}{X^2} = -\frac{bx+2\,a}{qX} - \frac{b}{q}\int \frac{dx}{X}.$$

$$\int \frac{xdx}{X^{n+1}} = -\frac{2\,a+bx}{nqX^n} - \frac{b(2\,n-1)}{nq}\int \frac{dx}{X^n}.$$

$$\int \frac{x^2}{X}dx = \frac{x}{c} - \frac{b}{2\,c^2}\log X + \frac{b^2-2\,ac}{2\,c^2}\int \frac{dx}{X}.$$

$$\int \frac{x^2}{X^2}dx = \frac{(b^2-2\,ac)x+ab}{cqX} + \frac{2\,a}{q}\int \frac{dx}{X}.$$

$$\int \frac{x^m dx}{X^{n+1}} = -\frac{x^{m-1}}{(2\,n-m+1)\,cX^n} - \frac{n-m+1}{2\,n-m+1}\cdot\frac{b}{c}\int \frac{x^{m-1}dx}{X^{n+1}} + \frac{m-1}{2\,n-m+1}\cdot\frac{a}{c}\int \frac{x^{m-2}dx}{X^{n+1}}.$$

$$\int \frac{ax}{xX} - \frac{1}{2\,a}\log\frac{x^2}{X} - \frac{b}{2\,a}\int \frac{dx}{X}.$$

$$\int \frac{dx}{x^2 X} = \frac{b}{2\,a^2} \log \frac{X}{x^2} - \frac{1}{ax} + \left(\frac{b^2}{2\,a^2} - \frac{c}{a}\right) \int \frac{dx}{X}.$$

$$\int \frac{dx}{x^m X^{n+1}} = -\frac{1}{(m-1)\,ax^{m-1}X^n} - \frac{n+m-1}{m-1} \cdot \frac{b}{a} \int \frac{dx}{x^{m-1}X^{n+1}} - \frac{2\,n+m-1}{m-1} \cdot \frac{c}{a} \int \frac{dx}{x^{m-2}X^{n+1}}.$$

D.—Rational Fractions

Every proper fraction can be represented by the general form:

$$\frac{f(x)}{F(x)} = \frac{g_1 x^{n-1} + g_2 x^{n-2} + g_3 x^{n-3} + \cdot\ \cdot\ \cdot + g_n}{x^n + k_1 x^{n-1} + k_2 x^{n-2} + \cdot\ \cdot\ \cdot + k_n}.$$

If a, b, c, etc. are the roots of the equation $F(x) = 0$, so that

$$F(x) = (x-a)^p\ (x-b)^q\ (x-c)^r \cdot\ \cdot\ \cdot,$$

then

$$\begin{aligned} \frac{f(x)}{F(x)} = & \frac{A_1}{(x-a)^p} + \frac{A_2}{(x-a)^{p-1}} + \frac{A_3}{(x-a)^{p-2}} + \cdot\ \cdot\ \cdot + \frac{A_p}{x-a} \\ & + \frac{B_1}{(x-b)^q} + \frac{B_2}{(x-b)^{q-1}} + \frac{B_3}{(x-b)^{q-2}} + \cdot\ \cdot\ \cdot + \frac{B_q}{x-b} \\ & + \frac{C_1}{(x-c)^r} + \frac{C_2}{(x-c)^{r-1}} + \frac{C_3}{(x-c)^{r-2}} + + \cdot\ \cdot\ \cdot + \frac{C_r}{x-c} \\ & + \cdot\ \cdot\ \cdot \quad \cdot\ \cdot\ \cdot \quad \cdot\ \cdot\ \cdot \quad \cdot\ \cdot\ \cdot \quad \cdot\ \cdot\ \cdot \quad \cdot\ \cdot\ \cdot, \end{aligned}$$

where the numerators of the separate fractions are constant.

If a, b, c, etc are single roots, then $p = q = r = \cdot\ \cdot\ \cdot = 1$, and

$$\frac{F(x)}{f(x)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c} \cdot\ \cdot\ \cdot, \quad \text{where} \quad A = \frac{f(a)}{F'(a)},\ B = \frac{f(b)}{F'(b)},\ \text{etc.}$$

The simpler fractions, into which the original fraction is thus divided, may be integrated by means of the following formulas:

$$\int \frac{h\,dx}{(mx+n)^l} = \int \frac{h\,d(mx+n)}{m(mx+n)^l} = \frac{h}{m(1-l)\ (mx+n)^{l-1}}.$$

$$\int \frac{h\,dx}{mx+n} = \frac{h}{m} \log\ (mx+n).$$

If any of the roots of the equation $f(x) = 0$ are imaginary, the parts of the integral which arise from conjugate roots can be combined, and the integral thus brought into a real form. The following formula, in which $i = \sqrt{-1}$, is often useful in combining logarithms of conjugate complex quantities:

$$\log\ (x \pm yi) = \tfrac{1}{2} \log\ (x^2 + y^2) \pm i \tan^{-1} \frac{y}{x}.$$

II.—IRRATIONAL ALGEBRAIC FUNCTIONS.

A.—Expressions involving $\sqrt{a+bx}$

The substitution of a new variable of integration, $y = \sqrt{a+bx}$, gives

$$\int \sqrt{a+bx}\,dx = \frac{2}{3b} \sqrt{(a+bx)^3}.$$

$$\int x\sqrt{a+bx}\,dx=-\frac{2(2a-3bx)\sqrt{(a+bx)^3}}{15\,b^2}.$$

$$\int x^2\sqrt{a+bx}\,dx=\frac{2(8\,a^2-12\,abx+15\,b^2x^2)\sqrt{(a+bx)^3}}{105\,b^3}.$$

$$\int\frac{\sqrt{a+bx}}{x}dx=2\sqrt{a+bx}+a\int\frac{dx}{x\sqrt{a+bx}}.$$

$$\int\frac{dx}{\sqrt{a+bx}}=\frac{2\sqrt{a+bx}}{b}.$$

$$\int\frac{xdx}{\sqrt{a+bx}}=-\frac{2(2\,a-bx)}{3\,b^2}\sqrt{a+bx}.$$

$$\int\frac{x^2dx}{\sqrt{a+bx}}=\frac{2(8\,a^2-4\,abx+3\,b^2x^2)}{15\,b^3}\sqrt{a+bx}.$$

$$\int\frac{dx}{x\sqrt{a+bx}}=\frac{1}{\sqrt{a}}\log\left(\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}\right).$$

$$\int\frac{dx}{x\sqrt{a+bx}}=\frac{-2}{\sqrt{a}}\tanh^{-1}\sqrt{\frac{a+bx}{a}}.$$

$$\int\frac{dx}{x^2\sqrt{a+bx}}=-\frac{\sqrt{a+bx}}{ax}-\frac{b}{2a}\int\frac{dx}{x\sqrt{a+bx}}.$$

$$\int(a+bx)^{\pm\frac{n}{2}}dx=\frac{2}{b}\int y^{1\pm n}dy=\frac{2\,(a+bx)^{\frac{2\pm n}{2}}}{b\,(2\pm n)}.$$

$$\int x(a+bx)^{\pm\frac{n}{2}}dx=\frac{2}{b^2}\left[\frac{(a+bx)^{\frac{4\pm n}{2}}}{4\pm n}-\frac{a(a+bx)^{\frac{2\pm n}{2}}}{2\pm n}\right].$$

$$\int\frac{x^m dx}{\sqrt{a+bx}}=\frac{2\,x^m\sqrt{a+bx}}{(2m+1)b}-\frac{2\,ma}{(2m+1)b}\int\frac{x^{m-1}dx}{\sqrt{a+br}}.$$

$$\int\frac{dx}{x^n\sqrt{a+bx}}=-\frac{\sqrt{a+bx}}{(n-1)ax^{n-1}}-\frac{(2\,n-3)b}{(2\,n-2)a}\int\frac{dx}{x^{n-1}\sqrt{a+bx}}.$$

$$\int\frac{(a+bx)^{\frac{n}{2}}\,dx}{x}=b\int(a+bx)^{\frac{n-2}{2}}dx+a\int\frac{(a+bx)^{\frac{n-2}{2}}}{x}dx.$$

$$\int\frac{dx}{x(a+bx)^{\frac{m}{2}}}-\frac{1}{a}\int\frac{dx}{x(a+bx)^{\frac{m-2l}{2}}}-\frac{b}{a}\int\frac{dx}{(a+bx)^{\frac{n}{2}}}.$$

B.—Expressions involving $\sqrt{x^2 \pm a^2}$ and $\sqrt{a^2 - x^2}$

$$\int \sqrt{x^2 \pm a^2}\,dx = \tfrac{1}{2}[x\sqrt{x^2 \pm a^2} \log (x + \sqrt{x^2 \pm a^2})].^*$$

$$\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right)\right].$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log (x + \sqrt{x^2 \pm a^2}).^*$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \text{ or } -\cos^{-1}\left(\frac{x}{a}\right).$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\cos^{-1}\left(\frac{a}{x}\right), \text{ or } \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right).$$

$$\int \frac{dx}{x\sqrt{a^2 \pm x^2}} = -\frac{1}{a} \log \left(\frac{a + \sqrt{a^2 \pm x^2}}{x}\right).^*$$

$$\int \frac{\sqrt{a^2 \pm x^2}}{x}\,dx = \sqrt{a^2 \pm x^2} - a \log\left(\frac{a + \sqrt{a^2 \pm x^2}}{x}\right).^*$$

$$\int \frac{\sqrt{x^2 - a^2}}{x}\,dx = \sqrt{x^2 - a^2} - a \cos^{-1}\frac{a}{x}.$$

$$\int \frac{x\,dx}{\sqrt{a^2 \pm x^2}} = \pm\sqrt{a^2 \pm x^2}.$$

$$\int \frac{x\,dx}{\sqrt{x^2 - a^2}} = \sqrt{x^2 - a^2}.$$

$$\int x\sqrt{x^2 \pm a^2}\,dx = \tfrac{1}{3}\sqrt{(x^2 \pm a^2)^3}.$$

$$\int x\sqrt{a^2 - x^2}\,dx = -\tfrac{1}{3}\sqrt{(a^2 - x^2)^3}.$$

$$\int \sqrt{(x^2 \pm a^2)^3}\,dx = \frac{1}{4}\left[x\sqrt{(x^2 \pm a^2)^3} \pm \frac{3\,a^2 x}{2}\sqrt{x^2 \pm a^2} + \frac{3\,a^4}{2}\log (x + \sqrt{x^2 \pm a^2})\right].^*$$

$$\int \sqrt{(a^2 - x^2)^3}\,dx = \frac{1}{4}\left[x\sqrt{(a^2 - x^2)^3} + \frac{3\,a^2 x}{2}\sqrt{a^2 - x^2} + \frac{3\,a^4}{2}\sin^{-1}\frac{x}{a}\right].$$

$$\int \frac{dx}{\sqrt{(x^2 \pm a^2)^3}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}.$$

$$\int \frac{dx}{\sqrt{(a^2 - x^2)^3}} = \frac{x}{a^2\sqrt{a^2 - x^2}}.$$

$^*\log\left(\frac{x + \sqrt{x^2 + a^2}}{a}\right) = \sinh^{-1}\left(\frac{x}{a}\right)$; $\log\left(\frac{x + \sqrt{x^2 - a^2}}{a}\right) = \cosh^{-1}\left(\frac{x}{a}\right)$.

$\log\left(\frac{a + \sqrt{a^2 - x^2}}{x}\right) \operatorname{sech}^{-1}\left(\frac{x}{a}\right)$; $\log\left(\frac{a + \sqrt{a^2 + x^2}}{x}\right) = \operatorname{csch}^{-1}\left(\frac{x}{a}\right)$.

$$\int \frac{xdx}{\sqrt{(x^2 \pm a^2)^3}} = \frac{-1}{\sqrt{x^2 \pm a^2}}.$$

$$\int \frac{xdx}{\sqrt{(a^2 - x^2)^3}} = \frac{1}{\sqrt{a^2 - x^2}}.$$

$$\int x\sqrt{(x^2 \pm a^2)^3}dx = \tfrac{1}{5}\sqrt{(x^2 \pm a^2)^5}.$$

$$\int x\sqrt{(a^2 - x^2)^3}dx = -\tfrac{1}{5}\sqrt{(a^2 - x^2)^5}.$$

$$\int x^2\sqrt{x^2 \pm a^2}dx = \frac{x}{4}\sqrt{(x^2 \pm a^2)^3} \mp \frac{a^2}{8}x\sqrt{x^2 \pm a^2} - \frac{a^4}{8}\log\,(x + \sqrt{x^2 \pm a^2}).^*$$

$$\int x^2\sqrt{a^2 - x^2}dx = -\frac{x}{4}\sqrt{(a^2 - x^2)^3} + \frac{a^2}{8}\left(x\sqrt{a^2 - x^2} + a^2\sin^{-1}\frac{x}{a}\right).$$

$$\int \frac{x^2dx}{\sqrt{x^2 \pm a^2}} = \frac{x}{2}\sqrt{x^2 \pm a^2} \mp \frac{a^2}{2}\log\,(x + \sqrt{x^2 \pm a^2}).^*$$

$$\int \frac{x^2dx}{\sqrt{a^2 - x^2}} = -\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}.$$

$$\int \frac{dx}{x^2\sqrt{x^2 \pm a^2}} = \mp\frac{\sqrt{x^2 \pm a^2}}{a^2x}.$$

$$\int \frac{dx}{x^2\sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2x}.$$

$$\int \frac{\sqrt{x^2 \pm a^2}dx}{x^2} = -\frac{\sqrt{x^2 \pm a^2}}{x} + \log\,(x + \sqrt{x^2 \pm a^2}).^*$$

$$\int \frac{\sqrt{a^2 - x^2}}{x^2}dx = -\frac{\sqrt{a^2 - x^2}}{x} - \sin^{-1}\frac{x}{a}.$$

$$\int \frac{x^2dx}{\sqrt{(x^2 \pm a^2)^3}} = \frac{-x}{\sqrt{x^2 \pm a^2}} + \log\,(x + \sqrt{x^2 \pm a^2}).^*$$

$$\int \frac{x^2dx}{\sqrt{(a^2 - x^2)^3}} = \frac{x}{\sqrt{a^2 - x^2}} - \sin^{-1}\frac{x}{a}.$$

C.—Expressions involving $\sqrt{a + bx + cx^2}$

Let $X = a + bx + cx^2$, $q = 4\,ac - b^2$, and $k = \frac{4c}{q}$. In order to rationalize the function $f(x)$, $\sqrt{a + bx + cx^2}$), we may put $\sqrt{a + bx + cx^2} = \sqrt{\pm c}\sqrt{A + Bx \pm x^2}$, according as c is positive or negative, and then substitute for x a new variable z, such that

$z = \sqrt{A + Bx + x^2} - x$, if $c > 0$;

$z = \dfrac{\sqrt{A + Bx - x^2} - \sqrt{A}}{x}$, if $c < 0$ and $\dfrac{a}{-c} > 0$;

$z = \sqrt{\dfrac{x - \beta}{\alpha - x}}$, where α and β are the roots of the equation $A + Bx - x^2 = 0$, if $c < 0$ and $\dfrac{a}{-c} < 0$.

*See note on page 100.

By rationalization, or by the aid of reduction formulas, may be obtained the values of the ollowing integrals:

$$\int\frac{dx}{\sqrt{X}}=\frac{1}{\sqrt{c}}\log\left(\sqrt{X}+x\sqrt{c}+\frac{b}{2\sqrt{c}}\right),\quad\text{or}\quad\frac{1}{\sqrt{c}}\sinh^{-1}\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right),\ \text{if } c>0.$$

$$\int\frac{dx}{\sqrt{X}}=\frac{1}{\sqrt{-c}}\sin^{-1}\left(\frac{-2cx-b}{\sqrt{b^2-4ac}}\right),\ \text{if } c<0.$$

$$\int\frac{dx}{X\sqrt{X}}=\frac{2(2cx+b)}{q\sqrt{X}}.$$

$$\int\frac{dx}{X^2\sqrt{X}}=\frac{2(2cx+b)}{3q\sqrt{X}}\left(\frac{1}{X}+2k\right).$$

$$\int\frac{dx}{X^n\sqrt{X}}=\frac{2(2cx+b)\sqrt{X}}{(2n-1)qX^n}+\frac{2k(n-1)}{2n-1}\int\frac{dx}{X^{n-1}\sqrt{X}}.$$

$$\int\sqrt{X}\,dx=\frac{(2cx+b)\sqrt{X}}{4c}+\frac{1}{2k}\int\frac{dx}{\sqrt{X}}.$$

$$\int X\sqrt{X}\,dx=\frac{(2cx+b)\sqrt{X}}{8c}\left(X+\frac{3}{2k}\right)+\frac{3}{8k^2}\int\frac{dx}{\sqrt{X}}.$$

$$\int X^2\sqrt{X}\,dx=\frac{(2cx+b)\sqrt{X}}{12c}\left(X^2+\frac{5X}{4k}+\frac{15}{8k^2}\right)+\frac{5}{16k^3}\int\frac{dx}{\sqrt{X}}.$$

$$\int X^n\sqrt{X}\,dx=\frac{(2cx+b)X^n\sqrt{X}}{4(n+1)c}+\frac{2n+1}{2(n+1)k}\int\frac{X^n dx}{\sqrt{X}}.$$

$$\int\frac{x\,dx}{\sqrt{X}}=\frac{\sqrt{X}}{c}-\frac{b}{2c}\int\frac{dx}{\sqrt{X}}.$$

$$\int\frac{x\,dx}{X\sqrt{X}}=-\frac{2(bx+2a)}{q\sqrt{X}}.$$

$$\int\frac{x\,dx}{X^n\sqrt{X}}=-\frac{\sqrt{X}}{(2n-1)cX^n}-\frac{b}{2c}\int\frac{dx}{X^n\sqrt{X}}.$$

$$\int\frac{x^2dx}{\sqrt{X}}=\left(\frac{x}{2c}-\frac{3b}{4c^2}\right)\sqrt{X}+\frac{3b^2-4ac}{8c^2}\int\frac{dx}{\sqrt{X}}.$$

$$\int\frac{x^2dx}{X\sqrt{X}}=\frac{(2b^2-4ac)x+2ab}{cq\sqrt{X}}+\frac{1}{c}\int\frac{dx}{\sqrt{X}}.$$

$$\int\frac{x^2dx}{X^n\sqrt{X}}=\frac{(2b^2-4ac)x+2ab}{(2n-1)cqX^{n-1}\sqrt{X}}+\frac{4ac+(2n-3)b^2}{(2n-1)cq}\int\frac{dx}{X^{n-1}\sqrt{X}}.$$

$$\int\frac{x^3dx}{\sqrt{X}}=\left(\frac{x^2}{3c}-\frac{5bx}{12c^2}+\frac{5b^2}{8c^3}-\frac{2a}{3c^2}\right)\sqrt{X}+\left(\frac{3ab}{4c^2}-\frac{5b^3}{16c^3}\right)\int\frac{dx}{\sqrt{X}}.$$

$$\int x\sqrt{X}\,dx=\frac{X\sqrt{X}}{3c}-\frac{b}{2c}\int\sqrt{X}\,dx.$$

$$\int xX\sqrt{X}\,dx=\frac{X^2\sqrt{X}}{5c}-\frac{b}{2c}\int X\sqrt{X}\,dx.$$

$$\int \frac{xX^n dx}{\sqrt{X}} = \frac{X^n\sqrt{X}}{(2n+1)c} - \frac{b}{2c}\int \frac{X^n dx}{\sqrt{X}}.$$

$$\int x^2\sqrt{X}\,dx = \left(x - \frac{5b}{6c}\right)\frac{X\sqrt{X}}{4c} + \frac{5b^2-4ac}{16c^2}\int \sqrt{X}\,dx.$$

$$\int \frac{x^2X^n dx}{\sqrt{X}} = \frac{xX^n\sqrt{X}}{2(n+1)c} - \frac{(2n+3)b}{4(n+1)c}\int \frac{xX^n dx}{\sqrt{X}} - \frac{a}{2(n+1)c}\int \frac{X^n dx}{\sqrt{X}}.$$

$$\int x^3\sqrt{X}dx = \left(x^2 - \frac{7bx}{8c} + \frac{35b^2}{48c^2} - \frac{2a}{3c}\right)\frac{X\sqrt{X}}{5c} + \left(\frac{3ab}{8c^2} - \frac{7b^3}{32c^3}\right)\int \sqrt{X}\,dx.$$

$$\int \frac{dx}{x\sqrt{X}} = -\frac{1}{\sqrt{a}}\log\left(\frac{\sqrt{X}+\sqrt{a}}{x} + \frac{b}{2\sqrt{a}}\right), \text{ if } a>0.$$

$$\int \frac{dx}{x\sqrt{X}} = \frac{1}{\sqrt{-a}}\sin^{-1}\left(\frac{bx+2a}{x\sqrt{b^2-4ac}}\right), \text{ if } a<0.$$

$$\int \frac{dx}{x\sqrt{X}} = -\frac{2\sqrt{X}}{bx}, \text{ if } a=0.$$

$$\int \frac{dx}{xX^n\sqrt{X}} = \frac{\sqrt{X}}{(2n-1)aX^n} + \frac{1}{a}\int \frac{dx}{xX^{n-1}\sqrt{X}} - \frac{b}{2a}\int \frac{dx}{X^n\sqrt{X}}.$$

$$\int \frac{dx}{x^2\sqrt{X}} = -\frac{\sqrt{X}}{ax} - \frac{b}{2a}\int \frac{dx}{x\sqrt{X}}.$$

$$\int \frac{\sqrt{X}dx}{x} = \sqrt{X} + \frac{b}{2}\int \frac{dx}{\sqrt{X}} + a\int \frac{dx}{x\sqrt{X}}.$$

$$\int \frac{X^n dx}{x\sqrt{X}} = \frac{X^n}{(2n-1)\sqrt{X}} + a\int \frac{X^{n-1}dx}{x\sqrt{X}} + \frac{b}{2}\int \frac{X^{n-1}dx}{\sqrt{X}}.$$

$$\int \frac{\sqrt{X}dx}{x^2} = -\frac{\sqrt{X}}{x} + \frac{b}{2}\int \frac{dx}{x\sqrt{X}} + c\int \frac{dx}{\sqrt{X}}.$$

$$\int \frac{x^m dx}{X^n\sqrt{X}} = \frac{1}{c}\int \frac{x^{m-2}dx}{X^{n-1}\sqrt{X}} - \frac{b}{c}\int \frac{x^{m-1}dx}{X^n\sqrt{X}} - \frac{a}{c}\int \frac{x^{m-2}dx}{X^n\sqrt{X}}.$$

$$\int \frac{x^mX^n dx}{\sqrt{X}} = \frac{x^{m-1}X^n\sqrt{X}}{(2n+m)c} - \frac{(2n+2m-1)b}{2c(2n+m)}\int \frac{x^{m-1}X^n dx}{\sqrt{X}} - \frac{(m-1)a}{(2n+m)c}\int \frac{x^{m-2}X^n dx}{\sqrt{X}}.$$

$$\int \frac{dx}{x^mX^n\sqrt{X}} = -\frac{\sqrt{X}}{(m-1)ax^{m-1}X^n} - \frac{(2n+2m-3)b}{2a(m-1)}\int \frac{dx}{x^{m-1}X^n\sqrt{X}} - \frac{(2n+m-2)c}{(m-1)a}\int \frac{dx}{x^{m-2}X^n\sqrt{X}}.$$

$$\int \frac{X^n dx}{x^m\sqrt{X}} = -\frac{X^{n-1}\sqrt{X}}{(m-1)x^{m-1}} + \frac{(2n-1)b}{2(m-1)}\int \frac{X^{n-1}dx}{x^{m-1}\sqrt{X}} + \frac{(2n-1)c}{m-1}\int \frac{X^{n-1}\,dx}{X^{m-2}\sqrt{X}}.$$

$$\int \frac{dx}{(a'+b'x)\sqrt{X}} = \frac{1}{\sqrt{-h}}\tan^{-1}\frac{2h+m(a'+b'x)}{2b'\sqrt{-hX}}, \text{ or } \frac{1}{\sqrt{h}}\log\frac{2h+m(a'+b'x)-2b'\sqrt{hX}}{a'+b'x},$$

where $m = bb' - 2a'c$ and $h = ab'^2 - a'bb' + ca'^2$.

If $h=0$, the value of the integral is $-2b'\sqrt{X}/[m(a'+b'x)]$.

D.—Miscellaneous Algebraic Expressions.

$$\int \sqrt{2\,ax-x^2}\,dx=\tfrac{1}{2}[(x-a)\sqrt{2\,ax-x^2}+a^2\sin^{-1}(x-a)/a].$$

$$\int \frac{dx}{\sqrt{2\,ax-x^2}}=\cos^{-1}a\left(\frac{a-x}{a}\right).$$

$$\int \frac{dx}{\sqrt{a+bx}\cdot\sqrt{a'+b'x}}=\frac{2}{\sqrt{-bb'}}\tan^{-1}\sqrt{\frac{-b'(a+bx)}{b(a'+b'x)}},\ \text{or}\ \frac{2}{\sqrt{bb'}}\tanh^{-1}\sqrt{\frac{b'(a+bx)}{b(a'+b'x)}}.$$

$$\int \sqrt{(a+bx)(a'+b'x)}dx=\frac{k+2\,b\sqrt{a'+b'x}}{4\,bb'}\sqrt{(a+bx)(a'+b'x)}$$
$$-\frac{k^2}{8\,bb'}\int\frac{dx}{\sqrt{a+bx}\cdot\sqrt{a'+b'x}}\ [k=ab'-a'b].$$

$$\int \sqrt{\frac{a'+b'x}{a+bx}}dx=\frac{\sqrt{a+bx}\cdot\sqrt{a'+b'x}}{b}-\frac{k}{2\,b}\int\frac{dx}{\sqrt{a+bx}\sqrt{a'+b'x}}.$$

$$\sqrt{\frac{1+x}{1-x}}\,dx=\sin^{-1}x-\sqrt{1-x^2}.$$

$$\int\sqrt{\frac{x+a}{x+b}}dx=\sqrt{(x+a)(x+b)}+(a-b)\log\left(\sqrt{x+a}+\sqrt{x+b}.\right)$$

$$\int\frac{dx}{\sqrt{(x-a)(a'-x)}}=2\sin^{-1}\sqrt{\frac{x-a}{a'-a}}.$$

$$\int\frac{(px+q)dx}{(x-a')(x-b')\sqrt{a+bx+cx^2}}=\frac{q+a'p}{a'-b'}\int\frac{dx}{(x-a')\sqrt{a+bx+cx^2}}-\frac{q+b'p}{a'-b'}\int\frac{dx}{(x-b')\sqrt{a+bx+cx^2}}.$$

$$\int\frac{dx}{(a'+b'x)\sqrt{a+bx+cx^2}}=\frac{1}{\sqrt{h}}.\log\left(\frac{2\,h+m(a'+b'x)-2\,b'\sqrt{h(a+bx+cx^2)}}{a'+b'x}\right),$$

or $\frac{1}{\sqrt{-h}}.\tan^{-1}\left(\frac{2\,h+m(a'+b'x)}{2\,b'\sqrt{-h(a+bx+cx^2)}}\right)$, where $m=bb'-2\,a'c$ and $h=ab'^2-\ 'bb'+ca'^2$.

$$\int f\left\{x,\sqrt[n]{\frac{a+bx}{a'+b'x}}\right\}dx=n(a'b-ab')\int f\left\{\frac{a-a'z^n}{b'z^n-b},z\right\}\cdot\frac{z^{n-1}dz}{(b'z^n-b)^2},\quad\text{where}\quad z^n(a'+b'x)=a+bx.$$

$$\int f(x,\sqrt{a+bx+cx^2})dx=2\int f\left(\frac{2\sqrt{a}.z-b}{1-z^2},\frac{z^2\sqrt{a}-bz+\sqrt{a}}{1-z^2}\right).\frac{z^2\sqrt{a}-bz+\sqrt{a}}{(1-z^2)^2}dz,$$

where $xz+\sqrt{a}=\sqrt{a+bx+cx^2}$.

III. TRANSCENDENTAL FUNCTIONS.

$$\int\sin x dx=-\cos x.$$

$$\int\sin^2 x\quad=-\tfrac{1}{2}\cos x\sin x+\tfrac{1}{2}x=\tfrac{1}{2}x-\tfrac{1}{4}\sin 2\,x.$$

$$\int\sin^3 xd\quad=-\tfrac{1}{3}\cos x(\sin^2 x+2).$$

$$\int \sin^n x dx = \frac{-\sin^{n-1}x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x dx.$$

$$\int \cos x dx = \sin x.$$

$$\int \cos^2 x dx = \tfrac{1}{2} \sin x \cos x + \tfrac{1}{2} x = \tfrac{1}{2} x + \tfrac{1}{4} \sin 2x.$$

$$\int \cos^3 x dx = \tfrac{1}{3} \sin x(\cos^2 x + 2).$$

$$\int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x dx.$$

$$\int \sin x \cos x dx = \tfrac{1}{2} \sin^2 x.$$

$$\int \sin^2 x \cos^2 x dx = -\tfrac{1}{8}(\tfrac{1}{4} \sin 4x - x).$$

$$\int \sin x \cos^m x dx = -\frac{\cos^{m+1} x}{m+1}.$$

$$\int \sin^m x \cos x dx = \frac{\sin^{m+1} x}{m+1}.$$

$$\int \cos^m x \sin^n x dx = \frac{\cos^{m-1} x \sin^{n+1} x}{m+n} + \frac{m-1}{m+n}\int \cos^{m-2} x \sin^n x dx.$$

$$\int \cos^m x \sin^n x dx = -\frac{\sin^{n-1} x \cos^{m+1} x}{m+n} + \frac{n-1}{m+n}\int \cos^m x \sin^{n-2} x dx.$$

$$\int \frac{\cos^m x dx}{\sin^n x} = -\frac{\cos^{m+1} x}{(n-1)\sin^{n-1} x} - \frac{m-n+2}{n-1}\int \frac{\cos^m x dx}{\sin^{n-2} x}.$$

$$\int \frac{\cos^m x dx}{\sin^n x} = \frac{\cos^{m-1} x}{(m-n)\sin^{n-1} x} + \frac{m-1}{m-n}\int \frac{\cos^{m-2} x dx}{\sin^n x}.$$

$$\int \frac{\sin^m x dx}{\cos^n x} = -\int \frac{\cos^m\left(\frac{\pi}{2} - x\right) d\left(\frac{\pi}{2} - x\right)}{\sin^n\left(\frac{\pi}{2} - x\right)}.$$

$$\int \frac{dx}{\sin^m x \cos^n x} = \frac{1}{n-1} \cdot \frac{1}{\sin^{m-1} x \cdot \cos^{n-1} x} + \frac{m+n-2}{n-1}\int \frac{dx}{\sin^m x \cdot \cos^{n-2} x}$$

$$= -\frac{1}{m-1} \cdot \frac{1}{\sin^{m-1} x \cdot \cos^{n-1} x} + \frac{m+n-2}{m-1}\int \frac{dx}{\sin^{m-2} x \cdot \cos^n x}. \int \frac{dx}{\sin x \cos x} = \log \tan x.$$

$$\int \frac{dx}{\sin^m x} = -\frac{1}{m-1} \cdot \frac{\cos x}{\sin^{m-1} x} + \frac{m-2}{m-1}\int \frac{dx}{\sin^{m-2} x}.$$

$$\int \frac{dx}{\cos^n x} = \frac{1}{n-1} \cdot \frac{\sin x}{\cos^{n-1} x} + \frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2} x}.$$

$$\int \tan x dx = -\log \cos x.$$

$$\int \tan^2 x dx = \tan x - x.$$

$$\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx.$$

$$\int \operatorname{ctn} x dx = \log \sin x.$$

$$\int \operatorname{ctn}^2 x dx = -\operatorname{ctn} x - x.$$

$$\int \operatorname{ctn}^n x dx = -\frac{\operatorname{ctn}^{n-1} x}{n-1} - \int \operatorname{ctn}^{n-2} x dx.$$

$$\int \sec x dx = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right).$$

$$\int \sec^2 x dx = \tan x.$$

$$\int \sec^n x dx = \int \frac{dx}{\cos^n x}.$$

$$\int \csc x dx = \log \tan \tfrac{1}{2} x.$$

$$\int \csc^2 x dx = -\operatorname{ctn} x.$$

$$\int \csc^n x dx = \int \frac{dx}{\sin^n x}.$$

$$\int \frac{dx}{a + b\cos x} = \frac{-1}{\sqrt{a^2 - b^2}} \cdot \sin^{-1}\left[\frac{b + a\cos x}{a + b\cos x}\right], \quad [a > b > 0],$$

or

$$\frac{1}{\sqrt{a^2 - b^2}} \cdot \sin^{-1}\left[\frac{\sqrt{a^2 - b^2} \cdot \sin x}{a + b\cos x}\right], \quad [a > b > 0],$$

or

$$\frac{1}{\sqrt{a^2 - b^2}} \cdot \tan^{-1}\left[\frac{\sqrt{a^2 - b^2} \cdot \sin x}{b + a\cos x}\right], \quad [a > b > 0],$$

or

$$\frac{1}{\sqrt{b^2 - a^2}} \log\left[\frac{b + a\cos x + \sqrt{b^2 - a^2} \cdot \sin x}{a + b\cos x}\right], \quad [a > 0,\ b^2 > a^2].$$

$$\int \frac{dx}{a + b\cos x + c\sin x} = \frac{-1}{\sqrt{a^2 - b^2 - c^2}} \cdot \sin^{-1}\left[\frac{b^2 + c^2 + a(b\cos x + c\sin x)}{\sqrt{b^2 + c^2}(a + b\cos x + c\sin x)}\right]$$

or

$$\frac{1}{\sqrt{b^2 + c^2 - a^2}} \cdot \log\left[\frac{b^2 + c^2 + a(b\cos x + c\sin x) + \sqrt{b^2 + c^2 - a^2}(b\sin x - c\cos x)}{\sqrt{b^2 + c^2}(a + b\cos x + c\sin x)}\right].$$

$$\int x\sin x dx = \sin x - x\cos x.$$

$$\int x^2 \sin x dx = 2x\sin x - (x^2 - 2)\cos x.$$

$$\int x^3 \sin x dx = (3x^2 - 6) \sin x - (x^3 - 6x) \cos x.$$

$$\int x^m \sin x dx = -x^m \cos x + m\int x^{m-1} \cos x dx.$$

$$\int x \cos x dx = \cos x + x \sin x.$$

$$\int x^2 \cos x dx = 2x \cos x + (x^2 - 2) \sin x.$$

$$\int x^3 \cos x dx = (3x^2 - 6) \cos x + (x^3 - 6x) \sin x.$$

$$\int x^m \cos x dx = x^m \sin x - m\int x^{m-1} \sin x dx.$$

$$\int \frac{\sin x}{x^m} dx = -\frac{1}{m-1} \cdot \frac{\sin x}{x^{m-1}} + \frac{1}{m-1}\int \frac{\cos x}{x^{m-1}} dx.$$

$$\int \frac{\cos x}{x^m} dx = -\frac{1}{m-1} \cdot \frac{\cos x}{x^{m-1}} - \frac{1}{m-1}\int \frac{\sin x}{x^{m-1}} dx.$$

$$\int \frac{\sin x}{x} dx = x - \frac{x^3}{3 \cdot 3!} + \frac{x^5}{5 \cdot 5!} - \frac{x^7}{7 \cdot 7!} + \frac{x^9}{9 \cdot 9!} \cdot \cdot \cdot$$

$$\int \frac{\cos x}{x} = \log x - \frac{x^2}{2 \cdot 2!} + \frac{x^4}{4 \cdot 4!} - \frac{x^6}{6 \cdot 6!} + \frac{x^8}{8 \cdot 8!} \cdot \cdot \cdot$$

$$\int \sin(mx+a) \cdot \sin(nx+b) dx = \frac{\sin(mx - nx + a - b)}{2(m-n)} - \frac{\sin(mx + nx + a + b)}{2(m+n)}.$$

$$\int \cos(mx+a) \cdot \cos(nx+b) dx = \frac{\sin(mx + nx + a + b)}{2(m+n)} + \frac{\sin(mx - nx + a - b)}{2(m-n)}.$$

$$\int \sin(mx+a) \cdot \cos(nx+b) dx = -\frac{\cos(mx + nx + a + b)}{2(m+n)} - \frac{\cos(mx - nx + a - b)}{2(m-n)}.$$

$$\int \sin(mx+a) \cdot \sin(mx+b) dx = \frac{x}{2} \cdot \cos(b-a) - \frac{\sin(mx+a) \cdot \cos(mx+b)}{2m}.$$

$$\int \sin(mx+a) \cdot \cos(mx+b) dx = \frac{\sin(mx+a) \cdot \sin(mx+b)}{2m} - \frac{x}{2} \cdot \sin(b-a).$$

$$\int \cos(mx+a) \cdot \cos(mx+b) dx = \frac{x}{2} \cdot \cos(b-a) + \frac{\sin(mx+a) \cos(mx+b)}{2m}.$$

$$\int \sin^{-1} x dx = x \sin^{-1} x + \sqrt{1-x^2}.$$

$$\int \cos^{-1} x dx = x \cos^{-1} x - \sqrt{1-x^2}.$$

$$\int \tan^{-1} x dx = x \tan^{-1} x - \tfrac{1}{2} \log(1+x^2).$$

$$\int \operatorname{ctn}^{-1} x\,dx = x \operatorname{ctn}^{-1} x + \tfrac{1}{2} \log (1+x^2).$$

$$\int \operatorname{versin}^{-1} x\,dx = (x-1) \operatorname{versin}^{-1} x + \sqrt{2x-x^2}.$$

$$\int (\sin^{-1} x)^2 dx = x (\sin^{-1} x)^2 - 2x + 2\sqrt{1-x^2} \sin^{-1} x.$$

$$\int x \cdot \sin^{-1} x\,dx = \tfrac{1}{4} [(2x^2-1) \sin^{-1} x + x\sqrt{1-x^2}].$$

$$\int x^n \sin^{-1} x\,dx = \frac{x^{n+1} \sin^{-1} x}{n+1} - \frac{1}{n+1}\int \frac{x^{n+1} dx}{\sqrt{1-x^2}}.$$

$$\int x^n \cos^{-1} x\,dx = \frac{x^{n+1} \cos^{-1} x}{n+1} + \frac{1}{n+1}\int \frac{x^{n+1} dx}{\sqrt{1-x^2}}.$$

$$\int x^n \tan^{-1} x\,dx = \frac{x^{n+1} \tan^{-1} x}{n+1} - \frac{1}{n+1}\int \frac{x^{n+1} dx}{1+x^2}.$$

$$\int \log x\,dx = x \log x - x.$$

$$\int \frac{(\log x)^n}{x} dx = \frac{1}{n+1} (\log x)^{n+1}.$$

$$\int \frac{dx}{x \log x} = \log (\log x).$$

$$\int \frac{dx}{x (\log x)^n} = -\frac{1}{(n-1)(\log x)^{n-1}}.$$

$$\int x^m \log x\,dx = x^{m+1} \left[\frac{\log x}{m+1} - \frac{1}{(m+1)^2}\right].$$

$$\int e^{ax} dx = \frac{e^{ax}}{a}.$$

$$\int x e^{ax} dx = \frac{e^{ax}}{a^2}(ax-1).$$

$$\int x^m e^{ax} dx = \frac{x^m e^{ax}}{a} - \frac{m}{a}\int x^{m-1} e^{ax} dx.$$

$$\int \frac{e^{ax}}{x^m} dx = -\frac{1}{m-1}\frac{e^{ax}}{x^{m-1}} + \frac{a}{m-1}\int \frac{e^{ax}}{x^{m-1}} dx.$$

$$\int e^{ax} \log x\,dx = \frac{e^{ax} \log x}{a} - \frac{1}{a}\int \frac{e^{ax}}{x} dx.$$

$$\int e^{ax} . \sin px\,dx = \frac{e^{ax}(a \sin px - p \cos px)}{a^2+p^2}.$$

$$\int e^{ax} . \cos px\,dx = \frac{e^{ax}(a \cos px + p \sin px)}{a^2+p^2}.$$

$$\int \sinh x\,dx = \cosh x; \quad \int \cosh x\,dx = \sinh x.$$

$$\int \tanh x\,dx = \log \cosh x; \int \operatorname{ctnh} x\,dx = \log \sinh x.$$

$$\int \operatorname{sech} x\,dx = 2\tan^{-1}(e^x).$$

$$\int \operatorname{csch} x\,dx = \log \tanh\left(\frac{x}{2}\right).$$

$$\int x \sinh x\,dx = x\cosh x - \sinh x.$$

$$\int x \cosh x\,dx = x\sinh x - \cosh x.$$

$$\int \cosh^2 x\,dx = \tfrac{1}{2}(\sinh x \cosh x + x).$$

$$\int \sinh x \cosh x\,dx = \tfrac{1}{4}\cosh(2x).$$

$$\int \sinh^2 x\,dx = \tfrac{1}{2}(\sinh x \cosh x - x).$$

IV.—MISCELLANEOUS DEFINITE INTEGRALS.

$$\int_0^\infty \frac{a\,dx}{a^2+x^2} = \frac{\pi}{2}, \text{ if } a>0;\ 0, \text{ if } a=0;\ -\frac{\pi}{2}, \text{ if } a<0.$$

$$\int_0^\infty x^{n-1}e^{-x}dx = \int_0^1 \left[\log\frac{1}{x}\right]^{n-1} dx = \Gamma(n).$$

$\Gamma(n+1) = n\cdot\Gamma(n)$, if $n>0$. $\qquad \Gamma(2)=\Gamma(1)=1.$

$\Gamma(n+1) = n!$, if n is an integer. $\qquad \Gamma(\tfrac{1}{2}) = \sqrt{\pi}.$

$\Gamma(n) = \Pi(n-1).$ $\qquad Z(y) = D_y[\log\Gamma(y)].$

$Z(1) = -0.577216.$

$$\int_0^1 x^{m-1}(1-x)^{n-1}dx = \int_0^\infty \frac{x^{m-1}dx}{(1+x)^{m+n}} = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$

$$\int_0^{\frac{\pi}{2}} \sin^n x\,dx = \int_0^{\frac{\pi}{2}} \cos^n x\,dx = \frac{1\cdot3\cdot5\cdots(n-1)}{2\cdot4\cdot6\cdots(n)}\cdot\frac{\pi}{2}, \text{ if } n \text{ is an even integer;}$$

$$= \frac{2\cdot4\cdot6\cdots(n-1)}{1\cdot3\cdot5\cdot7\cdots n}, \text{ if } n \text{ is an odd integer;}$$

$$= \frac{1}{2}\sqrt{\pi}\,\frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}+1\right)} \text{ for any value of } n \text{ greater than } -1.$$

$$\int_0^\infty \frac{\sin mx\, dx}{x} = \frac{\pi}{2}, \text{ if } m>0;\ 0, \text{ if } m=0;\ -\frac{\pi}{2}, \text{ if } m<0.$$

$$\int_0^\infty \frac{\sin x \cdot \cos mx\, dx}{x} = 0, \text{ if } m<-1 \text{ or } m>1;$$

$$\frac{\pi}{4}, \text{ if } m=-1 \text{ or } m=1;\ \frac{\pi}{2}, \text{ if } -1<m<1.$$

$$\int_0^\infty \frac{\sin^2 x\, dx}{x^2} = \frac{\pi}{2}.$$

$$\int_0^\infty \cos(x^2)\, dx = \int_0^\infty \sin(x^2)\, dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}.$$

$$\int_0^\pi \sin kx \sin mx\, dx = \int_0^\pi \cos kx \cos mx\, dx = 0,\ [k \neq m].$$

$$\int_0^\pi \sin kx \cos mx\, dx = \frac{2\ k,}{k^2-m^2} \text{ if } k-m \text{ is odd};\quad =0, \text{ if k}-m \text{ is even.}$$

$$\int_0^\pi \sin^2 mx\, dx = \int_0^\pi \cos^2 mx\, dx = \frac{\pi}{2}.$$

$$\int_0^\pi \sin kx \cos kx\, dx = 0.$$

$$\int_0^\pi \frac{dx}{a+b\cos x} = \frac{\pi}{\sqrt{a^2-b^2}},\ [a>b>0].$$

$$\int_0^\infty \frac{\cos mx\, dx}{1+x^2} = \frac{\pi}{2}.e^{-m}.$$

$$\int_0^\infty \frac{\cos x\, dx}{\sqrt{x}} = \int_0^\infty \frac{\sin x\, dx}{x} = \sqrt{\frac{\pi}{2}}.$$

$$\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{1-k^2\sin^2 x}} = \text{K} = \frac{\pi}{2}\left[1+\left(\frac{1}{2}\right)^2 k^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 k^4 + \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 k^6 + \cdots\right], \text{ if } k^2<1.$$

$$\int_0^{\frac{\pi}{2}} \sqrt{1-k^2\sin^2 x}\cdot dx = \text{E} = \frac{\pi}{2}\left[1-\left(\frac{1}{2}\right)^2 k^2 - \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 \frac{k^4}{3} - \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 \frac{k^6}{5} - \cdots\right], \text{ if } k^2<1.$$

$$\int_0^\infty e^{-a^2x^2}\, dx = \frac{1}{2a}\sqrt{\pi} = \frac{1}{2a}\Gamma\left(\frac{1}{2}\right).$$

$$\int_0^\infty x^n e^{-ax}\, dx = \frac{\Gamma(n+1)}{a^{n+1}} = \frac{n!}{a^{n+1}}.$$

$$\int_0^\infty x^{2n} e^{-ax^2}\, dx = \frac{1\cdot 3\cdot 5\cdots(2\,n-1)}{2^{n+1}a^n}\sqrt{\frac{\pi}{a}}.$$

$$\int_0^\infty e^{-x^2-\frac{a^2}{x^2}}dx=\frac{e^{-2a}\sqrt{\pi}}{2}.$$

$$\int_0^\infty e^{-ax}\cos mx\,dx=\frac{a}{a^2+m^2},\ \text{if } a>0.$$

$$\int_0^\infty e^{-ax}\sin mx\,dx=\frac{m}{a^2+m^2},\ \text{if } a>0.$$

$$\int_0 e^{-a^2x^2}\cos bx\,dx=\frac{\sqrt{\pi}\cdot e^{-\frac{b^2}{4a}}}{2a}.$$

$$\int_0^1\frac{\log x}{1-x}dx=-\frac{\pi^2}{6}.$$

$$\int_0^1\frac{\log x}{1+x}dx=-\frac{\pi^2}{12}.$$

$$\int_0^1\frac{\log x}{1-x^2}dx=-\frac{\pi^2}{8}.$$

$$\int_0^1\log\left(\frac{1+x}{1-x}\right)\cdot\frac{dx}{x}=\frac{\pi^2}{4}$$

$$\int_0^\infty\log\left(\frac{e^x+1}{e^x-1}\right)dx=\frac{\pi^2}{4}$$

$$\int_0^1\frac{dx}{\sqrt{\log\left(\frac{1}{x}\right)}}=\sqrt{\pi}.$$

$$\int_0^1 x^m\log\left(\frac{1}{x}\right)^n dx=\frac{\Gamma(n+1)}{(m+1)^{n+1}},\ [m+1>0,\ n+1>0].$$

$$\int_0^{\frac{\pi}{2}}\log\sin x\,dx=\int_0^{\frac{\pi}{2}}\log\cos x\,dx=-\frac{\pi}{2}.\log 2.$$

$$\int_0^\pi x\cdot\log\sin x\,dx=-\frac{\pi^2}{2}\log 2.$$

Values of the complete elliptic integrals, K and E, for different values of the modulus, k.

$$K=\int_0^{\frac{\pi}{2}}\frac{dz}{\sqrt{1-k^2\sin^2 z}};\quad E=\int_0^{\frac{\pi}{2}}\sqrt{1-k^2\sin^2 z}\cdot dz.$$

$\sin^{-1} k$	K	E	$\sin^{-1} k$	K	E	$\sin^{-1} k$	K	E
0°	1.5708	1.5708	50°	1.9356	1.3055	81.0°	3.2553	1.0338
1°	1.5709	1.5707	51°	1.9539	1.2963	81.2°	3.2771	1.0326
2°	1.5713	1.5703	52°	1.9729	1.2870	81.4°	3.2995	1.0313
3°	1.5719	1.5697	53°	1.9927	1.2776	81.6°	3.3223	1.0302
4°	1.5727	1.5689	54°	2.0133	1.2681	81.8°	3.3458	1.0290
5°	1.5738	1.5678	55°	2.0347	1.2587	82.0°	3.3699	1.0278
6°	1.5711	1.5665	56°	2.0571	1.2492	82.2°	3.3946	1.0267
7°	1.5767	1.5649	57°	2.0804	1.2397	82.4°	3.4199	1.0256
8°	1.5785	1.5632	58°	2.1047	1.2301	82.6°	3.4460	1.0245
9°	1.5805	1.5611	59°	2.1300	1.2206	82.8°	3.4728	1.0234
10°	1.5828	1.5589	60°	2.1565	1.2111	83.0°	3.5004	1.0223
11°	1.5854	1.5564	61°	2.1842	1.2015	83.2°	3.5288	1.0213
12°	1.5882	1.5537	62°	2.2132	1.1921	83.4°	3.5581	1.0202
13°	1.5913	1.5507	63°	2.2435	1.1826	83.6°	3.5884	1.0192
14°	1.5946	1.5476	64°	2.2754	1.1732	83.8°	3.6196	1.0182
15°	1.5981	1.5442	65°	2.3088	1.1638	84.0°	3.6519	1.0172
16°	1.6020	1.5405	65.5°	2.3261	1.1592	84.2°	3.6853	1.0163
17°	1.6061	1.5367	66.0°	2.3439	1.1546	84.4°	3.7198	1.0153
18°	1.6105	1.5326	66.5°	3.3622	1.1499	84.6°	3.7557	1.0144
19°	1.6151	1.5283	67.0°	2.3809	1.1454	84.8°	3.7930	1.0135
20°	1.6200	1.5238	67.5°	2.4001	1.1408	85.0°	3.8317	1.0127
21°	1.6252	1.5191	68.0°	2.4198	1.1362	85.2°	3.8721	1.0118
22°	1.6307	1.5141	68.5°	2.4401	1.1317	85.4°	3.9142	1.0110
23°	1.6365	1.5090	69.0°	2.4610	1.1273	85.6°	3.9583	1.0102
24°	1.6426	1.5037	69.5°	2.4825	1.1228	85.8°	4.0044	1.0094
25°	1.6490	1.4981	70.0°	2.5046	1.1184	86.0°	4.0528	1.0087
26°	1.6557	1.4924	70.5°	2.5273	1.1140	86.2°	4.1037	1.0079
27°	1.6627	1.4864	71.0°	2.5507	1.1096	86.4°	4.1574	1.0072
28°	1.6701	1.4803	71.5°	2.5749	1.1053	86.6°	4.2142	1.0065
29°	1.6777	1.4740	72.0°	2.5998	1.1011	86.8°	4.2744	1.0059
30°	1.6858	1.4675	72.5°	2.6256	1.0968	87.0°	4.3387	1.0053
31°	1.6941	1.4608	73.0°	2.6521	1.0927	87.2°	4.4073	1.0047
32°	1.7028	1.4539	73.5°	2.6796	1.0885	87.4°	4.4812	1.0041
33°	1.7119	1.4469	74.0°	2.7081	1.0844	87.6°	4.5619	1.0036
34°	1.7214	1.4397	74.5°	2.7375	1.0804	87.8°	4.6477	1.0031
35°	1.7312	1.4323	75.0°	2.7681	1.0764	88.0°	4.7427	1.0026
36°	1.7415	1.4248	75.5°	2.7998	1.0725	88.2°	4.8479	1.0022
37°	1.7522	1.4171	76.0°	2.8327	1.0686	88.4°	4.9654	1.0017
38°	1.7633	1.4092	76.5°	2.8669	1.0648	88.6°	5.0988	1.0014
39°	1.7748	1.4013	77.0°	2.9026	1.0611	88.8°	5.2527	1.0010
40°	1.7868	1.3931	77.5°	2.9397	1.0574	89.0°	5.4349	1.0008
41°	1.7992	1.3849	78.0°	2.9786	1.0538	89.1°	5.5402	1.0006
42°	1.8122	1.3765	78.5°	3.0192	1.0502	89.2°	5.6579	1.0005
43°	1.8256	1.3680	79.0°	3.0617	1.0468	89.3°	5.7914	1.0005
44°	1.8396	1.3594	79.5°	3.1064	1.0434	89.4°	5.9455	1.0003
45°	1.8541	1.3506	80.0°	3.1534	1.0401	89.5°	6.1278	1.0002
46°	1.8691	1.3418	80.2°	3.1729	1.0388	89.6°	6.3504	1.0001
47°	1.8848	1.3329	80.4°	3.1928	1.0375	89.7°	6.6385	1.0001
48°	1.9011	1.3238	80.6°	3.2132	1.0363	89.8°	7.0440	1.0000
49°	1.9180	1.3147	80.8°	3.2340	1.0350	89.9°	7.7371	1.0000

Common logarithm of Γ (n) for values of n between 1 and 2

$$\Gamma(n)=\int_0^\infty x^{n-1}\cdot e^{-x}dx=\int_0^1\left[\log\frac{1}{x}\right]^{n-1}dx.$$

n	$\log_{10}\Gamma(n)$	n	$\log_{10}\Gamma(n)$	n	$\log_{10}\Gamma(n)$	n	$\log_{10}\Gamma(n)$	n	$\log_{10}\Gamma(n)$
1.01	$\bar{1}.9975$	1.21	$\bar{1}.9617$	1.41	$\bar{1}.9478$	1.61	$\bar{1}.9517$	1.81	$\bar{1}.9704$
1.02	$\bar{1}.9951$	1.22	$\bar{1}.9605$	1.42	$\bar{1}.9476$	1.62	$\bar{1}.9523$	1.82	$\bar{1}.9717$
1.03	$\bar{1}.9928$	1.23	$\bar{1}.9594$	1.43	$\bar{1}.9475$	1.63	$\bar{1}.9529$	1.83	$\bar{1}.9730$
1.04	$\bar{1}.9905$	1.24	$\bar{1}.9583$	1.44	$\bar{1}.9473$	1.64	$\bar{1}.9536$	1.84	$\bar{1}.9743$
1.05	$\bar{1}.9883$	1.25	$\bar{1}.9573$	1.45	$\bar{1}.9473$	1.65	$\bar{1}.9543$	1.85	$\bar{1}.9757$
1.06	$\bar{1}.9862$	1.26	$\bar{1}.9564$	1.46	$\bar{1}.9472$	1.66	$\bar{1}.9550$	1.86	$\bar{1}.9771$
1.07	$\bar{1}.9841$	1.27	$\bar{1}.9554$	1.47	$\bar{1}.9473$	1.67	$\bar{1}.9558$	1.87	$\bar{1}.9786$
1.08	$\bar{1}.9821$	1.28	$\bar{1}.9546$	1.48	$\bar{1}.9473$	1.68	$\bar{1}.9566$	1.88	$\bar{1}.9800$
1.09	$\bar{1}.9802$	1.29	$\bar{1}.9538$	1.49	$\bar{1}.9474$	1.69	$\bar{1}.9575$	1.89	$\bar{1}.9815$
1.10	$\bar{1}.9783$	1.30	$\bar{1}.9530$	1.50	$\bar{1}.9475$	1.70	$\bar{1}.9584$	1.90	$\bar{1}.9831$
1.11	$\bar{1}.9765$	1.31	$\bar{1}.9523$	1.51	$\bar{1}.9477$	1.71	$\bar{1}.9593$	1.91	$\bar{1}.9846$
1.12	$\bar{1}.9748$	1.32	$\bar{1}.9516$	1.52	$\bar{1}.9479$	1.72	$\bar{1}.9603$	1.92	$\bar{1}.9862$
1.13	$\bar{1}.9731$	1.33	$\bar{1}.9510$	1.53	$\bar{1}.9482$	1.73	$\bar{1}.9613$	1.93	$\bar{1}.9878$
1.14	$\bar{1}.9715$	1.34	$\bar{1}.9505$	1.54	$\bar{1}.9485$	1.74	$\bar{1}.9623$	1.94	$\bar{1}.9895$
1.15	$\bar{1}.9699$	1.35	$\bar{1}.9500$	1.55	$\bar{1}.9488$	1.75	$\bar{1}.9633$	1.95	$\bar{1}.9912$
1.16	$\bar{1}.9684$	1.36	$\bar{1}.9495$	1.56	$\bar{1}.9492$	1.76	$\bar{1}.9644$	1.96	$\bar{1}.9929$
1.17	$\bar{1}.9669$	1.37	$\bar{1}.9491$	1.57	$\bar{1}.9496$	1.77	$\bar{1}.9656$	1.97	$\bar{1}.9946$
1.18	$\bar{1}.9655$	1.38	$\bar{1}.9487$	1.58	$\bar{1}.9501$	1.78	$\bar{1}.9667$	1.98	$\bar{1}.9964$
1.19	$\bar{1}.9642$	1.39	$\bar{1}.9483$	1.59	$\bar{1}.9506$	1.79	$\bar{1}.9679$	1.99	$\bar{1}.9982$
1.20	$\bar{1}.9629$	1.40	$\bar{1}.9481$	1.60	$\bar{1}.9511$	1.80	$\bar{1}.9691$	2.00	0.0000

$$\left\{\begin{array}{l}\Gamma(z+1)=z\cdot\Gamma(z),\text{ if } z>0;\ \Gamma(2)=\Gamma(1)=1;\\ {[\Gamma(x)\cdot\Gamma(1-x)]}=\pi/\sin \pi x,\text{ if } 1>x>0.\end{array}\right\}$$

If the values of an analytic function, $f(x)$, are given in a table for consecutive values of the argument, x, with the constant interval d, and if $h=kd$, where k is any desired fraction,

$$f(a+h)=f(a)+k\cdot\Delta_1+\frac{k(k-1)}{2\,!}\cdot\Delta_2+\frac{k(k-1)\,(k-2)}{3\,!}\cdot\Delta_3+\cdots,$$

where $f(a)$ is any tabulated value.

AIRCRAFT DESIGN DATA. NOTE NO. 2.

WEIGHTS OF MATERIAL.

I.—UNIT WEIGHTS.

Material.	Unit.	Pounds.
Acetone	per cubic foot	49
Acetylene [1]	do	0.073
Agate	do	162
Air [1]	do	0.081
Alcohol:		
Ethyl	do	50
Methyl	do	51
Aluminum:		
Bronze	do	485
Cast	do	162
Wrought	do	167
Amber	do	68
Antimony	do	418
Arsenic	do	358
Asbestos	do	153
Ash: [2]		
Commercial white	do	40
Black	do	35
Aspen [2]	do	28
Asphaltum	do	81
Balsa wood [2]	do	10
Basswood [2]	do	25
Beech [2]	do	41
Beeswax	do	61
Benzol	do	56
Birch [2]	do	43
Bismuth	do	612
Bone	do	116
Brass:		
Cast	do	505
Muntz	do	512
Naval rolled	do	525
Sheet	do	527
Wire	do	533
Brick	do	120
Bronze (gun metal)	do	550
Butternut [2]	do	27
Caoutchouc	do	60
Carbolic acid (crude)	do	60
Carbon dioxide [1]	do	0.123
Carbon monoxide [1]	do	0.078
Castor oil	do	61
Cedar: [2]		
Incense	do	26
Port Orford	do	31
Spanish	do	26
Western red	do	23
White northern	do	22
Celluloid	do	87
Cellulose, cone pith	do	8
Cement (set)	do	180
Chalk	do	137
Charcoal:		
Oak	per cubic foot	35
Pine	do	23
Cherry,[2] black	do	35
Chestnut [2]	do	31
Chlorine,[1] gas	do	0.198
Chloroform	do	92
Clay	do	137
Coal:		
Anthracite	do	97
Bituminous	do	84
Cocoanut oil	do	58
Coke	do	28
Copper:		
Cast	do	537
Hammered	do	556
Sheet, $\frac{1}{10}$ inch thick	per square foot	4.58
Wire	per cubic foot	554
Cork, compressed	do	14.4
Corundum	do	248
Cottonseed oil	do	58
Cottonwood [2]	do	28
Cottonwood, black [2]	do	25
Creosote oil	do	67
Cucumber wood [2]	do	34
Cylinder oil	do	56.2
Do	per gallon	7.5
Cypress: [2]		
Bald	per cubic foot	31
Yellow	do	30
Dogwood [2]	do	52
Elm,[2] rock and cork	do	44
White	do	35
Emery	do	250
Ether	do	50
Feldspar	do	166
Fiber (hard), vulcanite, bakelite, etc.	do	80
Fir: [2]		
Douglas	do	34
Alpine	do	23
Amabilis, grand and white.	do	29
Balsam and noble	do	27
Flint	do	164
Gasoline	do	46.7
Do	per gallon	6.1
Gelatine	per cubic foot	180
Glass:		
Common	do	165
Crystal	do	184

[1] 32° F. and 29.9 in. mercury.

[2] 15 per cent moisture.

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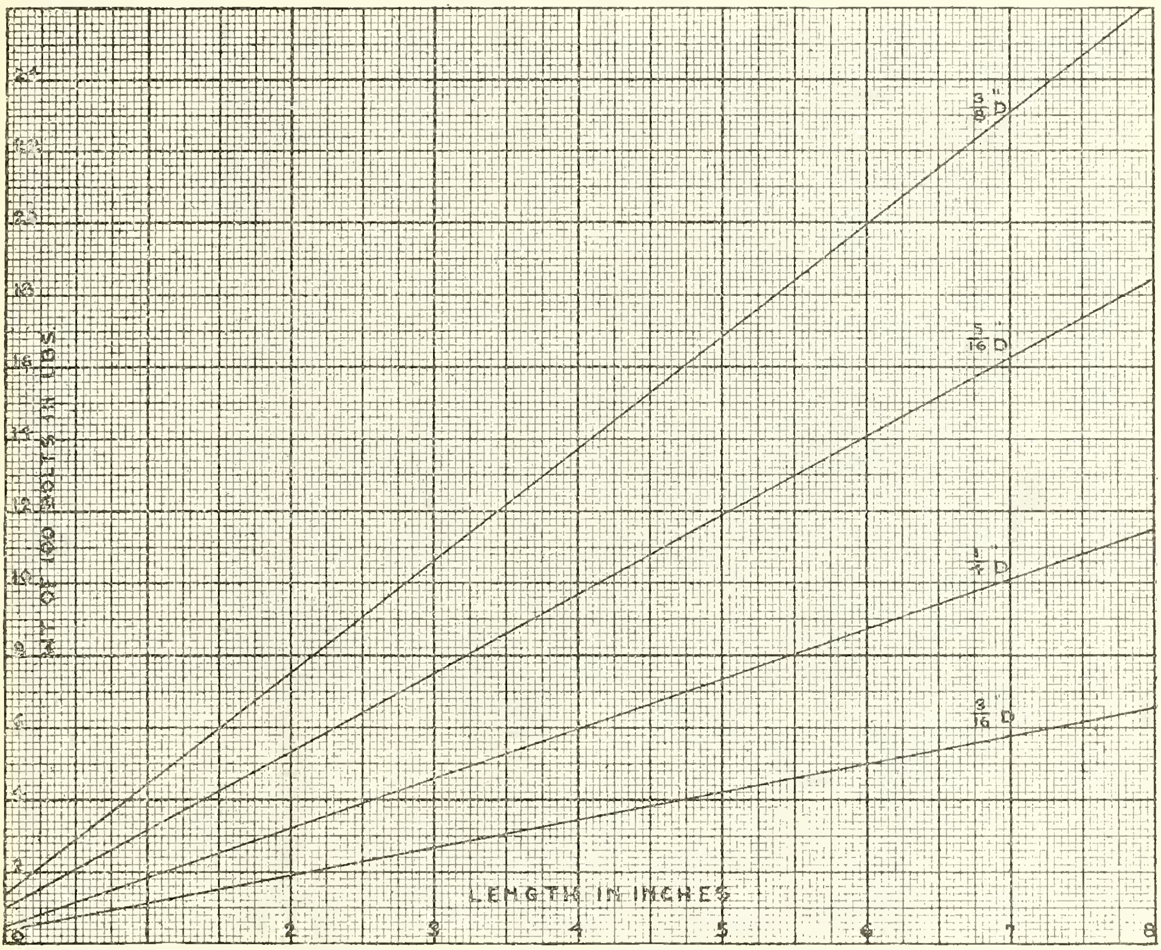
UNIT WEIGHTS—Continued.

Material.	Unit.	Pounds.
Glue	per cubic foot	80
Glycerine	do	79
Gold:		
Pure	do	1,203
Standard 22 carat fine	do	1,090
Granite	do	169
Graphite	do	157
Gum Arabic	do	83
Gum:[2]		
Red	do	34
Black	do	37
Cotton	do	35
Gum, rubber gasket material	do	100
Hemlock[2]	do	29
Hickory[2]	do	50
Holly[2]	do	40
Hornbeam[2]	do	50
Hydrogen[2]	do	0.0056
Ice	do	57.2
Iron:		
Cast	do	450
Wrought	do	480
Ivory	do	117
Kerosene	do	48
Larch[2]	do	39
Lard	do	57
Lead:		
Cast	do	710
Sheet, $\frac{1}{16}$ inch thick	per square foot	6
Leather:		
Dry	per cubic foot	54
Greased	do	64
Lime:		
Mortar	do	107
Slacked	do	84
Limestone	do	169
Linen, airplane, (4 coats dope and 1 coat paint)	per square foot	0.085
Linoleum, medium, $\frac{1}{8}$ inch thick	do	0.65
Linseed oil (boiled)	per cubic foot	59
Litharge, artificial	do	533
Litharge, natural	do	495
Locust:[2]		
Black	do	50
Honey	do	46
Mahogany:[2]		
True	do	36
African	do	34
Philippine	do	35
Manganese	do	499
Maple:[2]		
Hard	do	42
Red	do	37
Silver	do	35
Sugar	do	44
Marble	do	169
Meerchaum	do	71
Mercury	do	849
Methane[1]	do	0.045
Mica, $\frac{1}{16}$ inch thick	per square foot	153
Monel metal	per cubic foot	556
Naphtha:		
Wood	per cubic foot	53
Petroleum	do	42
Nickel:		
Cast	do	516
Rolled	do	541
Nitrogen[1]	do	0.078
Oak:[2]		
Commercial white	do	46
Red	do	44
Olive oil	do	57
Oxygen[1]	do	0.089
Palm oil	do	57
Paraffin wax	do	56
Paper	do	58
Peat	do	52
Persimmon[2]	do	51
Petroleum	do	55
Pine:[2]		
Norway	do	33
Sugar	do	27
Western	do	29
White	do	27
Pitch	do	67
Platinum	do	1,340
Poplar,[2] yellow	do	28
Porcelain	do	150
Quartz	do	165
Resin	do	67
Rubber goods	do	94
Rubber matting, $\frac{3}{16}$ inch thick	per square foot	0.57
Salt, rock	per cubic foot	136
Sandstone	do	141
Silver	do	655
Spruce,[2] red, white, and Sitka	do	27
Soapstone	do	169
Steam, 212° F.	do	0.036
Steel:		
Cast and wrought	do	490
Nickel, face-hardened	do	491
Sycamore[2]	do	36
Talc	do	171
Tallow	do	59
Tamarack[2]	do	38
Tar, bituminous	do	75
Tin	do	462
Turpentine	do	54
Walnut,[2] black	do	38
Water:		
Fresh	do	62.5
1 cubic foot=7.48 gallons	per gallon	8.35
Sea	per cubic foot	64
Whale oil	do	58
White metal (Babbitt)	do	456
Willow,[1] black	do	27
Yew[2]	do	46
Zinc:		
Cast	do	435
Sheet	do	449

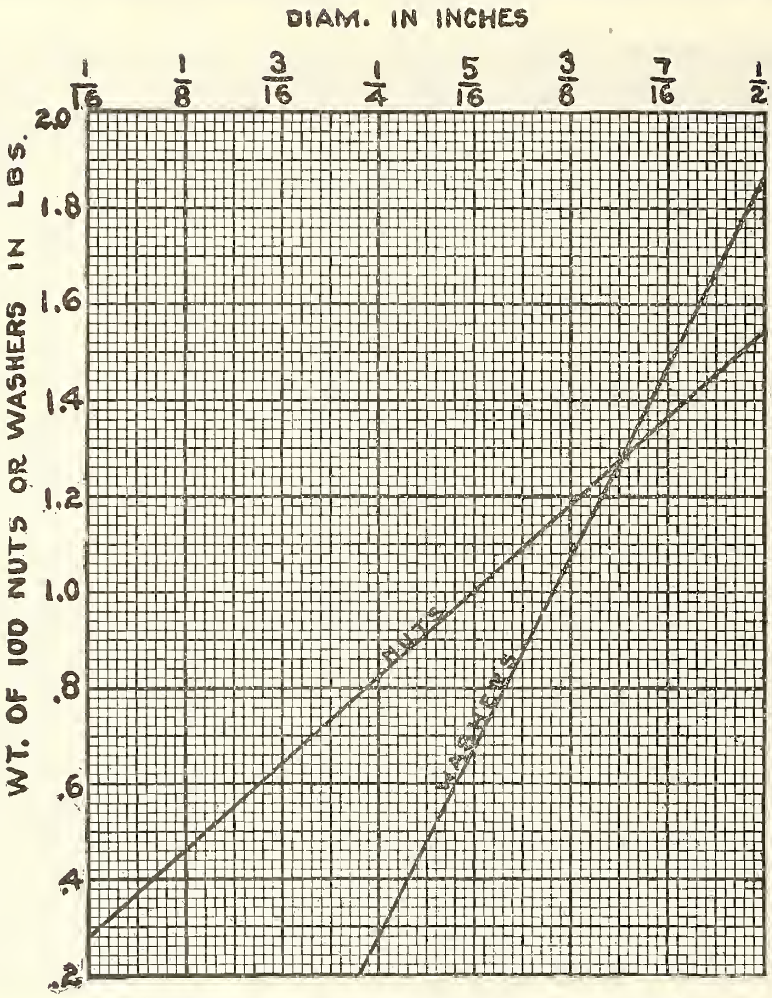
[1] 32° F. and 29.9 in mercury.   [2] 15 per cent moisture.

Ref.: Forest Service. Smithsonian Tables.

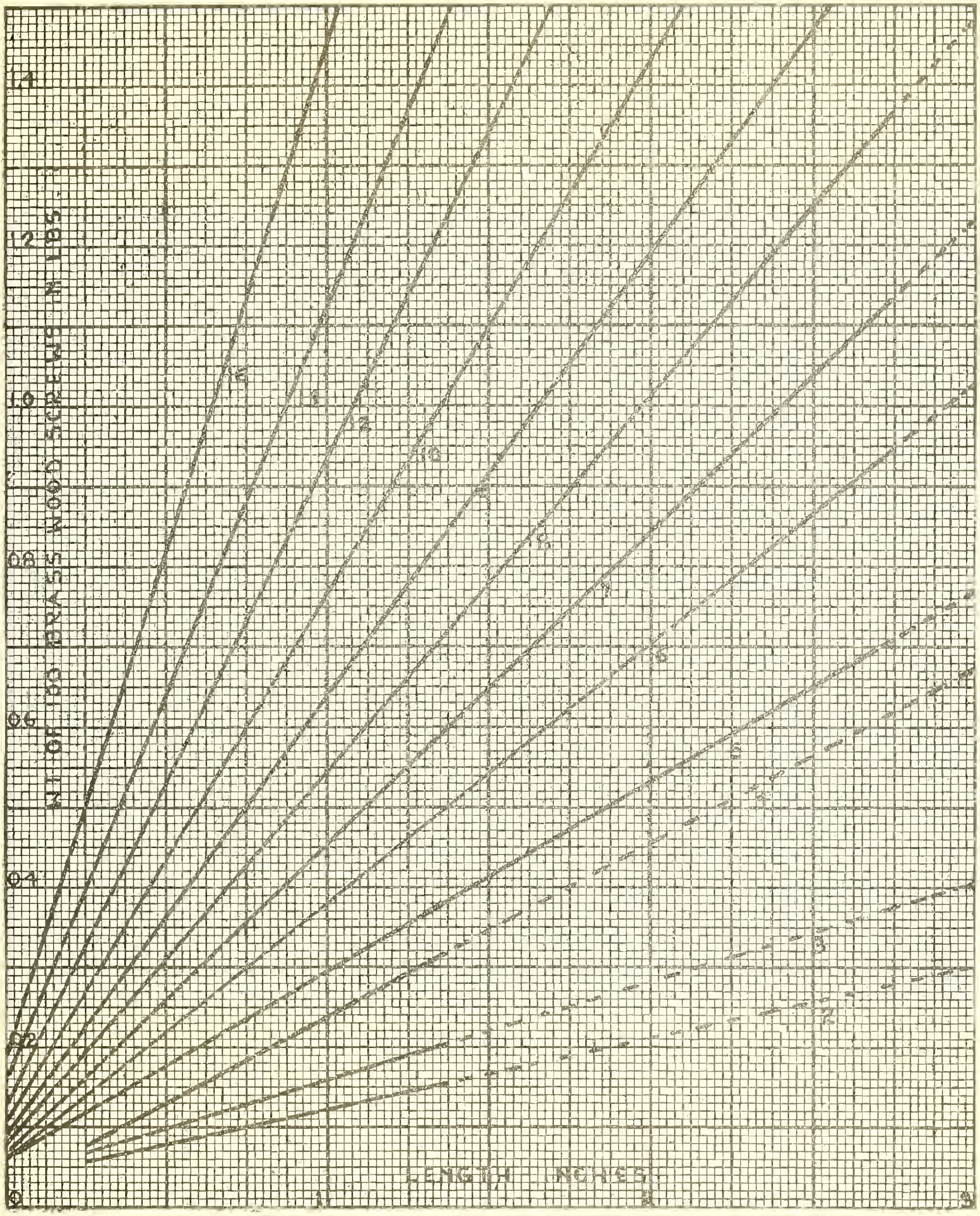
II.—WEIGHTS OF BOLTS, NUTS, WASHERS, SCREWS, NAILS, PINS, AND END FITTINGS.



WEIGHT OF AIRPLANE BOLTS.

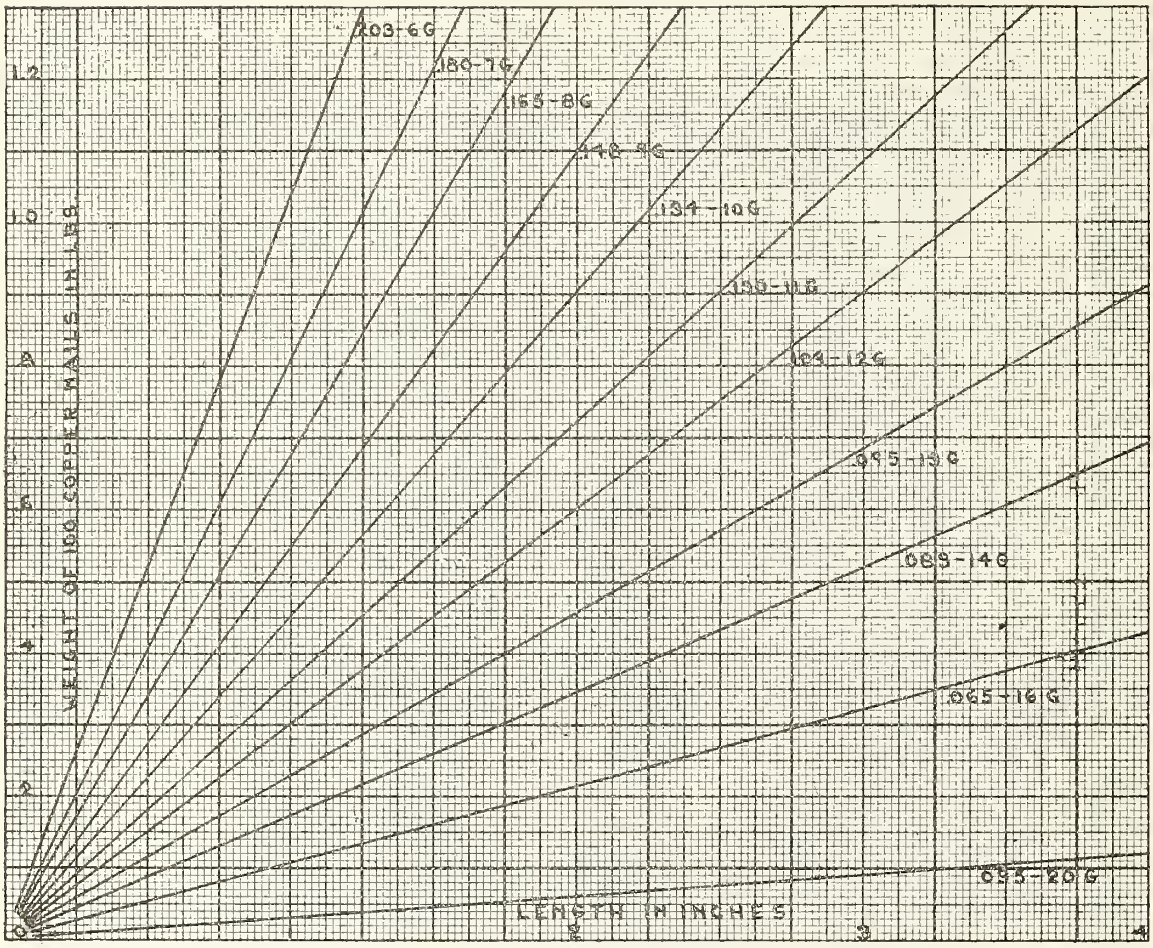


WEIGHTS OF AIRPLANE NUTS AND WASHERS.



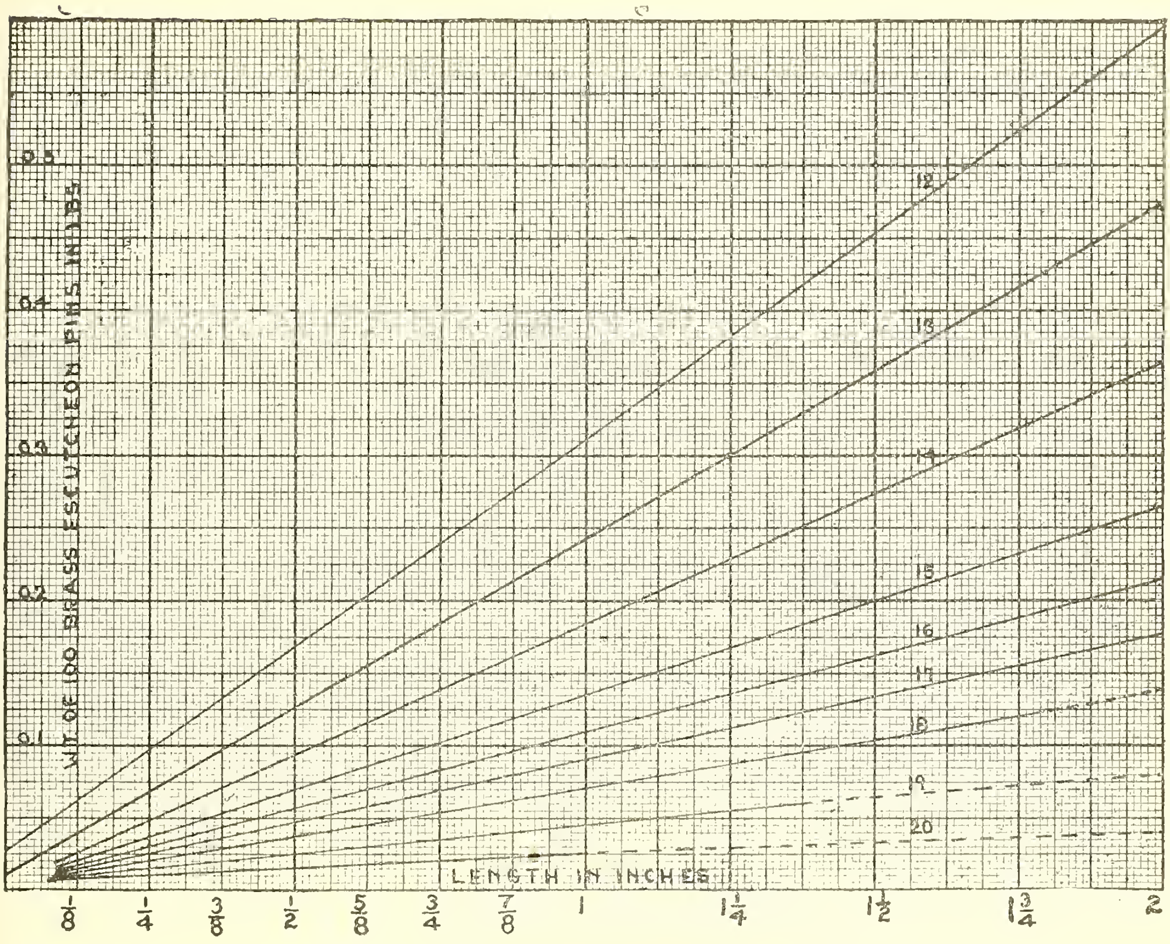
WEIGHT AND SIZE OF BRASS WOOD SCREWS IN POUNDS.

No. of screw.	2	3	4	5	6	8	10	12	14	16
Diameter across flat.........	0.163	0.189	0.216	0.242	0.268	0.321	0.374	0.426	0.479	0.532
Diameter of drill............	.048	.097	.110	.123	.136	.163	.189	.215	.242	.268



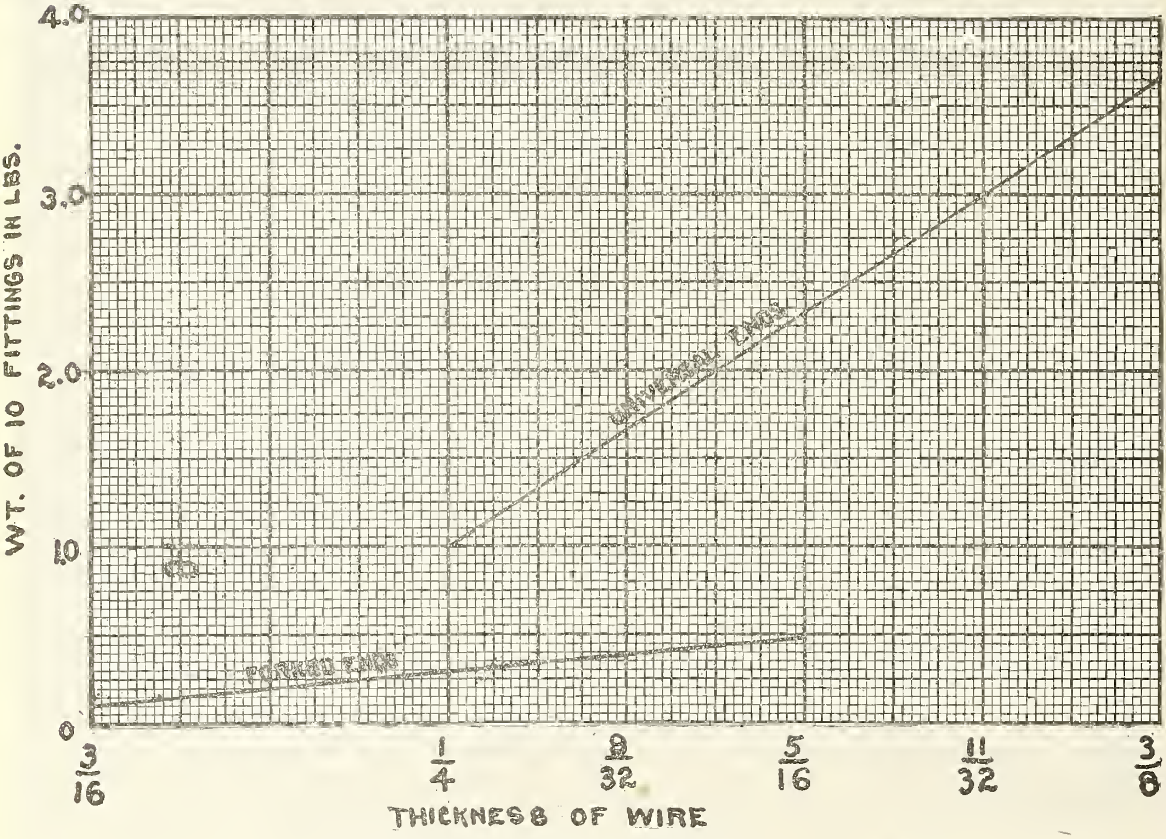
WEIGHT OF COPPER NAILS.

(Diameter of nails in B. W. Gauge.)



WEIGHT OF BRASS ESCUTCHEON PINS.

(Numbers on curves give size in B. W. Gauge.)



STREAMLINE WIRE END FITTINGS.

(C. & R. Aeronautical Specifications Nos. 62 and 68.)

WASHINGTON : GOVERNMENT PRINTING OFFICE : 1919

AIRCRAFT DESIGN DATA. NOTE NO. 3.

PROPERTIES OF SECTIONS.

I.—STANDARD SECTIONS.

98257—29—No. 3——1

TABLE 1.—*Standard sections.*

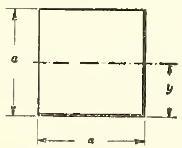
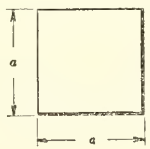
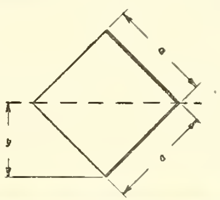
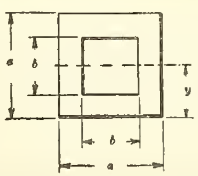
Section.	Area of section, A	Distance from neutral axis to extreme fiber. y	Moment of inertia, I	Section modulus, $S=\frac{I}{y}$	Radius of gyration, $K=\sqrt{\frac{I}{A}}$
	a^2	$\frac{1}{2}a$	$\frac{a^4}{12}$	$\frac{a^3}{6}$	$\frac{a}{\sqrt{12}}=0.289a$
	a^2	a	$\frac{a^4}{3}$	$\frac{a^3}{3}$	$\frac{a}{\sqrt{3}}=0.577a$
	a^2	$\frac{a}{\sqrt{2}}=0.707a$	$\frac{a^4}{12}$	$\frac{a^3}{6\sqrt{2}}=0.118a^3$	$\frac{a}{\sqrt{12}}=0.289a$
	a^2-b^2	$\frac{1}{2}a$	$\frac{a^4-b^4}{12}$	$\frac{a^4-b^4}{6a}$	$\sqrt{\frac{a^2+b^2}{12}}=0.289\sqrt{a^2+b^2}$

TABLE 1.—*Standard sections*—Continued.

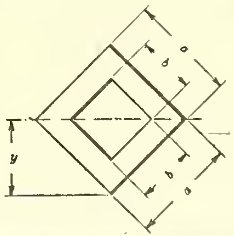
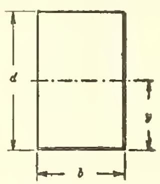
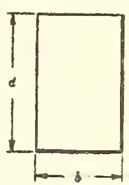
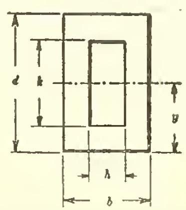
Section.	Area of section, A	Distance from neutral axis to extreme fiber, y	Moment of inertia, I	Section modulus, $S=\frac{I}{y}$	Radius of gyration, $K=\sqrt{\frac{I}{A}}$
	a^2-b^2	$\frac{a}{\sqrt{2}}=0.707a$	$\frac{a^4-b^4}{12}$	$\frac{\sqrt{2}(a^4-b^4)}{12a}=0.118\frac{a^4-b^4}{a}$	$\sqrt{\frac{a^2+b^2}{12}}=0.289\sqrt{a^2+b^2}$
	bd	$\frac{1}{2}d$	$\frac{bd^3}{12}$	$\frac{bd^2}{6}$	$\frac{d}{\sqrt{12}}=0.289d$
	bd	d	$\frac{bd^3}{3}$	$\frac{bd^2}{3}$	$\frac{d}{\sqrt{3}}=0.577d$
	$bd-hk$	$\frac{1}{2}d$	$\frac{bd^3-hk^3}{12}$	$\frac{bd^3-hk^3}{6d}$	$\sqrt{\frac{bd^3-hk^3}{12(bd-hk)}}$ $=0.289\sqrt{\frac{bd^3-hk^3}{bd-hk}}$

TABLE 1.—*Standard sections*—Continued.

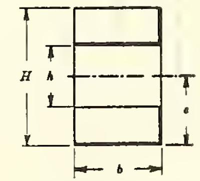
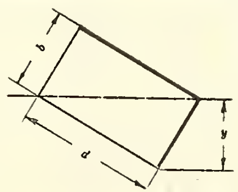
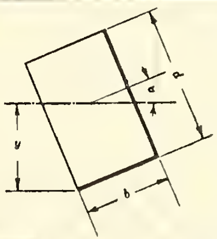
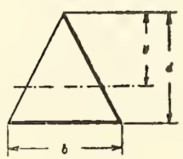
Section.	Area of section, A	Distance from neutral axis to extreme fiber, y	Moment of inertia, I	Section modulus, $S=\frac{I}{y}$	Radius of gyration, $K=\sqrt{\frac{I}{A}}$
	$bH-bh$	c	$\frac{b}{12}(H^3-h^3)$	$\frac{b}{6}\frac{H^3-h^3}{H}$	$\sqrt{\frac{H^3-h^3}{12(H-h)}}$
	bd	$\frac{bd}{\sqrt{b^2+d^2}}$	$\frac{b^3d^3}{6(b^2+d^2)}$	$\frac{b^2d^2}{6\sqrt{b^2+d^2}}$	$\frac{bd}{\sqrt{6(b^2+d^2)}}$ $=0.408\frac{bd}{\sqrt{b^2+d^2}}$
	bd	$\frac{1}{2}(d\cos\alpha+b\sin\alpha)$	$bd\ (d^2\cos^2\alpha+b^2\sin^2\alpha)$	$\frac{bd}{6}\left(\frac{d^2\cos^2\alpha+b^2\sin^2\alpha}{d\cos\alpha+b\sin\alpha}\right)$	$\sqrt{\frac{d^2\cos^2\alpha+b^2\sin^2\alpha}{12}}$ $=0.289\times$ $\sqrt{d^2\cos^2\alpha+b^2\sin^2\alpha}$
	$\frac{1}{2}bd$	$\frac{2}{3}d$	$\frac{bd^3}{36}$	$\frac{bd^2}{24}$	$\frac{d}{\sqrt{18}}=0.236d$

TABLE 1.—*Standard sections*—Continued.

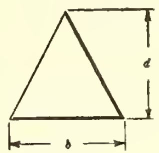
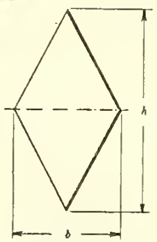
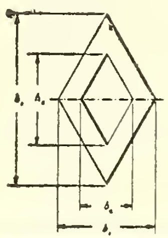
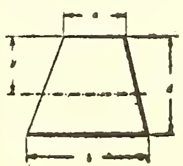
Section.	Area of section, A	Distance from neutral axis to extreme fiber, y	Moment of inertia, I	Section modulus, $S=\frac{I}{y}$	Radius of gyration, $K=\sqrt{\frac{I}{A}}$
	$\frac{1}{2}bd$	d	$\frac{bd^3}{12}$	$\frac{bd^2}{12}$	$\frac{d}{\sqrt{6}}=0.408d$
	$\frac{1}{2}bh$	$\frac{1}{2}h$	$\frac{1}{48}$	$\frac{1}{24}bh^2$	$0.2041h$
	$\frac{1}{2}(b_1h_1-b_2h_2)$	$\frac{h_1}{2}$	$\frac{1}{48}(b_1h_1^3-b_2h_2^3)$	$\frac{b_1h_1^3-b_2h_2^3}{24h_1}$	$\sqrt{\frac{I}{A}}$
	$\frac{d(a+b)}{2}$	$\frac{d(a+2b)}{3(a+b)}$	$\frac{d^3(a^2+4ab+b^2)}{36(a+b)}$	$\frac{d^2(a^2+4ab+b^2)}{12(a+2b)}$	$\sqrt{\frac{d^2(a^2+4ab+b^2)}{18(a+b)^2}}$

TABLE 1.—*Standard sections*—Continued.

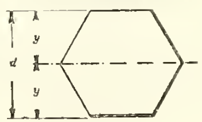
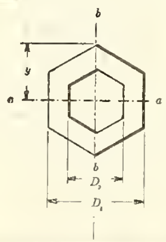
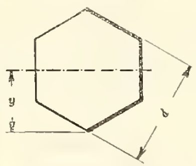
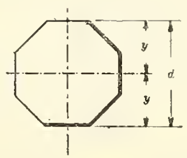
Section.	Area of section, A.	Distance from neutral axis to extreme fiber, y.	Moment of inertia, I.	Section modulus, $S=\frac{I}{y}$	Radius of gyration, $K=\sqrt{\frac{I}{A}}$.
	$\frac{3\ d^2 \tan 30°}{2}=0.866\ d^2$.	$\frac{d}{2}$	$\frac{A}{12}\left[\frac{d^2\ (1+2 \cos^2 30°)}{4 \cos^2 30°}\right]$ $=0.06\ d^4$	$\frac{A}{6}\left[\frac{d\ (1+2 \cos^2 30°)}{4 \cos^2 30°}\right]$ $=0.12\ d^3$	$\sqrt{\frac{d^2\ (1+2 \cos^2 30°)}{48 \cos^2 30°}}$ $=0.264\ d$
	$0.866\ (D_1^2-D_2^2)$ $=3.464\ (D_1-t)t$	$y=0.5774\ D_1$	$0.0601\ (D_1^4-D_2^4)$	$S\ aa=0.1042\ \frac{D_1^4-D_2^4}{D}$ $S\ bb=0.1203\ \frac{D_1^4-D_2^4}{D}$	$0.2635\sqrt{D_1^2+D_2^2}$
	$\frac{3\ d^2 \tan 30°}{2}=0.866\ d^2$	$\frac{d}{2 \cos 30°}=0.577\ d$	$\frac{A}{12}\left[\frac{d^2\ (1+2 \cos^2 30°)}{4 \cos^2 30°}\right]$ $=0.06\ d^4$	$\frac{A}{6}\left[\frac{d\ (1+2 \cos^2 30°)}{4 \cos^2 30°}\right]$ $=0.104\ d^3$	$\sqrt{\frac{d^2\ (1+2 \cos^2 30°)}{48 \cos^2 30°}}$ $=0.264\ d$
	$2\ d^2 \tan 22\tfrac{1}{2}°=0.828\ d^2$	$\frac{d}{2}$	$\frac{A}{12}\left[\frac{d^2\ (1+2 \cos^2 22\frac{1}{2}°)}{4 \cos^2 22\frac{1}{2}°}\right]$ $=0.055\ d^4$	$\frac{A}{6}\left[\frac{d\ (1+2 \cos^2 22\frac{1}{2}°)}{4 \cos^2 22\frac{1}{2}°}\right]$ $=0.109\ d^3$	$\sqrt{\frac{d^2\ (1+2 \cos^2 22\frac{1}{2}°)}{48 \cos^2 22\frac{1}{2}°}}$ $=0.257\ d$

TABLE 1.—*Standard sections*—Continued.

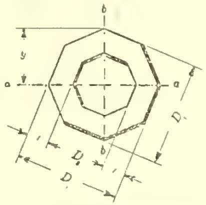
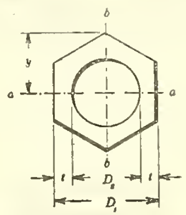
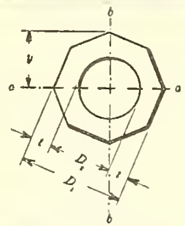
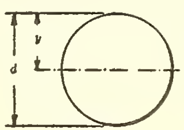
Section.	Area of section, A.	Distance from neutral axis to extreme fiber, y.	Moment of inertia, I.	Section modulus, $S=\frac{I}{y}$	Radius of gyration, $K=\sqrt{\frac{I}{A}}$.
	$0.8284\ (D_1^2-D_2^2)$ $=3.314\ (D_1-t)t$	$y=0.5412\ D_1$	$0.0547\ (D_1^4-D_2^4)$	$S\ aa=0.1011\ \frac{D_1^4-D_2^4}{D_1}$ $S\ bb=0.1095\ \frac{D_1^4-D_2^4}{D_1}$	$0.257\sqrt{D_1^2+D_2^2}$
	$0.866\ D_1^2-0.7854\ D_2^2$ $=0.0806\ D_1^2+3.1416\ (D_1-t)t$	$y=0.5774\ D_1$	$0.0601\ D_1^4-0.0491\ D_2^4$	$S\ aa=0.1042\ D_1^3-0.085\ \frac{D_2^4}{D_1}$ $S\ bb=0.1203\ D_1^3-0.0982\ \frac{D_2^4}{D_1}$	$\sqrt{\frac{I}{A}}$
	$0.8284\ D_1^4-0.7854\ D_2^2$ $=0.0430\ D_1^2+3.1416\ (D_1-t)t$	$y=0.5412\ D_1$	$0.0547\ D_1^4-0.0491\ D_2^4$	$S\ aa=0.1011\ D_1^3-0.0907\ \frac{D_2^4}{D_1}$ $S\ bb=0.1095\ D_1^3-0.0982\ \frac{D_2^4}{D_1}$	$\sqrt{\frac{I}{A}}$
	$\frac{\pi d^2}{4}=0.7854\ d^2$	$\frac{d}{2}$	$\frac{\pi d^4}{64}=0.049\ d^4$	$\frac{\pi d^3}{32}=0.098\ d^3$	$\frac{d}{4}$

TABLE 1.—*Standard sections*—Continued.

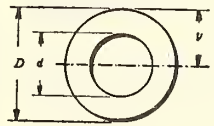
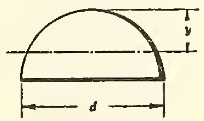
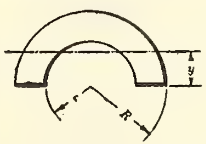
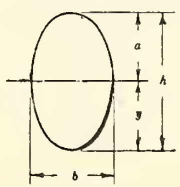
Section.	Area of section, A	Distance from neutral axis to extreme fiber, y	Moment of inertia, I	Section modulus, $S=\frac{I}{y}$	Radius of gyration, $K=\sqrt{\frac{I}{A}}$
	$\frac{\pi(D^2-d^2)}{4}$ $=0.7854\ (D^2-d^2)$	$\frac{D}{2}$	$\frac{\pi(D^4-d^4)}{64}$ $=0.049\ (D^4-d^4)$	$\frac{\pi(D^4-d^4)}{32\ D}$ $=0.098\ \frac{D^4-d^4}{D}$	$\frac{\sqrt{D^2+d^2}}{4}$
	$\frac{\pi d^2}{8}=0.393\ d^2$	$\frac{(3\pi-4)\ d}{6\pi}=0.288d$	$\frac{(9\pi^2-64)d^4}{1152\pi}=0.007\ d^4$	$\frac{(9\pi^2-64)d^3}{192(3\pi-4)}=0.024\ d^3$	$\frac{\sqrt{(9\pi^2-64)d^2}}{12\pi}=0.132d$
	$\frac{\pi(R^2-r^2)}{2}$ $=1.5708\ (R^2-r^2)$	$\frac{4(R^3-r^3)}{3\pi(R^2-r^2)}$ $=0.424\ \frac{R^3-r^3}{R^2-r^2}$	$0.1098\ (R^4-r^4)$ $-\frac{0.283\ R^2r^2\ (R-r)}{R+r}$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
	$0.7854\ bh$	a	$0.0491\ bh^3$	$S=0.0982\ bh^2$	$\frac{1}{4}\ h$

TABLE 1.—*Standard sections*—Continued.

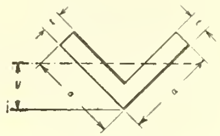
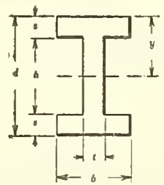
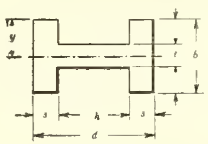
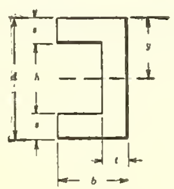
Section.	Area of section, A	Distance from neutral axis to extreme fiber, y	Moment of inertia, I	Section modulus, $S=\frac{I}{y}$	Radius of gyration, $K=\sqrt{\frac{I}{A}}$
	$t(2a-t)$	$\frac{a^2+at-t^2}{2(2a-t)\cos 45^2}$	$\frac{1}{3}\Big[2x^4-2(x-t)^4 + t[a-(2x-\frac{1}{2}t)]^3\Big]$ in which $x=\frac{a^2+at-t^2}{2(2a-t)}$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
	$bd-h(b-t)$	$\frac{d}{2}$	$\frac{bd^3-h^3(b-t)}{12}$	$\frac{bd^3-h^3(b-t)}{6d}$	$\sqrt{\frac{bd^3-h^3(b-t)}{12(bd-h(b-t)}}$
	$bd-h(b-t)$	$\frac{b}{2}$	$\frac{2sb^3+ht^3}{12}$	$\frac{2sb^3+ht^3}{6b}$	$\sqrt{\frac{2sb^3+ht^3}{12[bd-h(b-t)]}}$
	$bd-h(b-t)$	$\frac{d}{2}$	$\frac{bd^3-h^3(b-t)}{12}$	$\frac{bd^3-h^3(b-t)}{6d}$	$\sqrt{\frac{bd^3-h^3(b-t)}{12[bd-h(b-t)]}}$

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TABLE 1.—*Standard sections*—Continued.

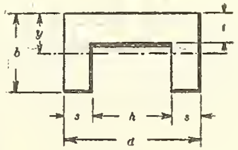
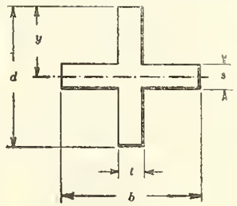
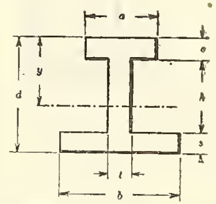
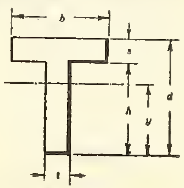
Section.	Area of section, A.	Distance from neutral axis to extreme fiber, y.	Moment of inertia, I.	Section modulus, $S=\frac{I}{y}$.	Radius of gyration, $K=\sqrt{\frac{I}{A}}$.
	$bd-h(b-t)$	$b-\frac{2b^2s-ht^2}{2bd-2h(b-t)}$	$\frac{2sb^3+ht^3}{3}-A(b-y)^2$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
	$dt+s(b-t)$	$\frac{d}{2}$	$\frac{td^3+s^3(b-t)}{12}$	$\frac{td^3+s^3(b-t)}{6d}$	$\sqrt{\frac{td^3+s^3(b-t)}{12[td+s(b-t)]}}$
	$bs+ht+as$	$d-[td^2+s^2(b-t)$ $+s(a-t)(2d-s)]\div 2A$	$\frac{1}{3}[b(d-y)^3+ay^3$ $-(b-t)(d-y-s)^3$ $-(a-t)(y-s)^3]$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
	$bs+ht$	$d-\frac{d^2t+s^2(b-t)}{2(bs+ht)}$	$\frac{1}{3}[ty^3-b(d-y)^3$ $-(b-t)(d-y-s)^3]$	$\frac{I}{y}$	$\sqrt{\frac{1}{3(bs+ht)}[ty^3+b(d-y)^3 -(b-t)(d-y-s)^3]}$

TABLE 1.—*Standard sections*—Continued.

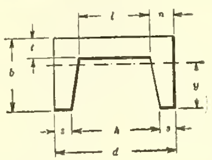
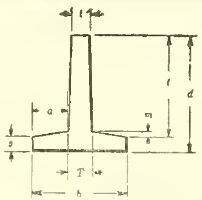
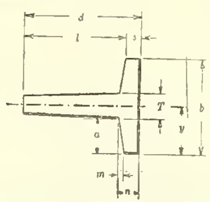
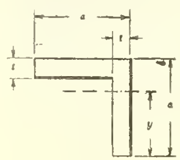
Section.	Area of section, A.	Distance from neutral axis to extreme fiber, y.	Moment of inertia, I.	Section modulus, $S=\frac{I}{y}$.	Radius of gyration, $K=\sqrt{\frac{I}{A}}$.
	$dt+a(s+n)$	$b-[b^2s+\frac{ht^2}{2}+\frac{g}{3}(b-t)^2$ $\times(b+2t)\div A$ In which g=slope of flange=$\frac{h-l}{2(b-t)}$	$\frac{1}{3}[2\ sb^3+lt^3+\ \frac{g}{2}(b^4-t^4)]$ $-A(b-y)^2$ In which g=slope of flange=$\frac{h-l}{2(b-t)}=\frac{1}{6}$ for standard channels.	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
	$\frac{l(T+t)}{2}+Tn+a\,(s+n)$	$d-[3s^2(b+T)$ $+2am(m+3s)+3Td^2$ $-l(T-t)(3d-l)]\div 6A$	$\frac{1}{12}[l^3(T+3t)+4bn^3$ $-2am^3]-A(d-y-n)^2$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
	$\frac{l(T+t)}{2}+Tn+a\,(s+n)$	$\frac{b}{2}$	$\frac{sb^3+mT^3+lt^3}{12}$ $+\frac{am[2a^2+(2a+3T)^2]}{36}$ $+l(T-t)\times$ $\frac{[(T-t)^2+2(T+2t)^2]}{144}$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
	$t(2a-t)$	$a-\frac{a^2+at-t^2}{2(2a-t)}$	$\frac{1}{3}[ty^3+a(a-y)^3$ $-(a-t)(a-y-t)^3]$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$

TABLE 1.—*Standard sections*—Continued.

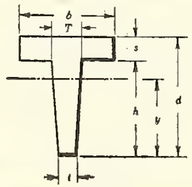
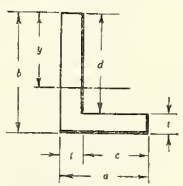
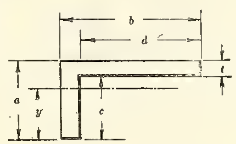
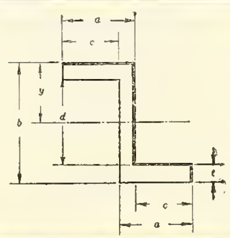
Section.	Area of section, A	Distance from neutral axis to extreme fiber,	Moment of inertia, I	Section modulus, $S=\frac{I}{y}$	Radius of gyration, $K=\sqrt{\frac{I}{A}}$
	$bs+\frac{h(T+t)}{2}$	$d-[3bs^2+3ht(d+s)$ $+h(T-t)(h+3s)]\div 6A$	$\frac{1}{12}[4bs^3+h^3(3t+T)]$ $-A(d-y-s)^2$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
	$t(a+b-t)$	$b-\frac{t(2d+a)+d^2}{2(d+a)}$	$\frac{1}{3}[ty^3+a(b-y)^3-(a-t)$ $(b-y-t)^3]$	$\frac{I}{y}$	$\sqrt{\frac{1}{3t(a+b-t)}[ty^3+a(b-y)^3-(a-t)(b-y-t)^3]}$
	$t(a+b-t)$	$a-\frac{t(2c+b)+c^2}{2(c+b)}$	$\frac{1}{3}[ty^3+b(a-y)^3-(b-t)$ $(a-y-t)^3]$	$\frac{I}{y}$	$\sqrt{\frac{1}{3t(a+b-t)}[ty^3+b(a-y)^3-(b-t)(a-y-t)^3]}$
	$t[b+2(a-t)]$	$\frac{b}{2}$	$\frac{ab^3-c(b-2t)^3}{12}$	$\frac{ab^3-c(b-2t)^3}{6b}$	$\sqrt{\frac{ab^3-c(b-2t)^3}{12t[b+2(a-t)]}}$

TABLE 1.—*Standard sections*—Continued.

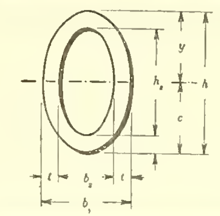
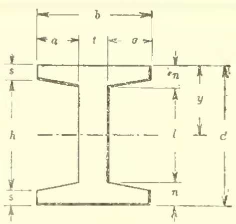
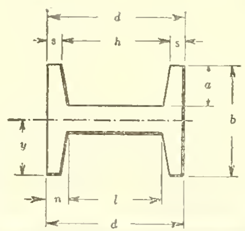
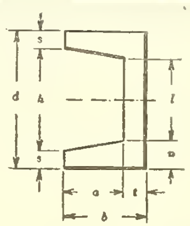
Section.	Area of section, A	Distance from neutral axis to extreme fiber, y	Moment of inertia, I	Section modulus, $S=\frac{I}{y}$	Radius of gyration, $K=\sqrt{\frac{I}{A}}$
	$0.7854(b_1h_1-b_2h_2)$ $=1.5708(b_1+h_1-2t)t$	c	$0.0491(b_1h_1^3-b_2h_2^3)$	$0.0982\left(\frac{b_1h_1^3-b_2h_2^3}{h_1}\right)$	$R=\sqrt{\frac{I}{A}}$
	$dt+2a(s+n)$	$\frac{d}{2}$	$\frac{1}{12}\left[bd^3-\frac{1}{4g}(h^4-l^4)\right]$ in which g=slope of flange$=\frac{h-l}{b-t}=\frac{1}{6}$ for standard I-beams.	$\frac{1}{6d}\left[bd^3-\frac{1}{4g}(h^4-l^4)\right]$	$\sqrt{\frac{\frac{1}{12}\left[bd^3-\frac{1}{4g}(h^4-l^4)\right]}{dt+2a(s+n)}}$
	$dt+2a(s+n)$	$\frac{b}{2}$	$\frac{1}{12}\left[b^3(d-h)+lt^3+\frac{g}{4}(b^4-t^4)\right]$ in which g=slope of flange (see above).	$\frac{1}{6b}\left[b^3(d-h)+lt^3+\frac{g}{4}(b^4-t^4)\right]$	$\sqrt{\frac{I}{A}}$
	$dt+a(s+n)$	$\frac{d}{2}$	$\frac{1}{12}\left[bd^3-\frac{1}{8g}(h^4-l^4)\right]$ in which g=slope of flange$=\frac{h-l}{2(b-t)}=\frac{1}{6}$ for standard channels.	$\frac{1}{6d}\left[bd^3-\frac{1}{8g}(h^4-l^4)\right]$	$\sqrt{\frac{\frac{1}{12}\left[bd^3-\frac{1}{8g}(h^4-l^4)\right]}{dt+a(s+n)}}$

TABLE 1.—*Standard sections*—Continued

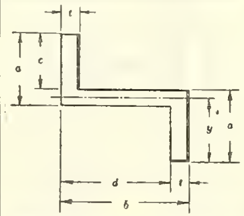
Section.	Area of section, A	Distance from neutral axis to extreme fiber, y	Moment of inertia, I	Section modulus, $S=\frac{I}{y}$	Radius of gyration, $K=\sqrt{\frac{I}{A}}$
	$t[b+2(a-t)]$	$\frac{2a-t}{2}$	$\frac{b(a+c)^3-2c^3d-6a^2cd}{12}$	$\frac{b(a+c)^3-2c^3d-6a^2cd}{6(2a-t)}$	$\sqrt{\frac{b(a+c)^3-2c^3d-6a^2cd}{12t[b+2(a-t)]}}$

TABLE 2.—*Standard sections.*

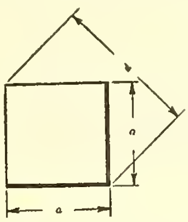
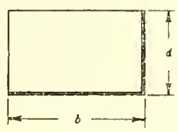
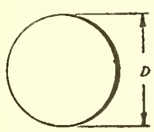
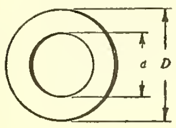
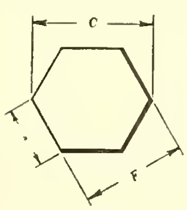
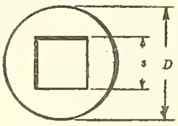
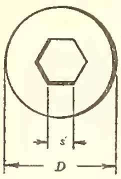
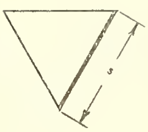
Section.	Polar moment of inertia, I_P	Polar section modulus, S_P
	$\frac{a^4}{6}=0.1667\ a^4$	$\frac{2}{9}\ a^3=0.22\ a^3=0.08\ d^3$
	$\frac{bd\ (b^2+d^2)}{12}$	$\frac{2}{9}\ bd^2=0.22\ bd^2$ (*d* is the shorter side)
	$\frac{\pi\ D^4}{32}=0.098\ D^4$	$\frac{\pi\ D^3}{32}=0.196\ D^3$
	$\frac{\pi}{32}\ (D^4-d^4)=0.098\ (D^4-d^4)$	$\frac{\pi}{16}\left(\frac{D^4-d^4}{D}\right)=0.196\ \left(\frac{D^4-d^4}{D}\right)$
	$\frac{5\sqrt{3}}{8}\ s^4=1.0825\ s^4$	$0.92\ s^3=0.115\ C^3=0.178\ F^3$

TABLE 2.—*Standard sections*—Continued.

Section.	Polar moment of inertia, I_P	Polar section modulus, S_P
	$\frac{\pi D^4}{32}-\frac{s^4}{6}=0.098\ D^4-0.167\ s^4$	$\frac{\pi D^3}{16}-\frac{s^4}{3D}=0.196\ D^3-0.333\ \frac{s^4}{D}$
	$\frac{\pi D^4}{32}-\frac{5\sqrt{3}}{8}\ s^4=0.098\ D^4-1.0825\ s^4$	$\frac{\pi D^3}{16}-\frac{5\sqrt{3}\ s^4}{4\ D}=0.196\ D^3-2.165\ \frac{s^4}{D}$
	$\frac{\sqrt{3}\ s^4}{48}=0.036\ s^4$	$\frac{s^3}{20}=0.05\ s^3$

II.—IRREGULAR SECTIONS—MOMENT OF INERTIA DETERMINATION.

The moment of inertia of irregular sections can readily be determined by the following method:

Draw the irregular section full size with the neutral axis passing through the center of gravity. Using the neutral axis as a base, cube all the ordinates. With a planimeter find the area inclosed by the curve drawn through these points. One-third of this area is equal to the moment of inertia of the section.

When it is inconvenient to draw the section full size, a convenient scale is used and later allowed for in the computation.

As an example, take a 5 by 3 inch I-beam section (fig. 1). Since the section is symmetrical about the neutral axis, only a quarter need be drawn.

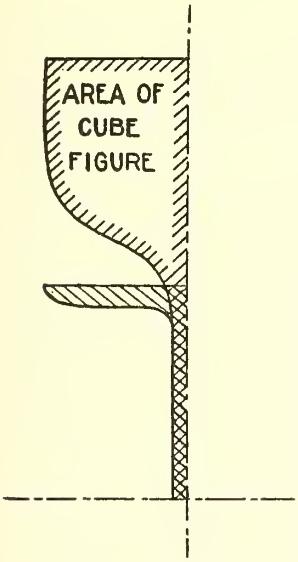


Fig. 1.—I-beam section.

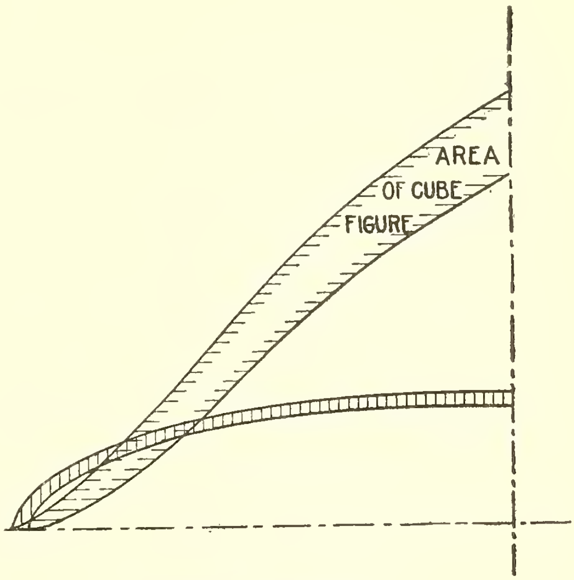


Fig. 2.—Tube section.

The ordinates from the neutral axis are cubed and for convenience are set out one-third of the cubed length (scale factor = 3); the cubed figure is distinguished by the open shading.

The area of this figure is found to be 3.4 square inches; then the total cubed area is 4 by 3.4 or 13.6 square inches. The moment of inertia then

$$=\frac{\text{scale factor}\times 13.6}{3}=13.6\text{ in.}^4$$

As a further example, take an oval-shaped tube (fig. 2). This is drawn four times actual size (scale factor $=\frac{1}{4}$), but again, because of symmetry, only one-quarter is drawn.

The ordinates are cubed and are drawn to a scale 100 times full size (scale factor $=\frac{1}{100}$). The total cubed area $=4\times 5.59$ or 22.36 square inches. The moment of inertia then

$$=\frac{22.36}{3\times 100\times 4}=.01863\text{ in.}^4$$

When the section is not symmetrical, the cubed figure is constructed in two separate portions—the one above the neutral axis and the other below it.

III.—STRUTS AND TUBE SECTIONS—AREAS AND RADII OF GYRATION.

STEEL TUBING.

$$A=\frac{\pi}{4}(D^2-d^2)$$

$$I=\frac{\pi}{64}(D^4-d^4)$$

$$K=\sqrt{\frac{I}{A}}$$

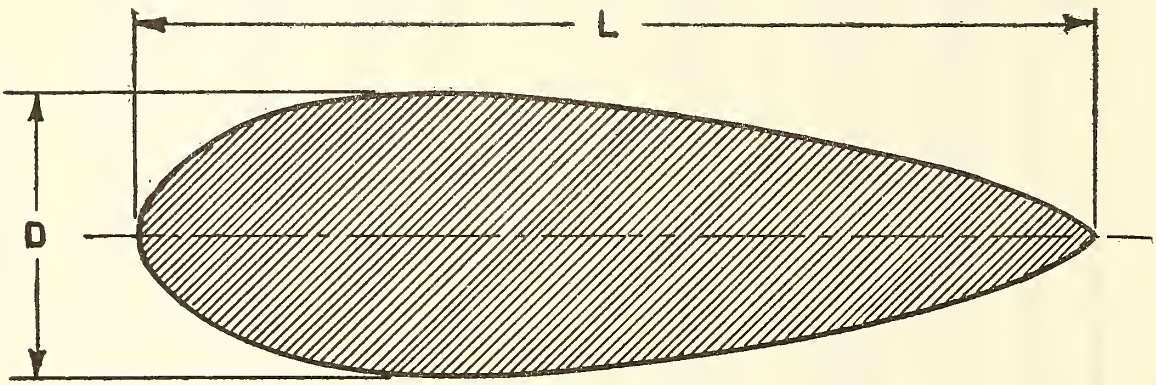
$$S=\frac{I}{y}$$

$$\frac{\text{Lbs.}}{\text{Ft.}}=3.4\times\text{area.}$$

Outside diameter, inches.	B. W. Gauge.	Area, square inches, A.	Moment of inertia, I.	Radius of gyration, K.	Section modulus, S.	Pounds per foot length.	Outside diameter, inches.	B. W. gauge.	Area, square inches, A.	Moment of inertia, I.	Radius of gyration, K.	Section modulus, S.	Pounds per foot length.
1/4	22	0.0196	0.000122	0.079	0.00098	0.0664	1 1/4	14	0.304	0.0521	0.414	0.0833	1.035
1/4	20	.0237	.000140	.077	.00112	.0806	1 3/8	22	.119	.0269	.475	.0392	.405
1/4	18	.0310	.000165	.073	.00132	.105	1 3/8	20	.147	.0331	.474	.0482	.501
1/4	17	.0350	.000176	.071	.00141	.119	1 3/8	18	.204	.0450	.470	.0654	.693
1/4	16	.0378	.000182	.069	.00146	.129	1 3/8	17	.240	.0524	.467	.0762	.816
1/4	14	.0425	.000189	.067	.00151	.148	1 3/8	16	.267	.0576	.465	.0838	.874
3/8	22	.0306	.000462	.123	.00246	.104	1 3/8	14	.337	.0706	.457	.1028	1.148
3/8	20	.0374	.000547	.121	.00292	.127	1 1/2	22	.129	.0351	.521	.0468	.440
3/8	18	.0501	.000682	.117	.00364	.170	1 1/2	20	.161	.0432	.518	.0576	.548
3/8	17	.0578	.000750	.114	.00400	.197	1 1/2	18	.223	.0590	.514	.0786	.758
3/8	16	.0633	.000795	.112	.00425	.215	1 1/2	17	.263	.0685	.510	.0913	.895
3/8	14	.0761	.000877	.107	.00467	.224	1 1/2	16	.293	.0755	.507	.1007	.996
1/2	22	.0416	.00116	.167	.00464	.141	1 1/2	14	.369	.0931	.502	.1240	1.254
1/2	20	.0511	.00139	.165	.00556	.174	1 5/8	22	.140	.0448	.565	.0552	.477
1/2	18	.0695	.00179	.161	.00716	.236	1 5/8	20	.175	.0552	.561	.0679	.595
1/2	17	.0805	.00200	.158	.00800	.274	1 5/8	18	.243	.0754	.557	.0928	.826
1/2	16	.0890	.00215	.155	.00860	.303	1 5/8	17	.286	.0879	.553	.1082	.972
1/2	14	.1087	.00245	.150	.00980	.369	1 5/8	16	.319	.0970	.551	.1195	1.085
3/4	22	.0635	.00415	.256	.0111	.216	1 5/8	14	.402	.1198	.546	.1415	1.368
3/4	20	.0792	.00504	.252	.0135	.269	1 3/4	22	.153	.0561	.606	.0641	.523
3/4	18	.1079	.00666	.248	.0178	.367	1 3/4	20	.190	.0693	.604	.0793	.646
3/4	17	.1261	.00760	.245	.0203	.432	1 3/4	18	.263	.0947	.600	.1082	.895
3/4	16	.1400	.00828	.243	.0221	.476	1 3/4	17	.308	.110	.598	.1259	1.050
3/4	14	.1739	.00982	.237	.0262	.592	1 3/4	16	.345	.122	.593	.1395	1.172
7/8	22	.0745	.00669	.300	.0153	.263	1 3/4	14	.436	.151	.589	.1725	1.472
7/8	20	.0924	.00816	.297	.0186	.314	1 7/8	22	.164	.0697	.652	.0744	.557
7/8	18	.1271	.0109	.293	.0249	.432	1 7/8	20	.205	.0856	.645	.0913	.697
7/8	17	.1489	.0125	.290	.0286	.507	1 7/8	18	.282	.117	.644	.1248	.925
7/8	16	.1654	.0136	.285	.0311	.563	1 7/8	17	.332	.137	.642	.1462	1.128
7/8	14	.2065	.0164	.282	.0375	.700	1 7/8	16	.371	.152	.639	.1622	1.262
1	22	.086	.0101	.343	.0201	.293	1 7/8	14	.467	.188	.635	.2007	1.586
1	20	.106	.0124	.342	.0248	.360	2	22	.174	.084	.696	.084	.592
1	18	.147	.0166	.336	.0332	.500	2	20	.216	.104	.694	.104	.735
1	17	.172	.0191	.333	.0382	.585	2	18	.300	.143	.691	.143	1.019
1	16	.191	.0210	.331	.0420	.650	2	17	.354	.167	.685	.167	1.203
1	14	.239	.0253	.326	.0506	.810	2	16	.395	.186	.684	.186	1.347
1 1/8	22	.097	.0145	.386	.0258	.330	2	14	.500	.230	.678	.230	1.700
1 1/8	20	.120	.0178	.385	.0316	.408	2 1/8	22	.185	.101	.739	.095	.629
1 1/8	18	.166	.0241	.381	.0428	.564	2 1/8	20	.230	.126	.738	.118	.782
1 1/8	17	.195	.0278	.377	.0495	.563	2 1/8	18	.320	.172	.733	.162	1.087
1 1/8	16	.216	.0305	.375	.0542	.732	2 1/8	17	.377	.201	.730	.189	1.282
1 1/8	14	.272	.0371	.369	.0660	.922	2 1/8	16	.425	.223	.724	.210	1.444
1 1/4	22	.107	.0201	.433	.0321	.365	2 1/8	14	.533	.277	.720	.261	1.810
1 1/4	20	.134	.0247	.429	.0395	.454	2 1/4	22	.196	.121	.785	.108	.666
1 1/4	18	.185	.0333	.424	.0533	.629	2 1/4	20	.244	.149	.781	.131	.830
1 1/4	17	.217	.0386	.422	.0618	.738	2 1/4	18	.339	.205	.778	.182	1.153
1 1/4	16	.240	.0426	.420	.0681	.816	2 1/4	17	.399	.240	.776	.213	1.356

TUBING—Continued.

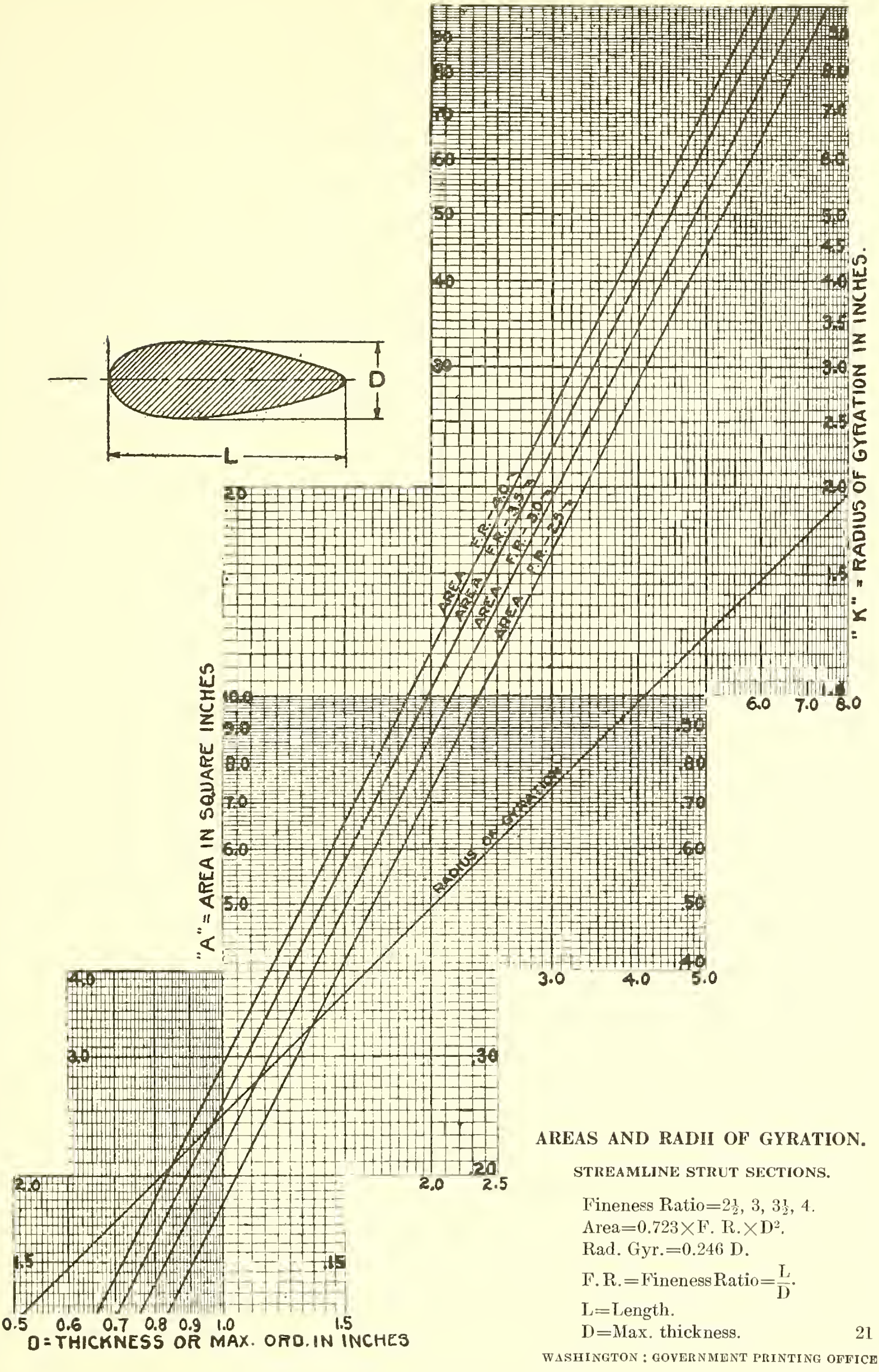
Outside diameter, inches.	B. W. gauge.	Area, square inches, A.	Moment of inertia, I.	Radius of gyration, K.	Section modulus, S.	Pounds per foot length.	Outside diameter, inches.	B. W. gauge.	Area, square inches, A.	Moment of inertia, I.	Radius of gyration, K.	Section modulus, S.	Pounds per foot length.
2¼	16	0. 446	0. 267	0. 774	0. 237	1. 515	2⅝	14	0. 663	0. 536	0. 899	0. 408	2. 254
2¼	14	. 565	. 332	. 767	. 295	1. 920	2¾	22	. 239	. 222	. 963	. 161	. 812
2⅜	22	. 206	. 142	. 830	. 120	. 701	2¾	20	. 299	. 275	. 960	. 200	1. 018
2⅜	20	. 257	. 176	. 827	. 148	. 874	2¾	18	. 416	. 382	. 958	. 278	1. 414
2⅜	18	. 358	. 242	. 822	. 204	1. 217	2¾	17	. 491	. 445	. 953	. 324	1. 669
2⅜	17	. 422	. 283	. 819	. 238	1. 434	2¾	16	. 548	. 494	. 949	. 359	1. 863
2⅜	16	. 472	. 315	. 816	. 265	1. 604	2¾	14	. 695	. 619	. 943	. 450	2. 374
2⅜	14	. 598	. 393	. 811	. 331	2. 035	2⅞	22	. 250	. 254	1. 008	. 177	. 850
2½	22	. 217	. 166	. 875	. 133	. 738	2⅞	20	. 312	. 315	1. 005	. 219	1. 062
2½	20	. 271	. 206	. 872	. 165	. 922	2⅞	18	. 435	. 434	. 999	. 302	1. 479
2½	18	. 377	. 283	. 867	. 226	1. 282	2⅞	17	. 513	. 508	. 996	. 354	1. 744
2½	17	. 445	. 332	. 864	. 265	1. 513	2⅞	16	. 574	. 566	. 993	. 394	1. 953
2½	16	. 497	. 369	. 862	. 295	1. 689	2⅞	14	. 728	. 710	. 987	. 494	2. 480
2½	14	. 630	. 461	. 855	. 368	2. 141	3	22	. 261	. 289	1. 053	. 193	. 887
2⅝	22	. 228	. 193	. 920	. 147	. 775	3	20	. 326	. 359	1. 049	. 239	1. 108
2⅝	20	. 285	. 239	. 916	. 182	. 970	3	18	. 454	. 495	1. 044	. 330	1. 545
2⅝	18	. 397	. 329	. 910	. 251	1. 350	3	17	. 536	. 581	1. 041	. 387	1. 823
2⅝	17	. 468	. 385	. 907	. 294	1. 592	3	16	. 599	. 646	1. 038	. 431	2. 037
2⅝	16	. 523	. 428	. 904	. 326	1. 780	3	14	. 761	. 810	1. 031	. 540	2. 588



ELEMENTS OF STREAMLINE STRUT SECTION.

Distance from leading edge, Decimal part of L.	Offsets from center line, Decimal part of D.	Distance from leading edge, Decimal part of L.	Offsets from center line, Decimal part of D.	Distance from leading edge, Decimal part of L.	Offsets from center line, Decimal part of D.	Distance from leading edge, Decimal part of L.	Offsets from center line, Decimal part of D.
0. 000	0. 0000	0. 100	0. 3637	0. 400	0. 4912	0. 900	0. 1887
. 005	. 0638	. 150	. 4350	. 500	. 4675	. 950	. 1237
. 010	. 1000	. 200	. 4763	. 600	. 4225	. 980	. 0762
. 020	. 1437	. 250	. 4900	. 700	. 3612	. 990	. 0500
. 040	. 2150	. 300	. 5000	. 800	. 2850	1. 000	. 0000
. 070	. 3000	. 350	. 4950				

Ref.: Best strut section—N. P. L. R. M. 183.



AREAS AND RADII OF GYRATION.

STREAMLINE STRUT SECTIONS.

Fineness Ratio=$2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4.
Area=$0.723 \times$ F. R. $\times D^2$.
Rad. Gyr.=0.246 D.

F. R.=Fineness Ratio=$\frac{L}{D}$.
L=Length.
D=Max. thickness.

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AIRCRAFT DESIGN DATA. NOTE NO. 4.

ULTIMATE STRENGTH OF STRUCTURAL MEMBERS.

I. STRUTS.

EULER'S THEORY FOR LONG COLUMNS.

Use Euler's formula where $\frac{L}{K}=100$ or greater

$$p=\frac{M\ \pi^2\ E}{\left\{\frac{L}{K}\right\}^2} \text{ or } P=\frac{M\ \pi^2\ E\ I}{L^2}$$

p = allowable crippling stress per square inch for variable $\frac{L}{K}$

P = total crippling load or p A

M = 1 for round or pinned ends.

$M=\frac{16}{9}$ for one pinned and one fixed end.

M = 4 for fixed ends.

L = length of column in inches.

I = moment of inertia of the section.

$K=\sqrt{\frac{I}{A}}$ = least radius of gyration.

E = modulus of elasticity.

J. B. JOHNSON'S FORMULA FOR SHORT COLUMNS

$$p=C\left[1-\left(\frac{C}{4M\ \pi^2\ E}\left\{\frac{L}{K}\right\}^2\right)\right]$$

Use Johnson's formula where

$$\frac{C}{4M\ \pi^2\ E}\left\{\frac{L}{K}\right\}^2<\frac{1}{2}$$

C = compression strength of material per square inch.

NOTE.—When plotting a curve of allowable ultimate strength $\left(\frac{P}{A}=p \text{ against } \frac{L}{K}\right)$ use both Johnson's and Euler's formulæ. Two curves will be obtained which can be joined and made into one mean curve.

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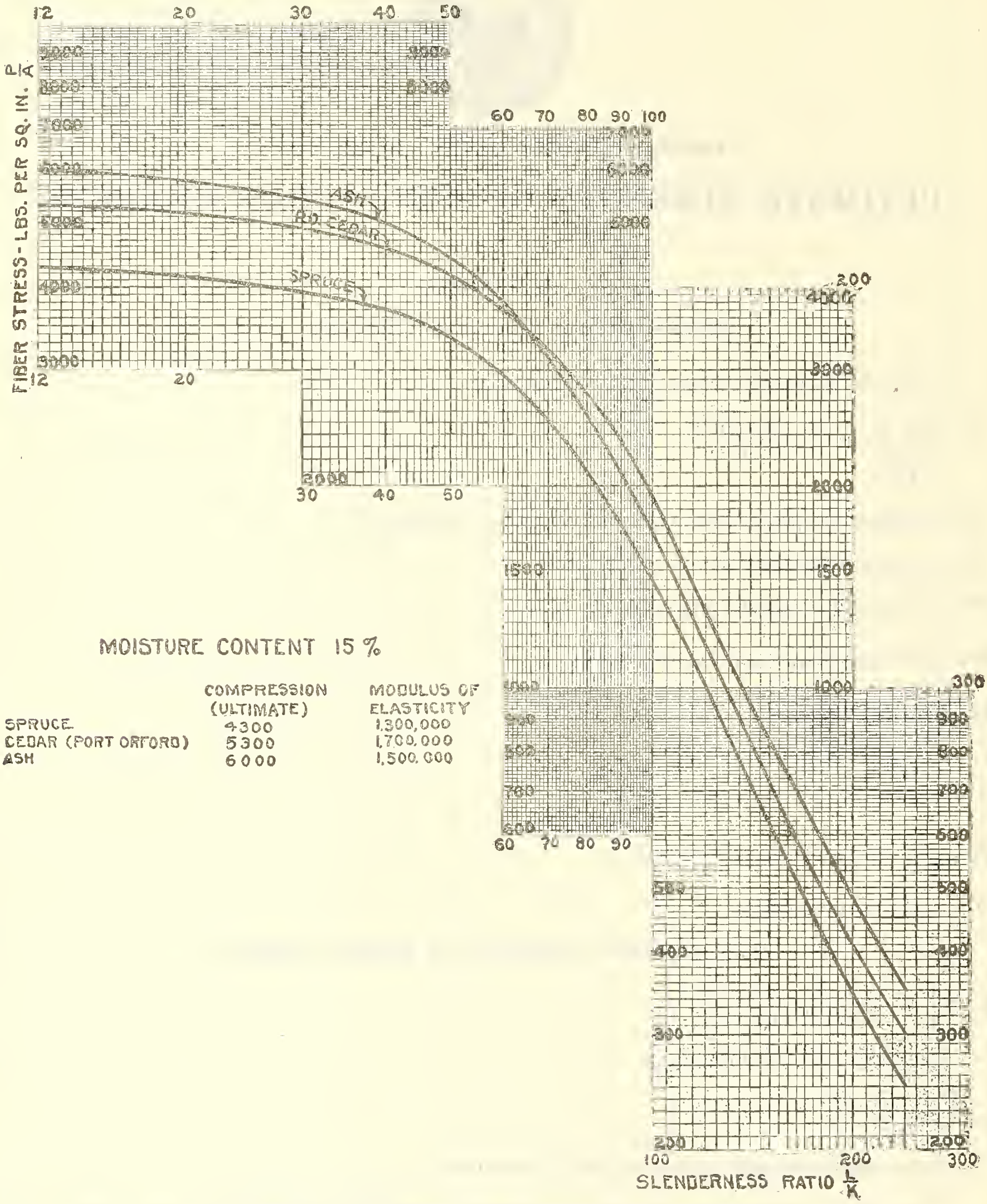
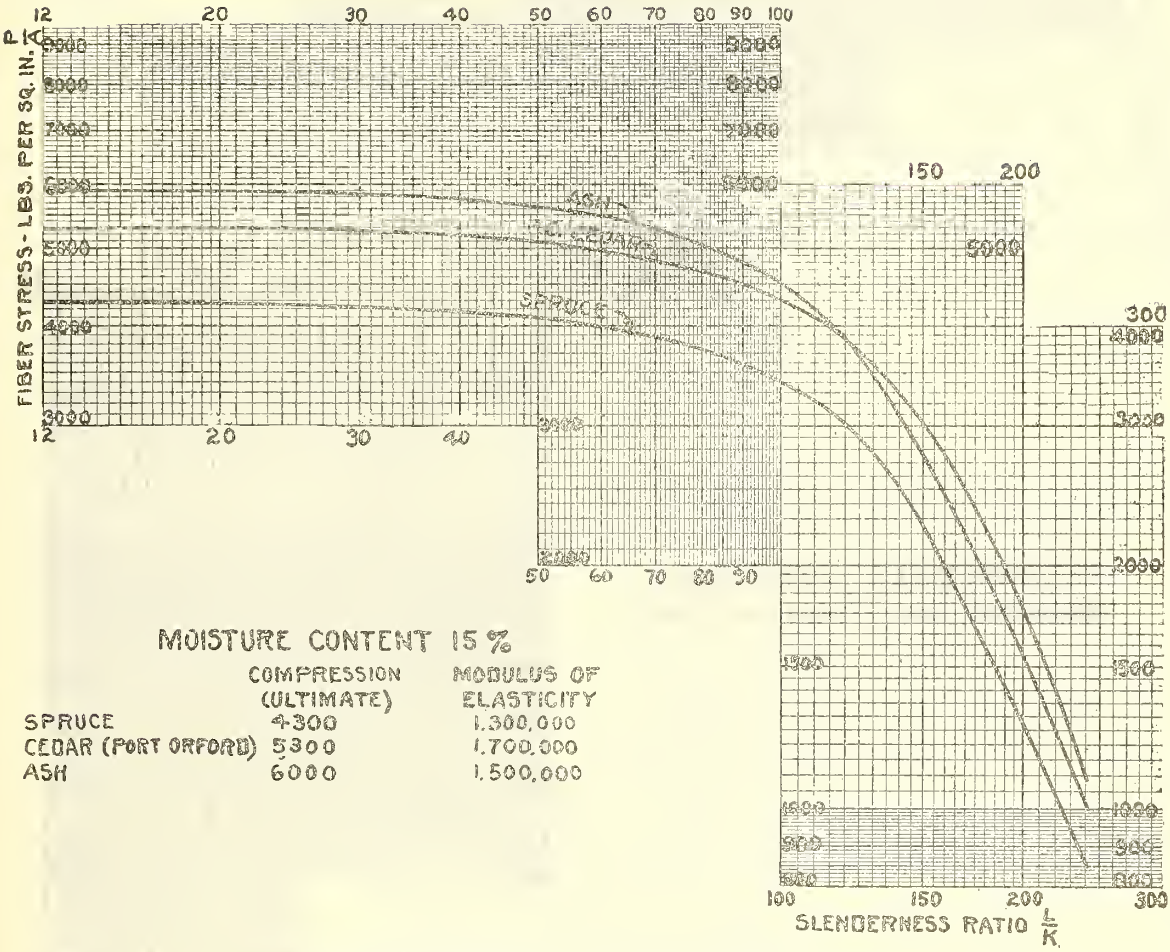


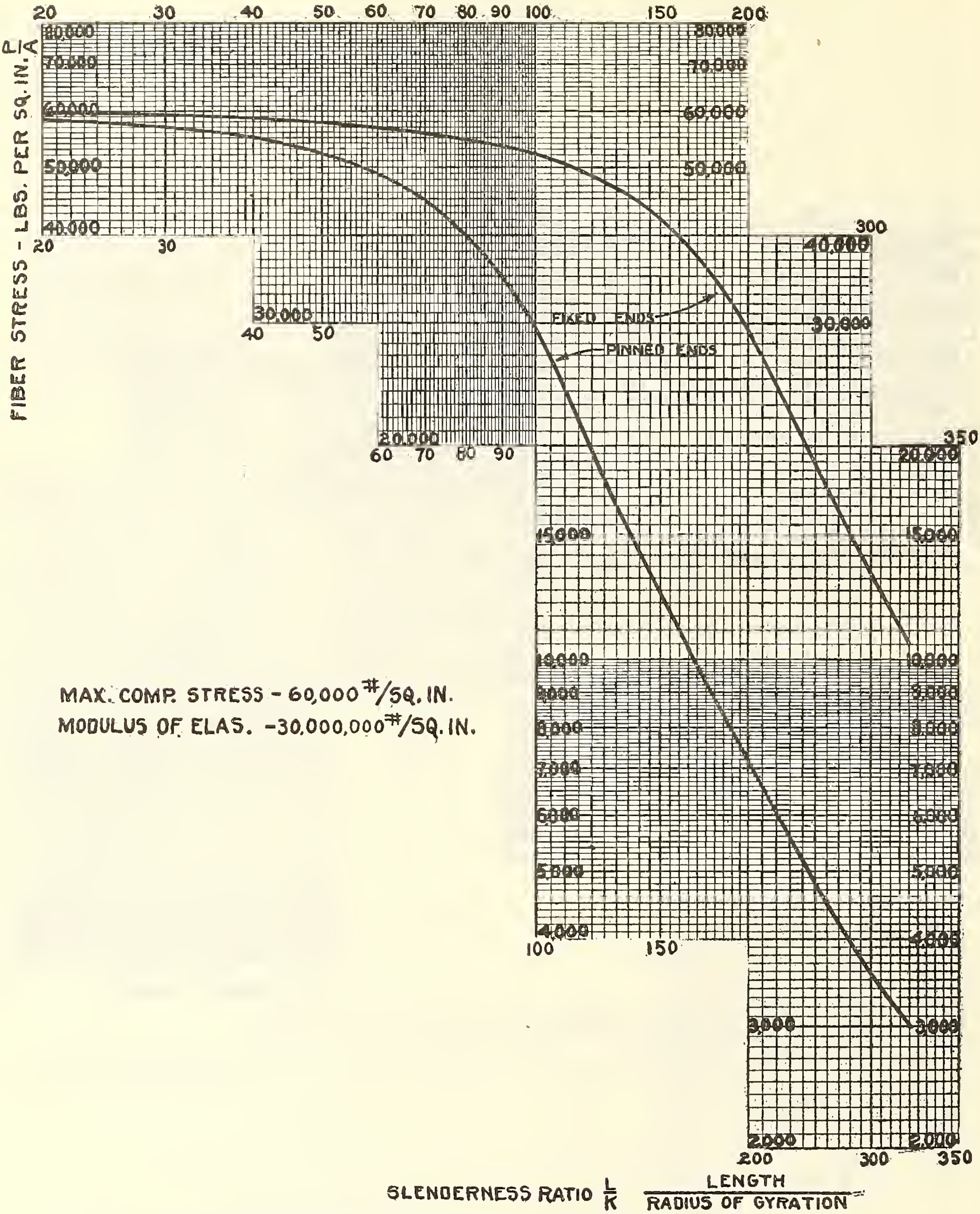
ULTIMATE STRENGTH FOR PINNED END STRUTS.

Curves calculated from Euler's and Johnson's formulæ, slightly modified in accordance with experimental tests.



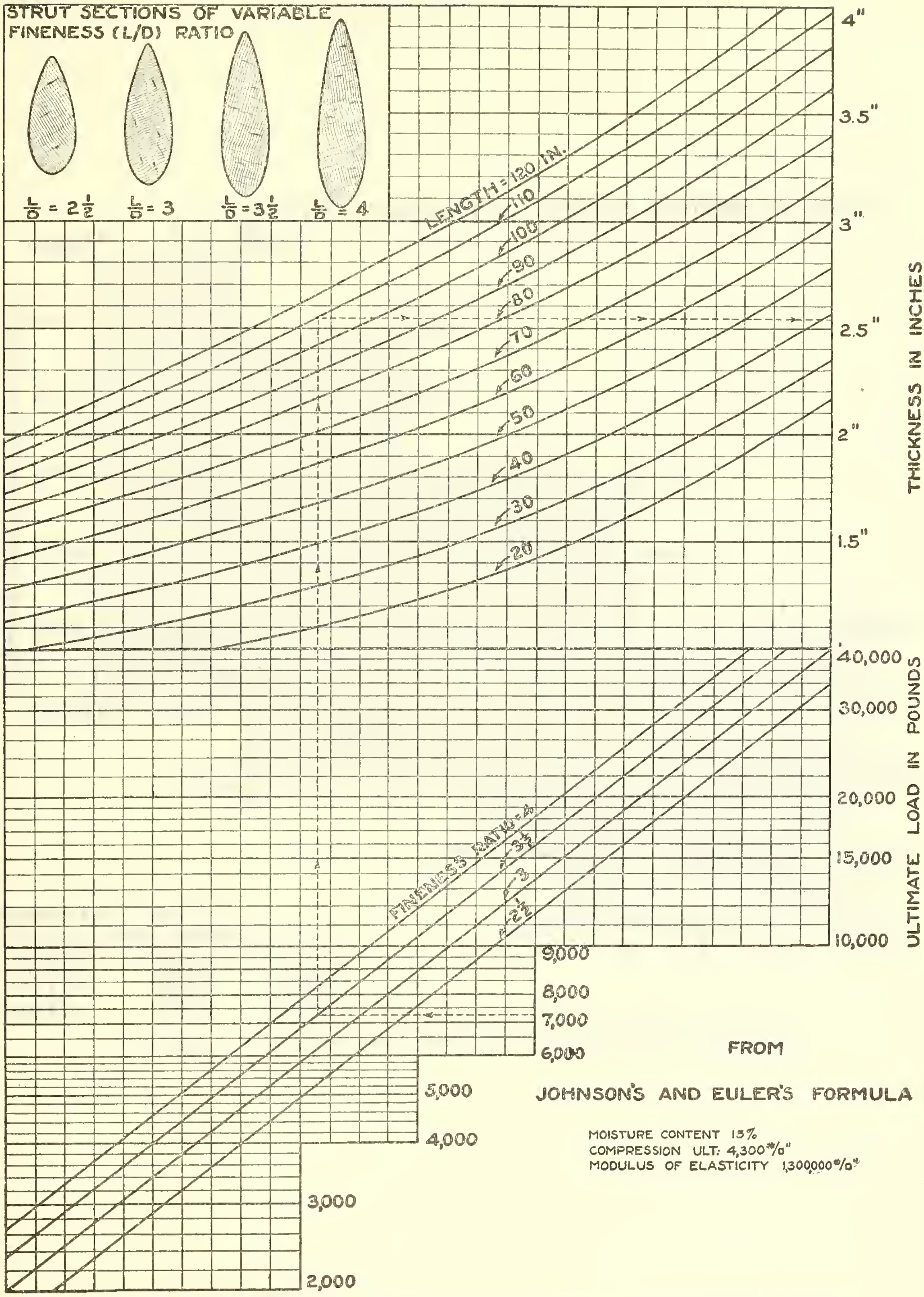
ULTIMATE STRENGTH FOR FIXED END STRUTS.

Curves calculated from Euler's and Johnson's formulæ, slightly modified in accordance with experimental tests.

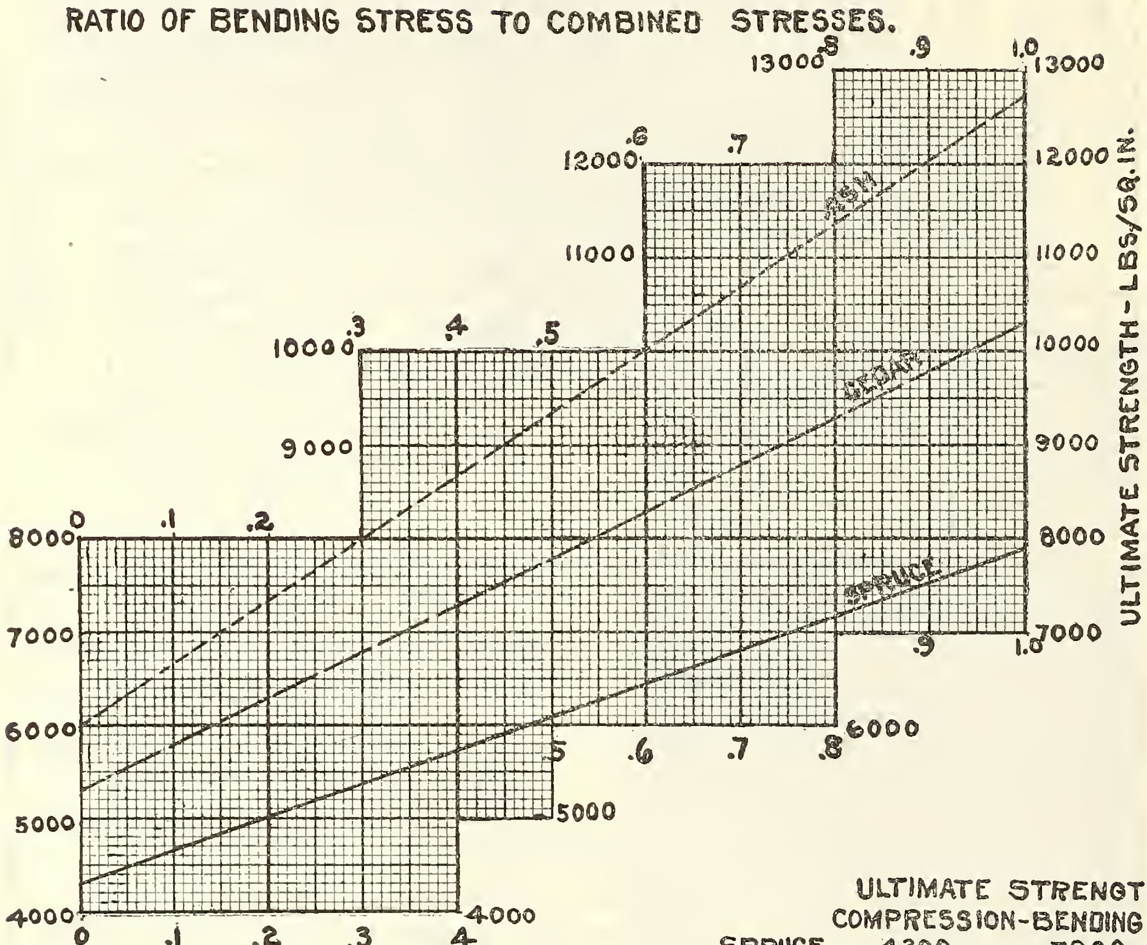


ULTIMATE STRENGTH OF CENTRALLY LOADED STEEL TUBES.

(Euler's and Johnson's formulæ.)



ULTIMATE STRENGTH OF STREAMLINE PINNED END SOLID SPRUCE STRUTS.



	ULTIMATE STRENGTH COMPRESSION	BENDING
SPRUCE	4300	7900
ASH	6000	12700
CEDAR	5300	10300

ULTIMATE STRENGTH FOR COMBINED COMPRESSION AND BENDING.

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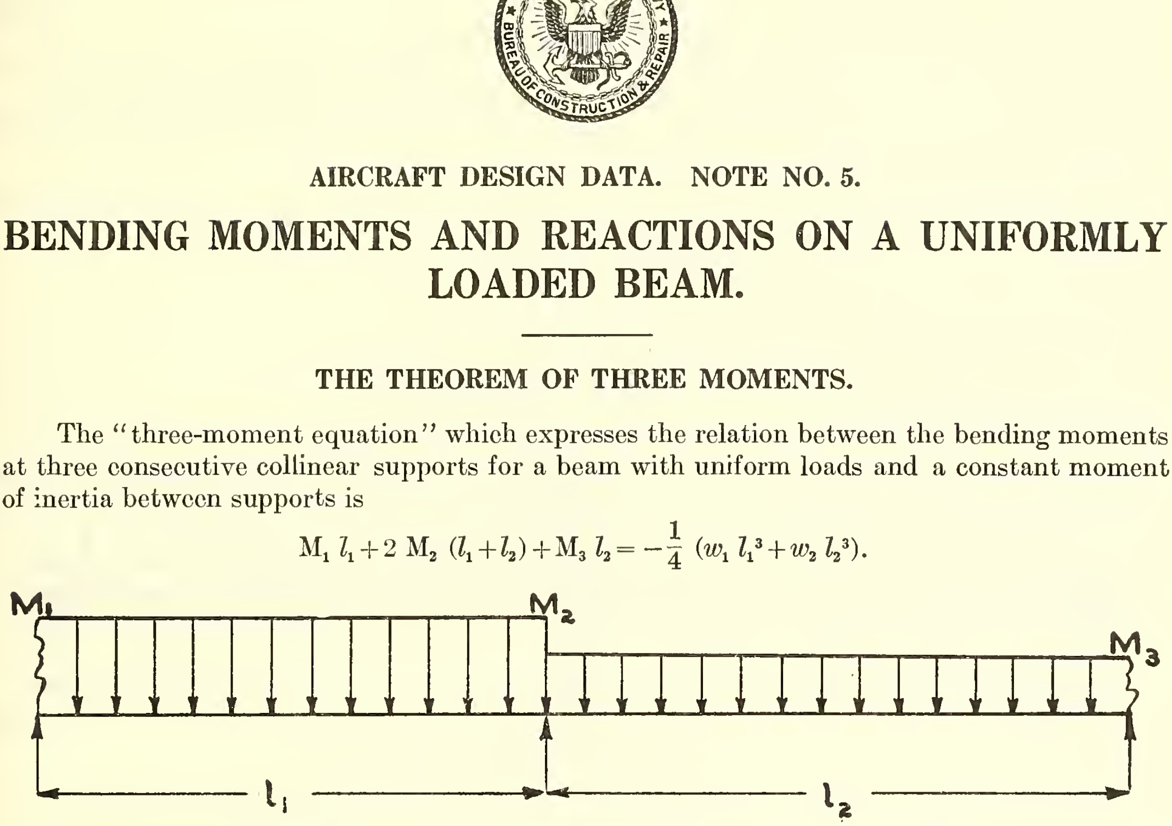
AIRCRAFT DESIGN DATA. NOTE NO. 5.

BENDING MOMENTS AND REACTIONS ON A UNIFORMLY LOADED BEAM.

THE THEOREM OF THREE MOMENTS.

The "three-moment equation" which expresses the relation between the bending moments at three consecutive collinear supports for a beam with uniform loads and a constant moment of inertia between supports is

$$M_1\, l_1 + 2\, M_2\, (l_1 + l_2) + M_3\, l_2 = -\frac{1}{4}\, (w_1\, l_1^3 + w_2\, l_2^3).$$



M_1, M_2, and M_3 are the moments at the supports; w_1 and w_2 are the values of the uniform loads for each unit of the span l_1 and l_2, respectively.

If w_1 and w_2 act upward, they are positive forces and the right-hand member of the equation becomes $+\frac{1}{4}\, (w_1\, l_1^3 + w_2\, l_2^3)$.

Positive bending moments give compression in the top fiber; negative bending moments give compression in the bottom fiber.

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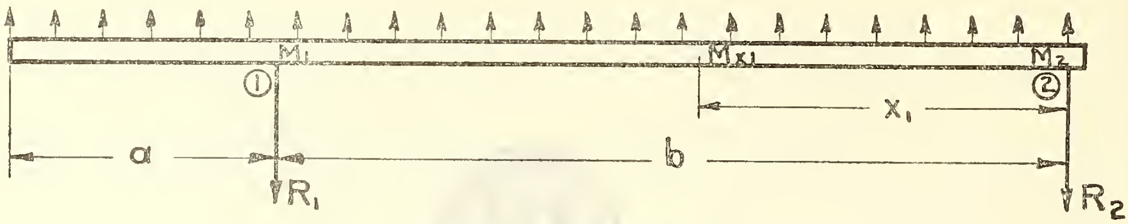
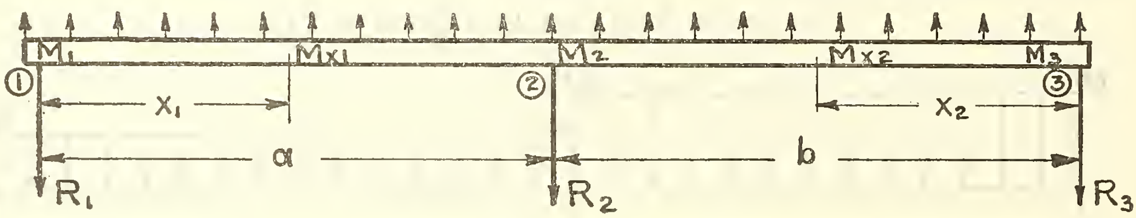
BENDING MOMENTS AND REACTIONS ON A UNIFORMLY LOADED BEAM SUPPORTED AT TWO POINTS AND ONE END OVERHANGING.



Running load assumed=unity.

Name.	Symbol.	Formulæ.
Joint moment	M_1	$\frac{a^2}{2}$
Reactions	R_1	$\frac{-(a+b)^2}{2b}$
	R_2	$\frac{-(b^2-a^2)}{2b}$.
Maximum moment in span b	M_{x1}	$\frac{-R_2^2}{2}$.
Abscissa of maximum moment	x_1	$-R_2$ measured from (2).

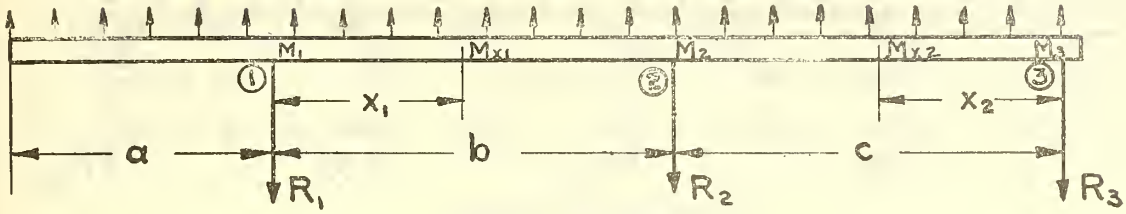
BENDING MOMENTS AND REACTIONS ON A UNIFORMLY LOADED BEAM SUPPORTED AT THREE POINTS.



Running load assumed=unity.

Name.	Symbol.	Formulæ.
Joint moment	M_2	$\frac{(a^3+b^3)}{8(a+b)}$.
Reactions	R_1	$-\left(\frac{a}{2}-\frac{M_2}{a}\right)$.
	R_2	$-(a+b+R_1+R_3)$.
	R_3	$-\left(\frac{b}{2}-\frac{M_2}{b}\right)$.
Maximum moments between joints	M_{x1}	$\frac{-R_1^2}{2}$.
	M_{x1}	$\frac{-R_3^2}{2}$.
Abscissae of maximum moments	x_1	$-R_1$ measured from (1).
	x_2	$-R_3$ measured from (3).

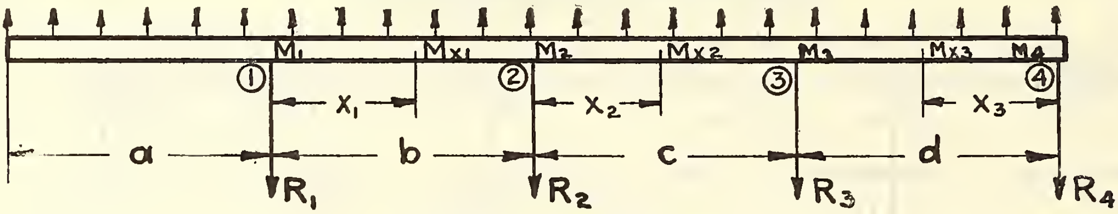
BENDING MOMENTS AND REACTIONS ON A UNIFORMLY LOADED BEAM SUPPORTED AT THREE POINTS AND ONE END OVERHANGING.



Running load assumed=unity.

Name.	Symbol.	Formulæ.
Joint moments	M_1	$\frac{a^2}{2}$.
	M_2	$\frac{(b^3+c^3-4\,b\,M_1)}{8(b+c)}$.
Reactions	R_1	$\frac{-[\frac{(b+a)^2}{2}-M_2]}{b}$.
	R_2	$-(a+b+c+R_1+R_3)$.
	R_3	$-\left(\frac{c}{2}-\frac{M_2}{c}\right)$.
Maximum moments between joints	M_{x1}	$M_1-\frac{(R_1+a)^2}{2}$.
	M_{x2}	$-\frac{R_3^2}{2}$
Abscissae of maximum moments	x_1	$-(R_1+a)$ measured from (1).
	x_2	$-R_3$ measured from (3).

BENDING MOMENTS AND REACTIONS ON A UNIFORMLY LOADED BEAM SUPPORTED AT FOUR POINTS AND ONE END OVERHANGING.



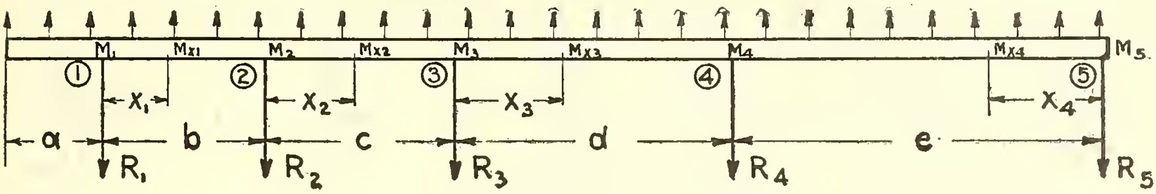
Running load assumed=unity.

In the table $l=\frac{(b^3+c^3)}{4}-\frac{ba^2}{2}$; $m=\frac{c^3+d^3}{4}$

$h=2(b+c)$; $j=2(c+d)$.

Name.	Symbol.	Formulae.
Joint moments	M_1	$\frac{a^2}{2}$
	M_2	$\frac{(jl-cm)}{(jh-c^2)}$
	M_3	$\frac{(m-cM_2)}{j}$
Reactions	R_1	$\frac{-\frac{(a+b)^2}{2}-M_2]}{b}$
	R_2	$\frac{-\frac{(a+b+c)^2}{2}+R_1(b+c)-M_3}{c}$
	R_3	$\frac{-[\frac{(c+d)^2}{2}+R_4(c+d)-M_2}{c}$
	R_4	$\frac{-\frac{d^2}{2}-M_3}{d}$
Maximum bending moments between joints.	M_{x1}	$M_1-\frac{(R_1+a)^2}{2}$
	M_{x2}	$M_2-\frac{(x_2^2)}{2}$
	M_{x3}	$-\frac{R_4^2}{2}$
Abscissae of maximum bending moments.	x_1	$-(R_1+a)$ measured from (1).
	x_2	$-(R_1+R_2+a+b)$ measured from (2).
	x_3	$-R_4$ measured from (4).

BENDING MOMENTS AND REACTIONS ON A UNIFORMLY LOADED BEAM SUPPORTED AT FIVE POINTS AND ONE END OVERHANGING.



Running load assumed=unity.

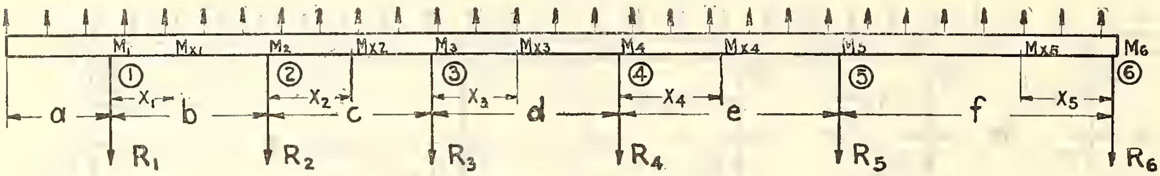
$l=\frac{b^3+c^3}{4}-\frac{a^2b}{2}$ $\qquad h=2(b+c)$

$m=\frac{c^3+d^3}{4}$ $\qquad j=2(c+d)$

$n=\frac{d^3+e^3}{4}$ $\qquad k=2(d+e)$

Name.	Symbol.	Formulae.
Bending moments	M_1	$\frac{a^2}{2}$
	M_2	$\frac{(d^2-jk)(jl-cm)+dc(dm-jn)}{(d^2-jk)(jh-c^2)+c^2d^2}$
	M_3	$\frac{(l-hM_2)}{c}$
	M_4	$\frac{n-dM_3}{k}$
Reactions	R_1	$-\frac{\frac{(a+b)^2}{2}-M_2}{b}$
	R_2	$-\frac{\frac{(a+b+c)^2}{2}+R_1(b+c)-M_3}{c}$
	R_3	$-\frac{\frac{(a+b+c+d)^2}{2}+R_1(b+c+d)+R_2(c+d)-M_4}{d}$
	R_4	$-\frac{\frac{a+b+c+d+e)^2}{2}+R_1(b+c+d+e)+R_2(c+d+e)+(R_3(d+e)}{e}$
	R_5	$-\frac{\frac{e^2}{2}-M_4}{e}$
Maximum bending moments between joints.	M_{x1}	$M_1-\frac{x_1^2}{2}$
	M_{x2}	$M_2-\frac{x_2^2}{2}$
	M_{x3}	$M_3-\frac{x_3^2}{2}$
	M_{x4}	$-\frac{R_5^2}{2}$
Abscissae of maximum bending moments.	x_1	$-(R_1+a)$ measured from (1)
	x_2	$-(R_1+R_2+a+b)$ measured from (2)
	x_3	$-(R_1+R_2+R_3+a+b+c)$ measured from (3)
	x_4	$-R_5$ measured from (5)

BENDING MOMENTS AND REACTIONS ON A UNIFORMLY LOADED BEAM SUPPORTED AT SIX POINTS AND ONE END OVERHANGING.



Running load assumed=unity.

$l=\frac{b^3-c^3}{4}-\frac{a^2b}{2}$ $\qquad g=2(b+c)$

$m=\frac{c^3+d^3}{4}$ $\qquad h=2(c+d)$

$n=\frac{d^3+e^3}{4}$ $\qquad j=2(d+e)$

$p=\frac{e^3+f^3}{4}$ $\qquad k=2(e+f)$

Name.	Symbol.	Formulae.
Bending moments	M_1	$\frac{a^2}{2}$
	M_2	$\frac{l-cM_3}{g}$
	M_3	$\frac{n-jM_4-eM_5}{d}$
	M_4	$\frac{kd(cl-mg)+(c^2-gh)(pe-kn)}{(gh-c^2)(kj-e^2)-kd^2g}$
	M_5	$\frac{P-eM_4}{k}$
Reactions	R_1	$-\frac{\frac{(a+b)^2}{2}-M_2}{b}$
	R_2	$-\frac{\frac{(a+b+c)^2}{2}+R_1(b+c)-M_3}{c}$
	R_3	$-\frac{\frac{(a+b+c+d)^2}{2}+R_1(b+c+d)+R_2(c+d)-M_4}{d}$
	R_4	$-\frac{\frac{(a+b+c+d+e)^2}{2}+R_1(b+c+d+e)+R_2(c+d+e)+R_3(d+e)-M_5}{e}$
	R_5	$-\frac{\frac{(e+f)^2}{2}+R_6(e+f)-M_4}{e}$
	R_6	$-\frac{\frac{f^2}{2}-M_5}{f}$
Maximum bending moments between joints.	M_{x1}	$M_1-\frac{x_1^2}{2}$
	M_{x2}	$M_2-\frac{x_2^2}{2}$
	M_{x3}	$M_3-\frac{x_3^2}{2}$
	M_{x4}	$M_4-\frac{x_4^2}{2}$
	M_{x5}	$M_5-\frac{x_5^2}{2}$
Abscissae of maximum bending moments.	x_1	$-(R_1+a)$ measured from (1)
	x_2	$-(R_1+R_2+a+b)$ measured from (2)
	x_3	$-(R_1+R_2+R_3+a+b+c)$ measured from (3)
	x_4	$-(R_1+R_2+R_3+R_4+a+b+c+d)$ measured from (4)
	x_5	$-R_6$ measured from (6)

WASHINGTON : GOVERNMENT PRINTING OFFICE : 1919

AIRCRAFT DESIGN DATA. NOTE NO. 6.

PERFORMANCE.

I.—PERFORMANCE PREDICTION FROM AN AVERAGE FROM BRITISH TRIALS.

APPROXIMATE PREDICTION OF PERFORMANCE.

Given: Weight, Area, Horsepower.

The curves in figures 1, 2, 3, and 4 are deduced from the reduction of British trials and represent average performance of all machines tested at Martlesham Heath during the last three years.

These curves are general ones (separation into types, such as tractors, twin engine, etc., does not lead to appreciably greater accuracy at the present time since the differences found between types are not great compared with the difference between designs of the same type), and are to be regarded only as a first approximation. Any new designs will differ from average design and these curves are only of value as a guide in initial layout.

The curves of figures 1, 2, and 3 may be used directly when the loading is 7 pounds per square foot; for other loadings use the formulæ on the curve.

Figure 4 gives an estimation of the diameter of the propeller to be used and is based on the following formula:

$$\text{H. P.} = \frac{1.73 K_y}{10^9 \text{ A. R.}} D^4 N^2 V_1 \left\{\frac{N\ D}{V_1}\right\}^{0.1}$$

K_y = absolute lift coefficient, — taken equal to 0.25.

$$\text{A. R.} = \frac{\text{Tip Radius}}{\text{Max. Blade Width}}, \text{ — taken equal to 5.75.}$$

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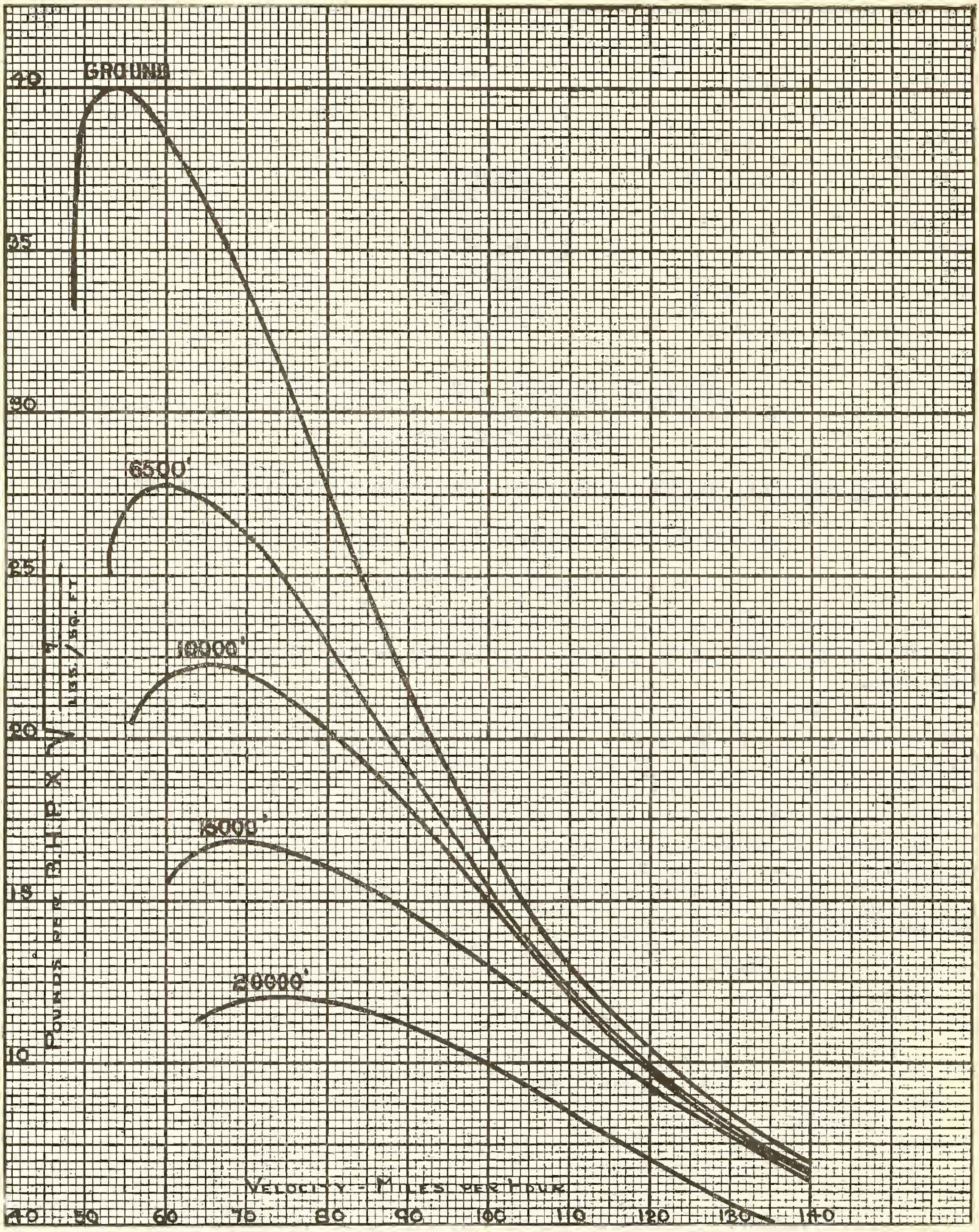


Fig. 1.—CURVES FOR APPROXIMATE PREDICTION OF AIRPLANE PERFORMANCE.

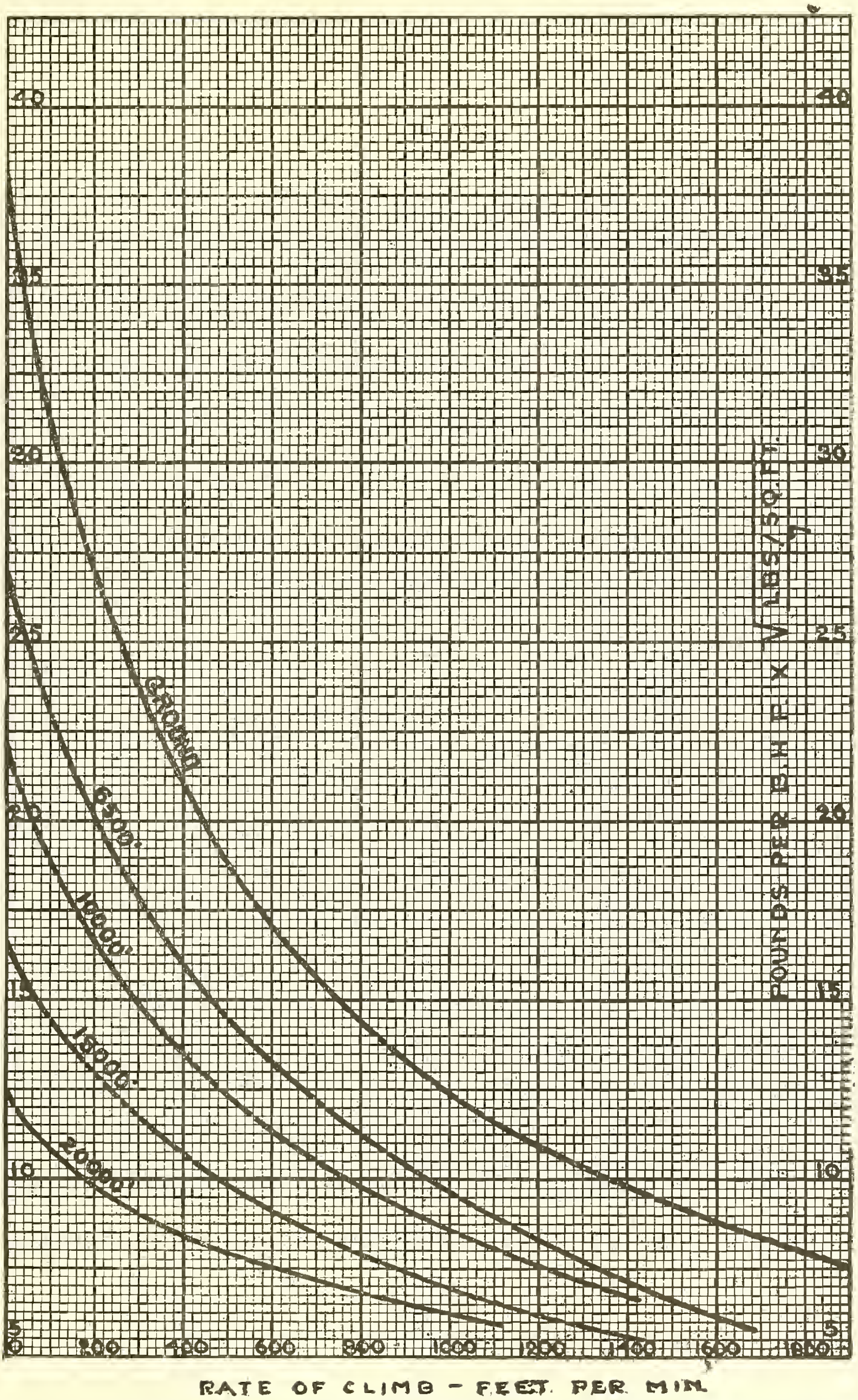


Fig. 2.—CURVES FOR APPROXIMATE PREDICTION OF AIRPLANE PERFORMANCE.

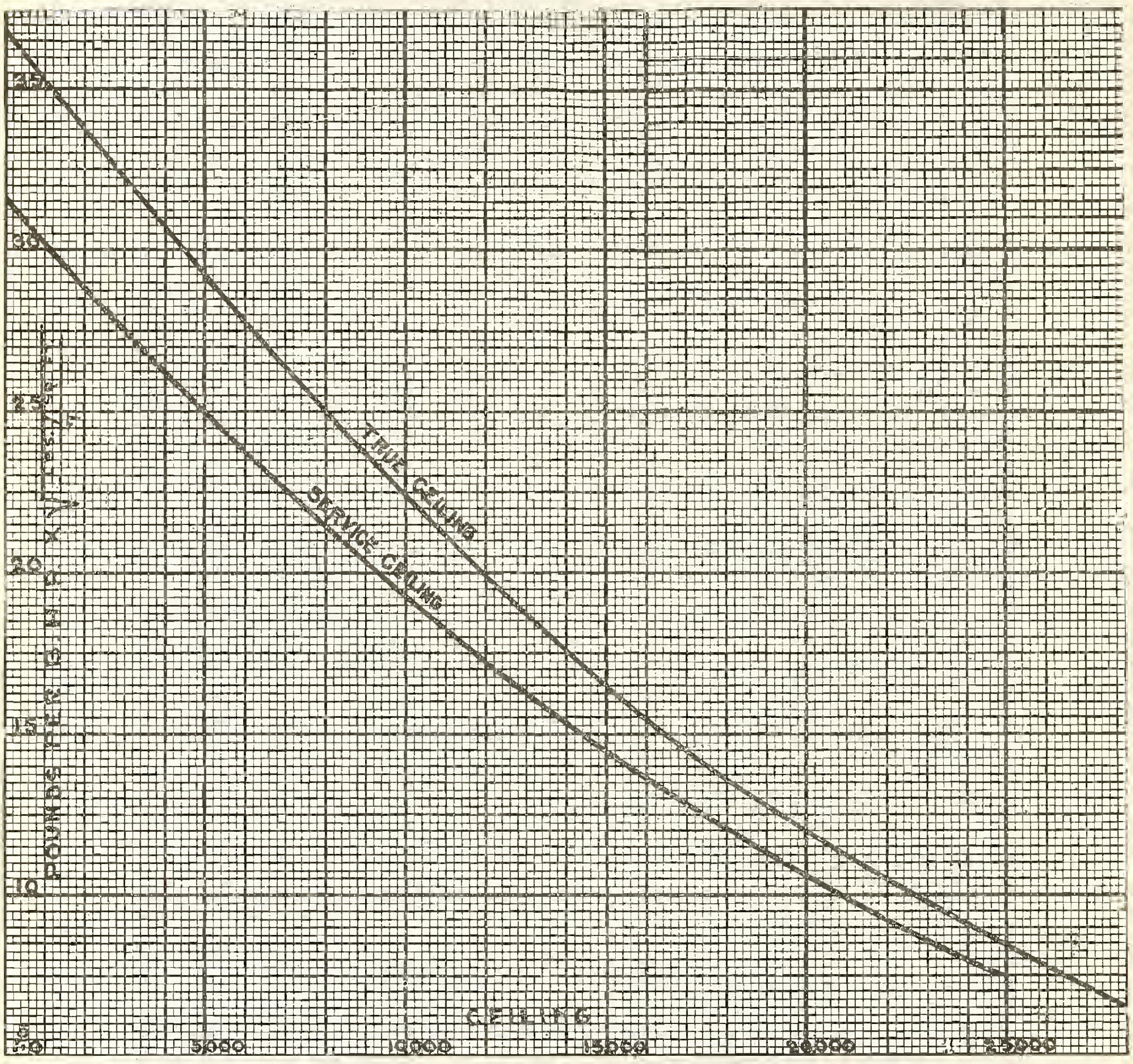


Fig. 3.—CURVES FOR APPROXIMATE PREDICTION OF AIRPLANE PERFORMANCE.

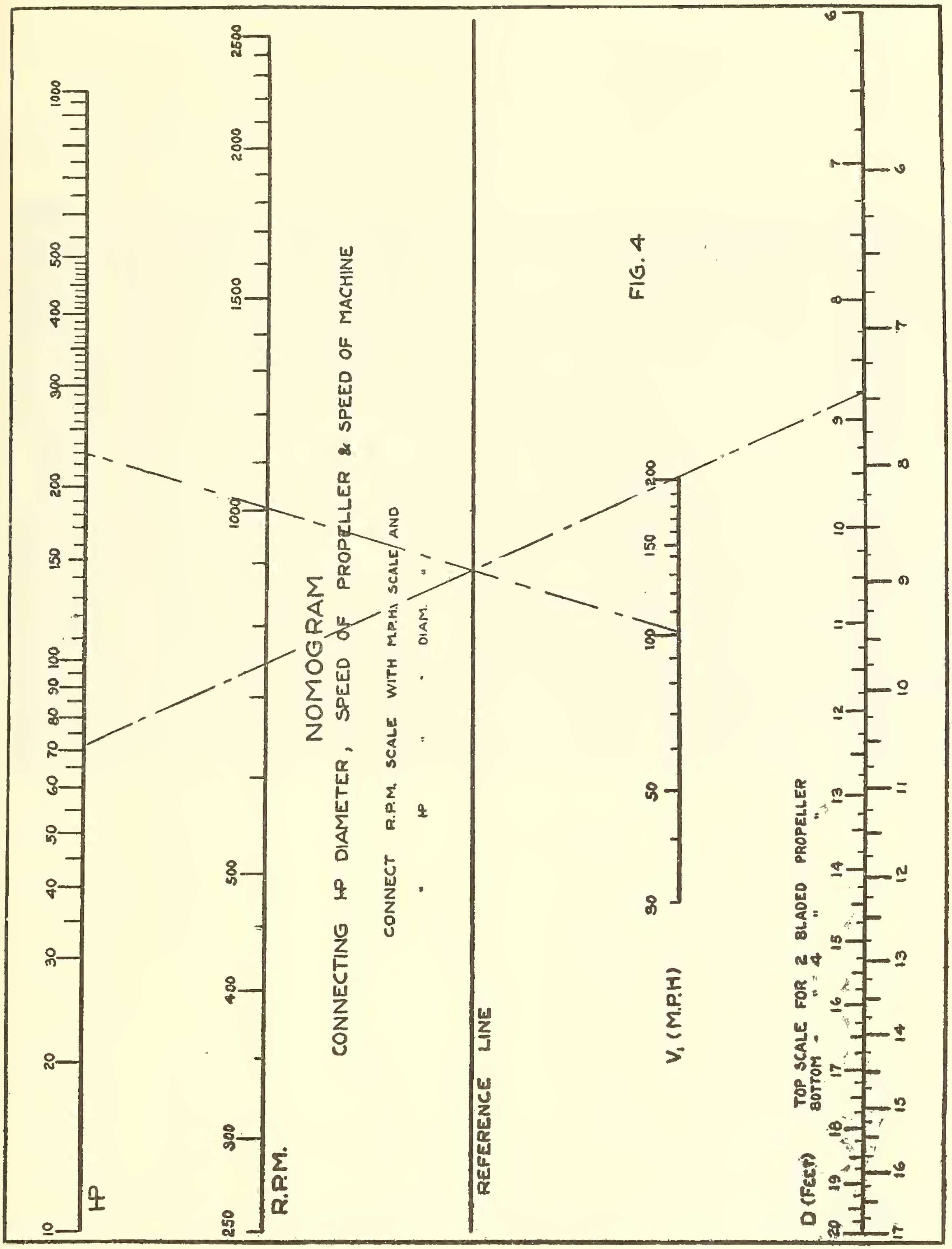
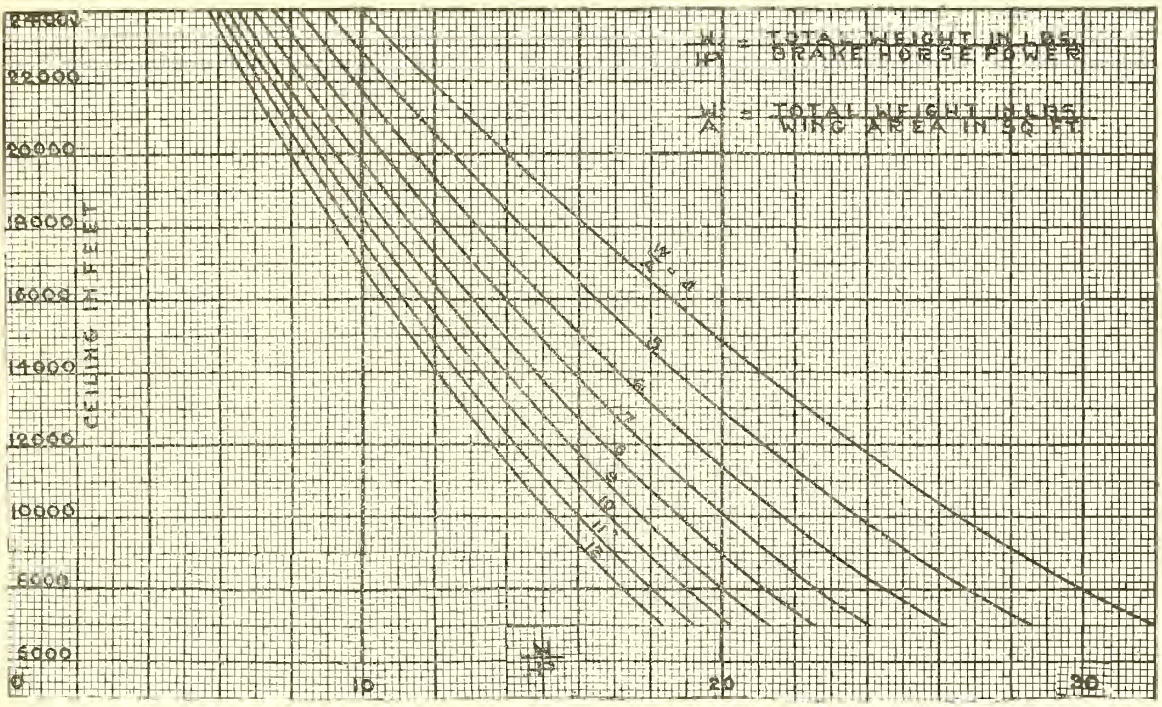


Figure 4.
Ref. A. C. A.

II.—CHART OF CEILINGS OF AIRPLANES.

These curves are based on average found from the latest available data on the performance of 74 machines of all types.

Their value lies in giving the ceiling of a machine in terms of its loading per horsepower, and loading per square foot of wing surface, which *ceiling* will be that expected from the average machine of that supporting area, weight, and power.



III.—THE DETERMINATION OF AIRPLANE CEILINGS.

The following method of ceiling determination is based on the condition that the rate of climb curve is a straight line. Climb test data from all available information warrants this conclusion.

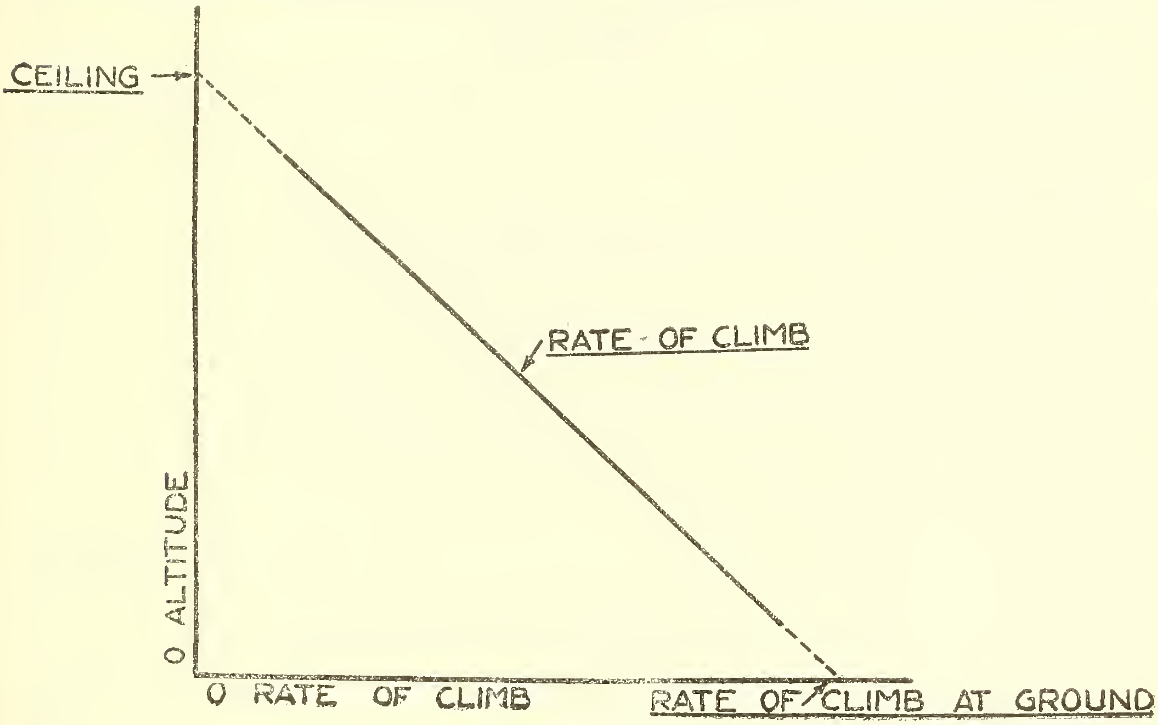
Where the rate of climb (altitude *v.* rate of climb) curve has been plotted its intersection with the altitude axis will give the ceiling of that particular machine, and the intersection with the rate of climb axis will give the rate of climb at the ground.

Or where the climb (altitude *v.* time) curve only has been plotted any two points chosen from it at equal time intervals and substituted in equation (II) below will give the ceiling.

(To allow for any inaccuracies in the climb curve plot, several pairs of points should be taken from the curve and an average value for the ceiling found.)

$$\text{(II)}\quad \text{Ceiling} = \frac{h_1^2}{2\,h_1 - h_2}$$

Where h_1 = height at a time t_1 and h_2 = height at a time t_2 so chosen that $t_2 = 2\,t_1$.



THEORY.

Let

h = height at any time, t.
H = ceiling, $t = \infty$.
R = initial rate of climb, $t = o$, $h = o$.
a = a constant.

then $\frac{dh}{dt}$ is the rate of climb at any altitude h.

Since the rate of climb falls off uniformly with the altitude,

$$(1)\qquad \frac{dh}{dt} = \text{R} - ah \int \frac{dh}{\text{R} - ah} = \int dt$$

integrating

(2) $$\log_e (R - ah) = -at + C$$

determining the value of the constant of integration (C), when $t = o$. $h = o$ substitute in (2)
$C = \log_e R$

From (2)

(3) $$\frac{R - ah}{R} = \epsilon^{-at}$$

(4) $$h = \frac{R}{a} (1 - \epsilon^{-at})$$

Determining the value of a, at the ceiling $h = H$, $t = \infty$, $a = \frac{R}{H}$

From (4)

(5) $$h = H (1 - \epsilon^{-at})$$

Taking two values h_1, and h_2 of h at times t_1 and t_2 where $t_2 = 2 t_1$

(6) $$h_1 = H (1 - \epsilon^{-at_1})$$

(7) $$h_2 = H (1 - \epsilon^{-2at_1})$$

Dividing

(8) $$\frac{h_2}{h_1} = 1 + \epsilon^{-at_1} \quad \text{or} \quad \epsilon^{-at_1} = \frac{h_2}{h_1} - 1.$$

Substitute in (6) the value of ϵ^{-at_1}

(9) $$h_1 = H \left(1 + 1 - \frac{h_2}{h_1}\right)$$

Solving for H

(10) $$H = \frac{h_1}{2 - \frac{h_2}{h_1}} = \frac{h_1^2}{2h_1 - h_2}$$

IV.—THE COMPUTATION OF AIRPLANE CEILINGS, RATE OF CLIMB, AND TIME TO AN ALTITUDE.

The simplifying assumptions upon which this calculation is based are taken up in the order of their entrance into the discussion.

Air density is an essential factor in the performanee of an airplane. Local atmospheric conditions determine both this density and its rate of change with altitude. Since performanees, ealeulated or determined experimentally, are of value in that they are capable of comparison with other performances, the present computation is in terms of a standard density.

This standard density is taken as 0.00237 slugs per cubic foot, and other densities are expressed as a percentage of this standard.

For the convenience of pilots, the change of density with altitude is expressed in terms of a standard height corresponding to an assumed standard atmosphere where the density varies with height according to a fixed law.

POWER REQUIRED.

The minimum power required for flight at standard density is obtained from wind tunnel test data on a model of the maehine, or from calculated performance curves (horsepower required *v.* miles per hour).

(1) $$P_r = \rho\, V^3\, (K_x A_w + K_p A_p).$$

ρ = air density at any altitude h.
P_r = power required for flight at altitude h.
V = forward velocity of the machine.
A_w = area of the supporting surface.
A_p = equivalent area of the parasite surface.
K_x = resistance coefficient of the supporting surface.
K_p = resistance coeffieient of the parasite surface.
K_y = lift coefficient of the supporting surface.
W = weight of the machine.

(2) $$W = \rho\, K_y A_w V^2. \quad V = \sqrt{\frac{W}{\rho A_w K_y}}$$

Substituting (2) in (1).

(3) $$P_r = (\rho)^{-0.5}\, W^{1.5}\, A_w^{-0.5} \left[\frac{K_x}{K_y^{1.5}} + \frac{K_p A_p}{K_y^{1.5} A_w}\right]$$

To express the power required at any altitude in terms of the power required at standard density,

(4) $$P_r = \left(\frac{\rho}{\rho_o}\right)^{-0.5} Pr_o.$$

Pr_o = power required at standard density.
ρ_o = standard density (0.00237 slugs per cubic foot).

In the transition from equation (3) to (4) it is assumed that the speed of the machine has been increased in the ratio $\left(\frac{\rho}{\rho_o}\right)^{-0.5}$, allowing the machine to maintain the attitude of flight for whieh Pr_o was measured, which constant angle of incidence keeps the quantity within the brackets in (3) a constant.

POWER AVAILABLE.

Motor tests under reduced pressure show the power to fall off as

$$\left(\frac{\rho}{\rho_o}\right)^{1.1}$$

Propeller tests show a change in efficiency of

$$\frac{\eta}{\eta_o}=\frac{1}{2}\left[1+\left(\frac{\rho}{\rho_o}\right)^{-0.5}\right]$$

η_o = propeller efficiency at standard density.

η = propeller efficiency at altitude h.

Then

$$P_a=\left(\frac{\rho_o}{\rho_o}\right)^{1.1}\frac{1}{2}\left[1+\left(\frac{\rho}{\rho_o}\right)^{-0.5}\right]P_{ao}.$$

P_a = power available at h.

P_{ao} = power available at standard density.

(5) $$P_a=\frac{1}{2}P_{ao}\left[\left(\frac{\rho}{\rho_o}\right)^{1.1}+\left(\frac{\rho}{\rho_o}\right)^{0.6}\right]$$

The machine has reached its ceiling when there is no excess power left for climbing, i. e., when the horsepower available and the horsepower required for flight have become equal.

$$\therefore P_a=P_r$$

From (4) and (5)

$$\frac{P_a}{P_r}=1=\frac{1}{2}\frac{P_{ao}\left[\left(\frac{\rho}{\rho_o}\right)^{1.1}+\left(\frac{\rho}{\rho_o}\right)^{0.6}\right]}{P_{ro}\left(\frac{\rho}{\rho_o}\right)^{0.5}}$$

Whence

(6) $$\frac{P_{ro}}{P_{ao}}=0.5\left[\left(\frac{\rho}{\rho_o}\right)^{1.6}+\left(\frac{\rho}{\rho_o}\right)^{1.1}\right]$$

Which gives the ceiling (in terms of standard density) for a machine when the power available and the power required for flight are known.

Equation (6) assumes that the conditions for equation (4) will obtain at the ceiling of the machine.

RATE OF CLIMB.

In feet per minute

$$\frac{dh}{dt}=\frac{\text{Excess horsepower }(33000)}{W}$$

$$=\frac{(P_a-P_r)33000}{W}$$

(7) $$=\frac{33000}{W}\left\{\frac{P_{ao}\left[\left(\frac{\rho}{\rho_o}\right)^{1.1}+\left(\frac{\rho}{\rho_o}\right)^{0.6}\right]}{2}-P_{ro}\left(\frac{\rho}{\rho_o}\right)^{-0.5}\right\}$$

$$=16500\frac{P_{ao}}{W}\left[\left(\frac{\rho}{\rho_o}\right)^{1.1}+\left(\frac{\rho}{\rho_o}\right)^{0.6}-2\frac{P_{ro}}{P_{ao}}\left(\frac{\rho}{\rho_o}\right)^{-0.5}\right]$$

EXPLANATION OF CURVES.

Curve (2) gives the ceiling of the machine in per cent of standard density when the thrust horsepower available and the thrust horsepower required (both referred to standard density) are known.

To obtain the curve, —values of ρ, the air density, as a per cent of standard density were taken and equation (6) was solved for the corresponding values of r_o, the ratio of power available, at standard density to power required at standard density.

Curve (3) gives the ceiling and the rate of climb of the machine when the thrust horsepower available and the thrust horsepower required (both referred to standard density) and the weight of the machine are known.

The intersection of the r_o curve with the altitude axis will give the ceiling (the rate of climb has become zero).

The intersection of the r_o curve with the $\left(\frac{dh}{dt}\right)\left(\frac{W}{P_{ao}}\right)$ axis when multiplied by $\left(\frac{P_{ao}}{W}\right)$ will give the rate of climb at the ground (the altitude is zero).

The abscissa of any point on the r_o curve when multiplied by $\frac{Pa_o}{W}$ will give the rate of climb at the altitude of the point taken.

To obtain the curve, —values of ρ and for each value a series of values for $\frac{P_{ao}}{P_{ro}}$, were taken and equation (7) was solved for $\left(\frac{dh}{dt}\right)\left(\frac{W}{P_{ao}}\right)$.

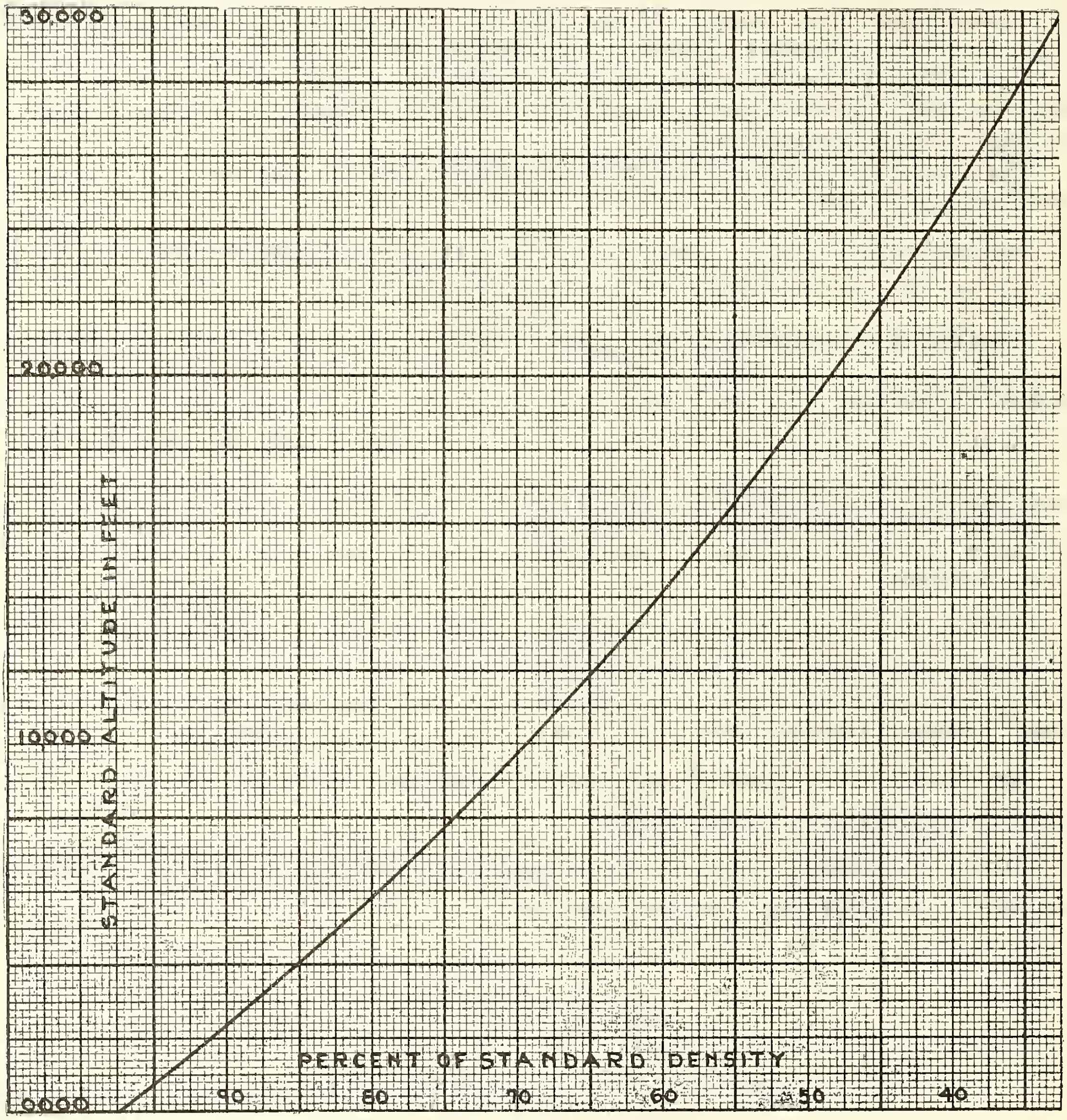
Curve (4) is similar to curve (3) except that the altitude is expressed in feet corresponding to standard density.

Curve (5) is also a rate of climb curve where r_o is plotted against $\left(\frac{dh}{dt}\right)\left(\frac{P_{ao}}{W}\right)$ and the curves are altitude curves taken at 2000-foot intervals.

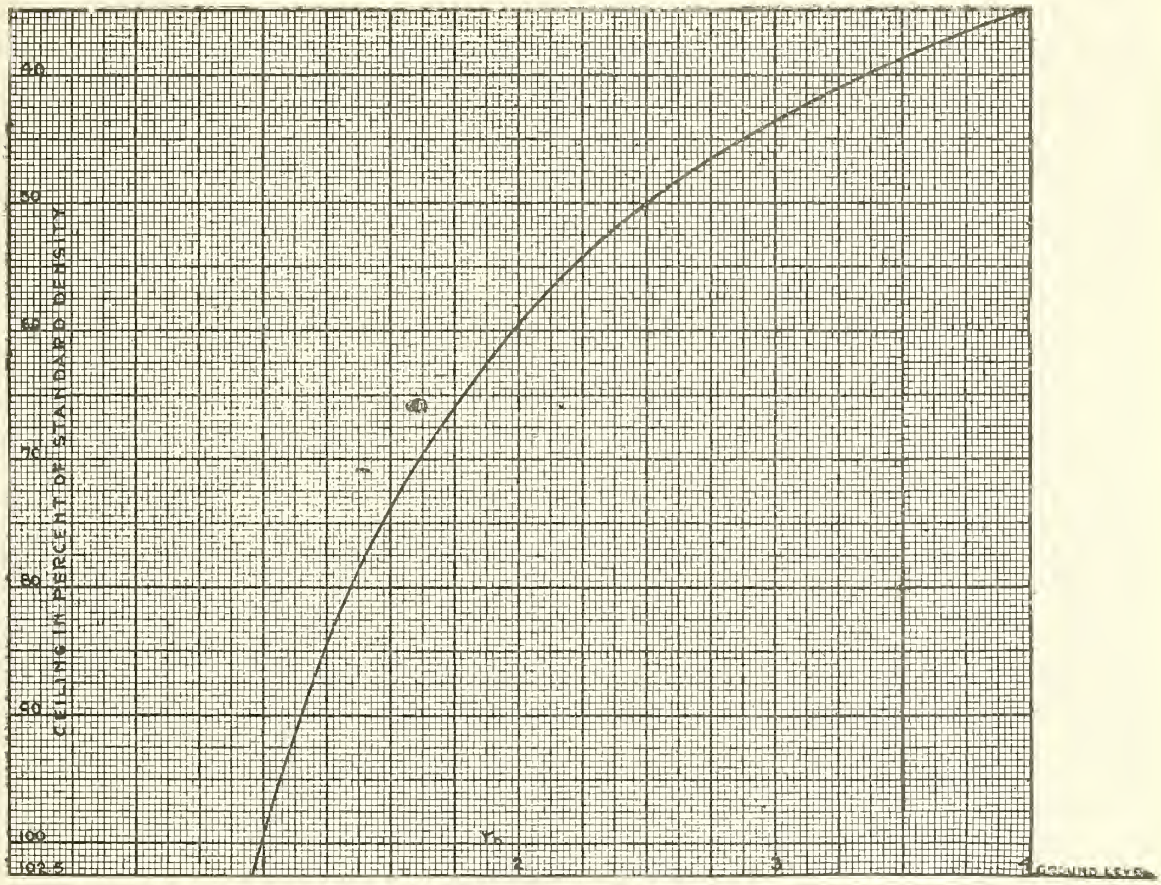
Curve (6) gives the time to any altitude when the thrust horsepower available and the thrust horsepower required (both at standard density) and the weight are known.

The abscissa of any point on the r_o curve when multiplied by $\frac{W}{Pa_o}$ will give the time in minutes to the altitude of the point taken.

Curve (7) is similar to curve (6) except that r_o is plotted against $T\left(\frac{P_{ao}}{W}\right)$ and the curves are altitude curves taken at 2,000-foot intervals.

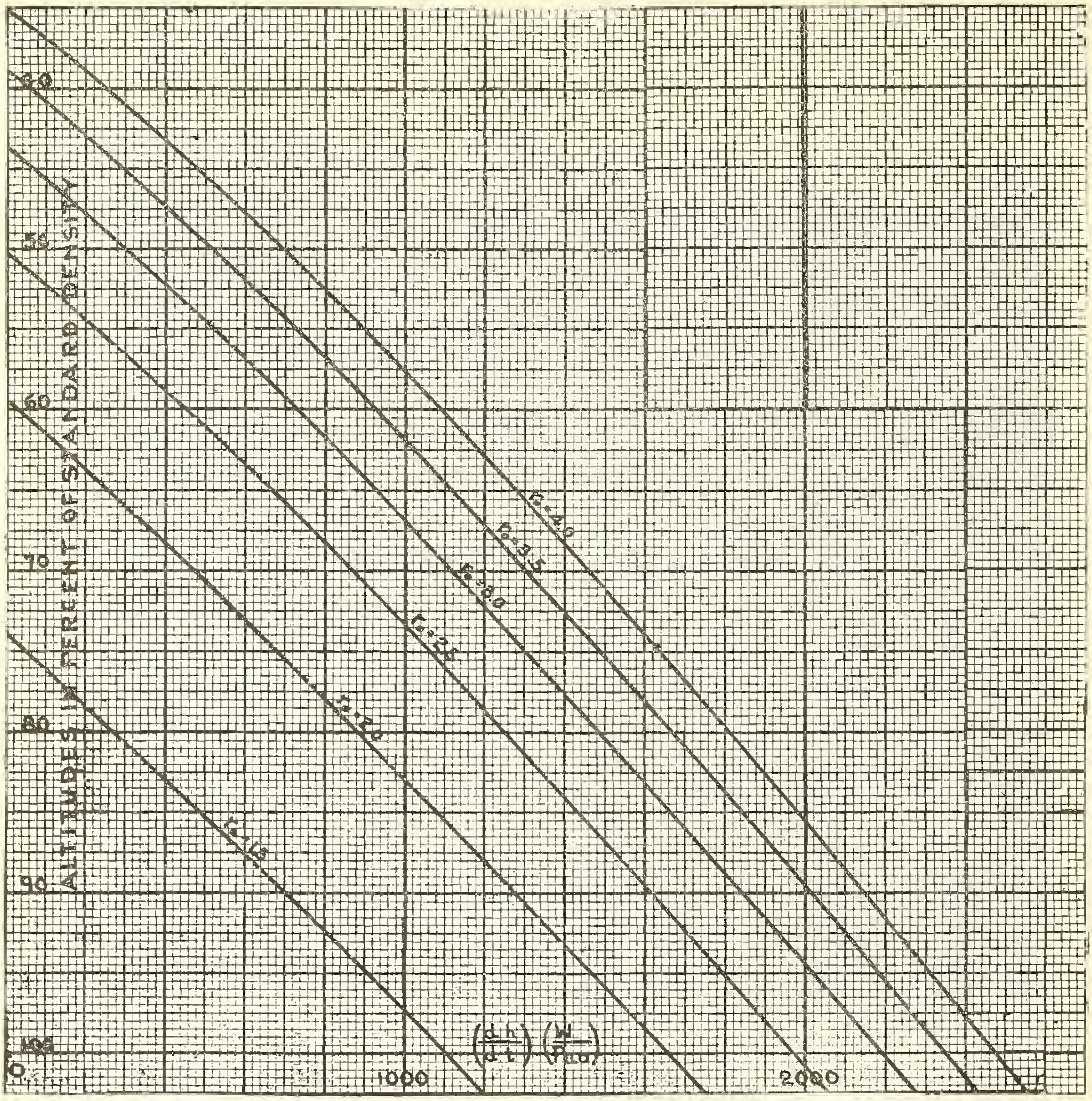


CURVE 1.—STANDARD DENSITY TO STANDARD ALTITUDE.



Curve 2.—CEILING.

$$r_0=\frac{\text{Power available}}{\text{Power required}} \text{ at standard density.}$$



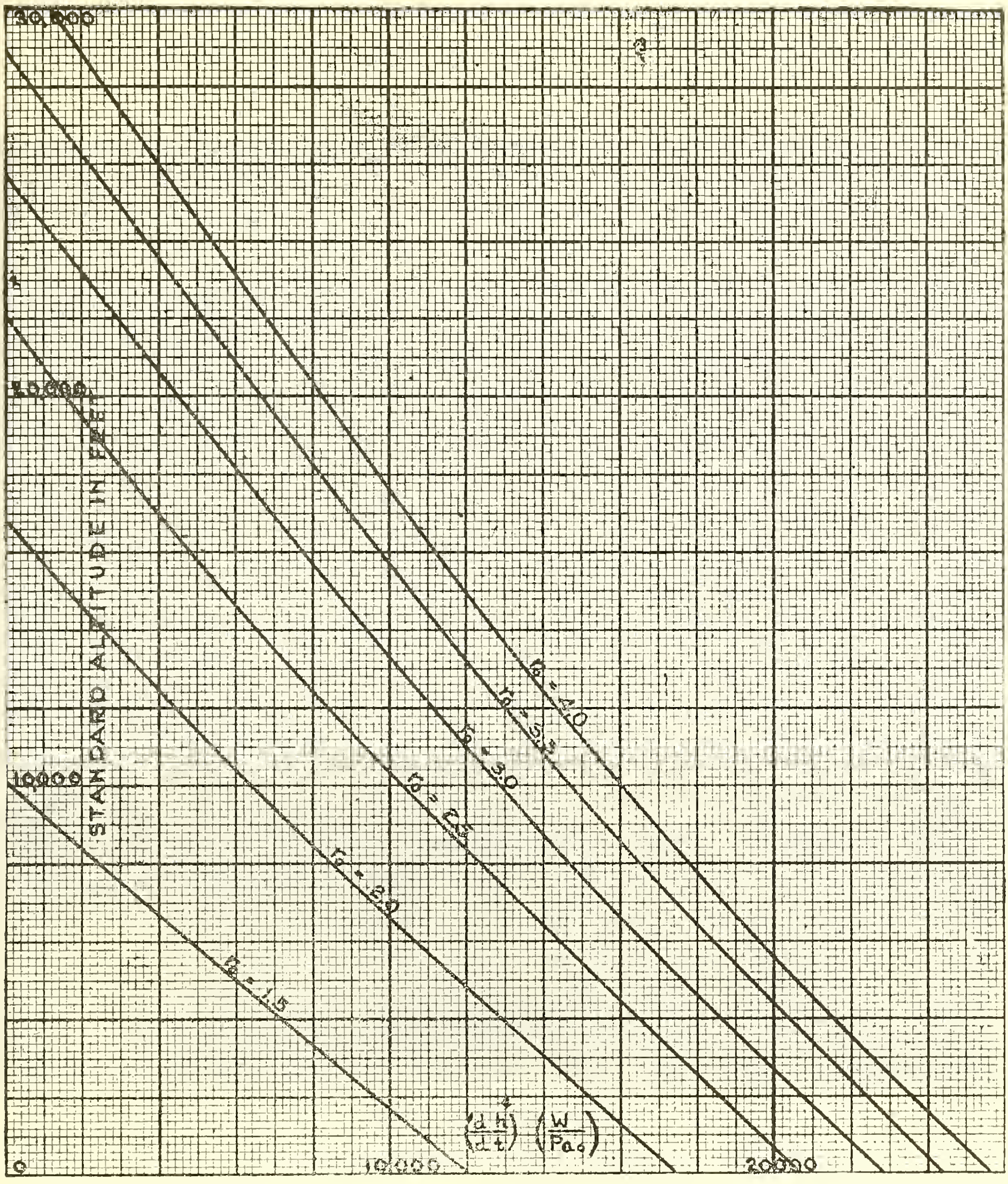
Curve 3.—CEILING AND RATE OF CLIMB.

$r_o = \frac{\text{Power available}}{\text{Power required}}$ at standard density.

$\frac{dh}{dt}$ = Rate of climb in feet per minute.

$\frac{W}{P_{ao}} = \frac{\text{Weight of machine}}{\text{Power available}}$ at standard density.

Intersection of curve with density axis gives ceiling in terms of standard density. Intersection of curve with ground level gives rate of climb multiplied by $\frac{W}{P_{ao}}$, at the ground.



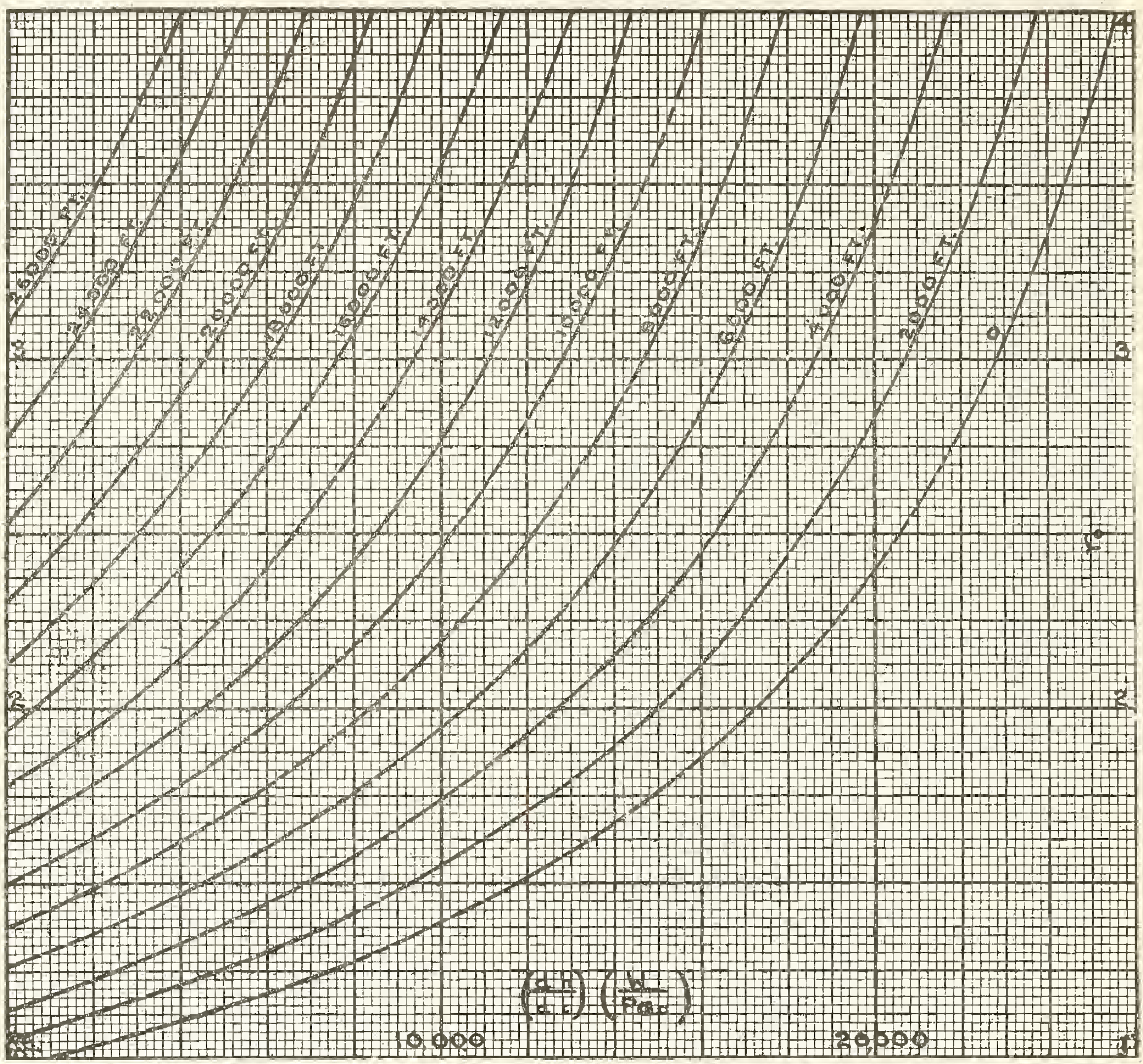
Curve 4.—CEILING AND RATE OF CLIMB.

$r_o = \frac{\text{Power available}}{\text{Power required}}$ at standard density.

$\frac{dh}{dt}$ = Rate of climb in feet per minute.

$\frac{W}{P_{ao}} = \frac{\text{Weight of machine}}{\text{Power available}}$ at standard density.

Intersection of curve with altitude axis gives ceiling in feet. Intersection of curve with $\left(\frac{dh}{dt}\right)\left(\frac{W}{P_{ao}}\right)$ axis gives rate of climb multiplied by $\frac{W}{P_{ao}}$, at the ground.

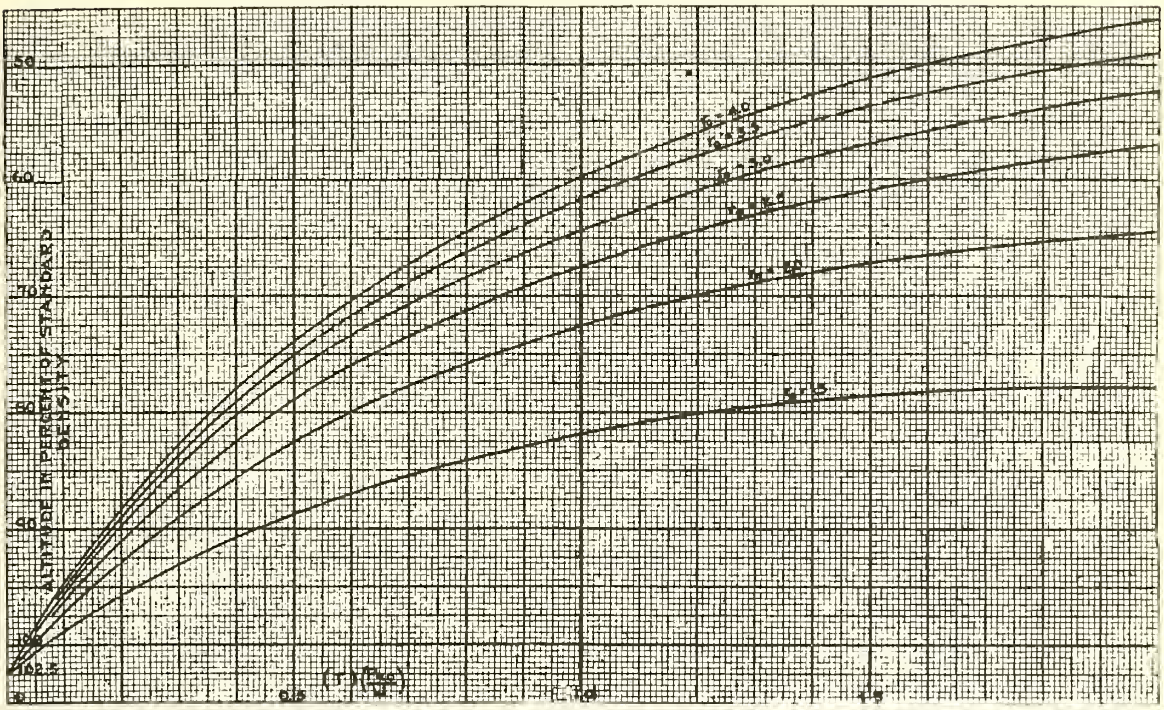


Curve 5.—RATE OF CLIMB.

$r_o = \frac{\text{Power available}}{\text{Power required}}$ at standard density.

$\frac{dh}{dt}$ = Rate of climb in feet per minute.

$\frac{W}{P_{ao}} = \frac{\text{Weight of machine}}{\text{Power availabl}}$ at standard density.

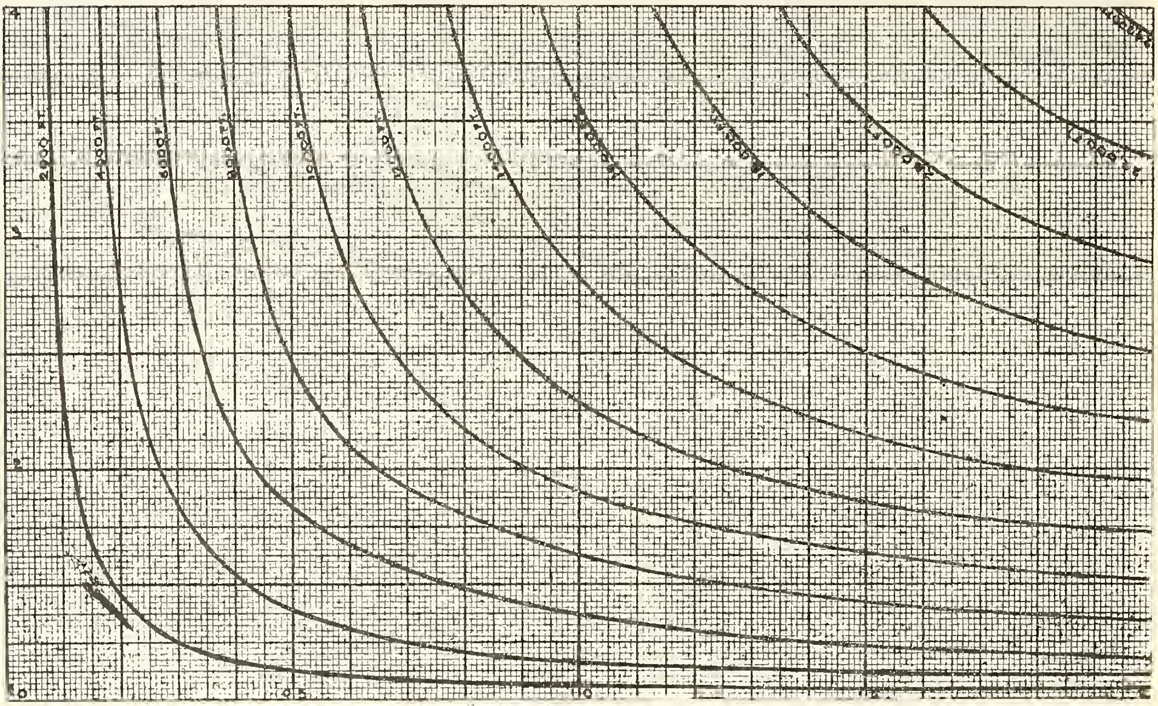


Curve 6.—TIME TO AN ALTITUDE.

$r_o = \frac{\text{Power available}}{\text{Power required}}$ at standard density.

$\frac{P_{ao}}{W} = \frac{\text{Power available}}{\text{Power required}}$ at standard density.

T=Time in minutes to an altitude.



$\left(\frac{r_o}{T}\right)\left(\frac{P_{ao}}{W}\right)$

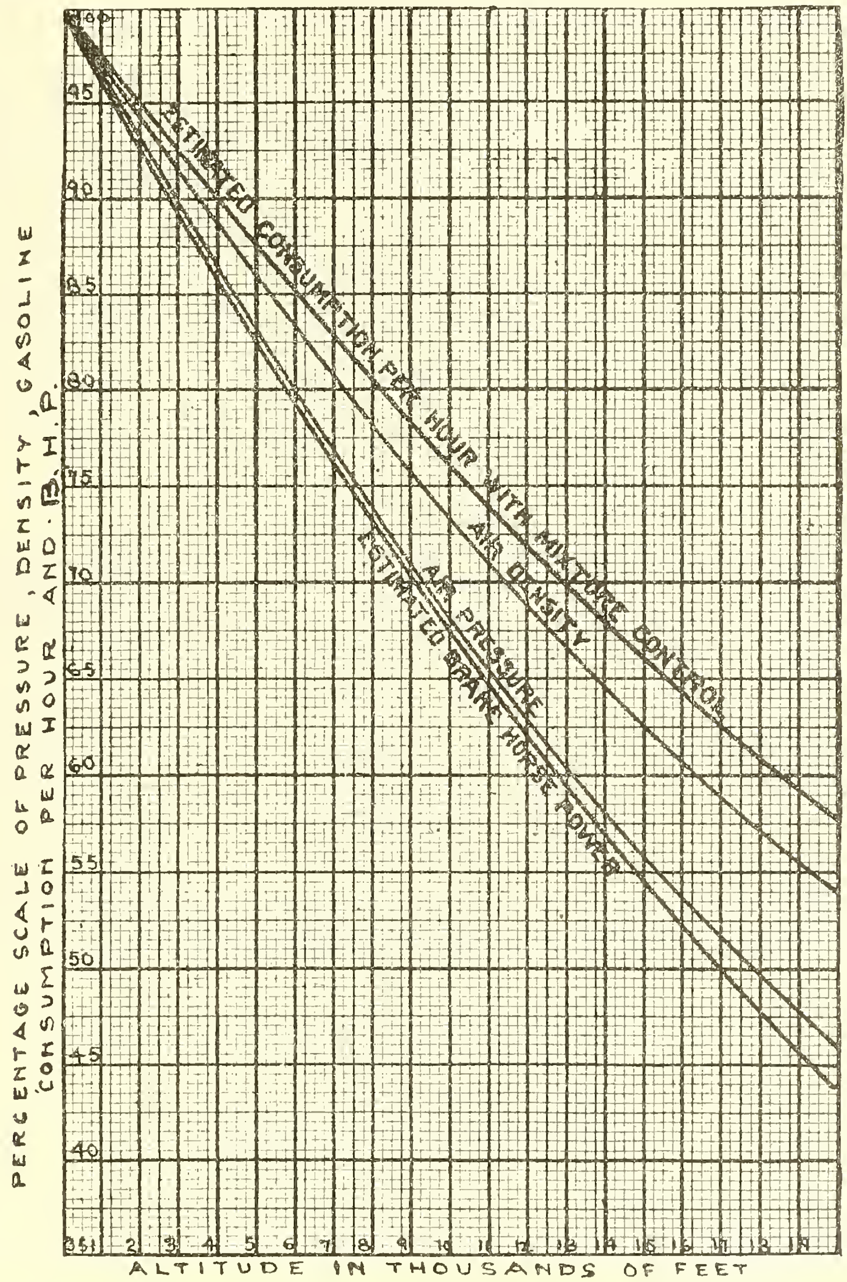
Curve 7.—TIME TO AN ALTITUDE.

$r_o = \frac{\text{Power available}}{\text{Power required}}$ at standard density.

$\frac{P_{ao}}{W} = \frac{\text{Power available}}{\text{Weight of machine}}$ at standard density.

T=Time in minutes to an altitude.

V.—ESTIMATED DECREASE IN BRAKE HORSEPOWER, GASOLINE CONSUMPTION AND AIR DENSITY WITH ALTITUDE.



ESTIMATED PERCENTAGE DECREASE IN GASOLINE CONSUMPTION, B. H. P. AIR PRESSURE, AND AIR DENSITY FOR ALTITUD UP TO 20,000 FEET.

(Starting or ground temperature—59° F.)

VI.—METHOD OF CALCULATING PERFORMANCE.

ALCULATIONS OF PERFORMANCE.

The calculations involved in the prediction of the performance of an airplane usually include:

Minimum horizontal flying speed.
Angle of incidence *v.* horizontal speed.
Horsepower required *v.* horizontal speed.
Ceiling *v.* horizontal speed.
Altitude *v.* rate of climb.
Radius of action under various conditions.

MINIMUM LANDING SPEED.

The minimum landing speed may be found approximately by the formula:

$$V=\sqrt{\frac{W}{A\ K'_y}}\ \text{(max.)}$$

Where

W = Weight of machine (lbs.).

K'_y (max.) = Maximum K_y (corrected).

A = Wing area. (sq. ft.)

For an accurate value it is necessary to use a weight which is corrected for lift of body and tail, and an area which is corrected for slip stream. The lift of the body is known from model tests and the lift of the tail may be calculated by the method outlined below. The method of correcting area for slip stream is given in Note 10. (P. 2).

LIFT OF TAIL SURFACES.

Owing to the down-wash from the wings and to the slip stream from the propeller, the calculations of lift and drag are more complicated for tail surfaces than for wings. However, the same corrections are to be applied in either case.

ANGLE OF INCIDENCE OF TAIL SURFACES.

The angle of incidence of the tail surfaces is determined by the down-wash from the main wings. This down-wash is a function of the angle of incidence of the wings and is not appreciably affected by the increased relative velocity of the propeller slip stream. Tests have shown that the angle of incidence of the tail is given by: $i_t = i - \Theta - \alpha$.

. Where

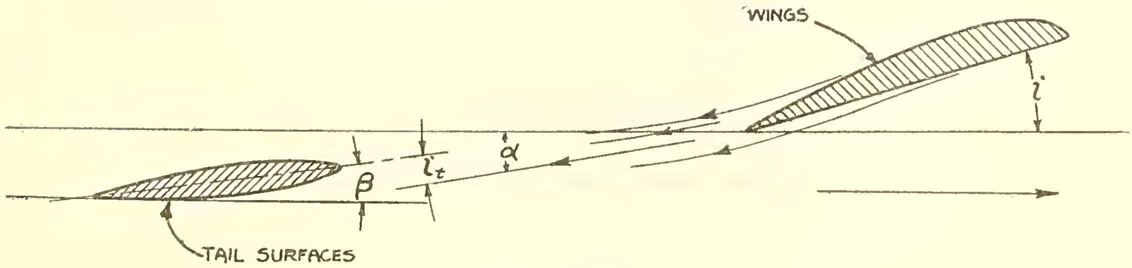
α = angle of down-wash = $0.5\ i + 1°$.

i_t = angle of incidence of tail.

i = angle of incidence of wings.

Θ = angle between wings and tail.

$\therefore\ i_t = 0.5\ i - \Theta - 1°$.



DOW -VASH.

Ref.: Eiffel; A. C. A.

EFFECT OF THE INCREASED RELATIVE VELOCITY IN PROPELLER SLIP STREAM.

This effect is to increase the equivalent area of the tail surfaces. It is accounted for, as in calculations on the wings, by multiplying the area inside the slip stream by a factor (see note 10) and adding to the area outside the slip stream to obtain the equivalent total area.

AEROFOIL SECTIONS EMPLOYED IN TAIL SURFACES.

These sections are usually double cambered and have characteristics quite different from those used in wings. The proper values of lift and drag coefficients should be taken from curves for the particular section used.

VARIATION OF SPEED WIT ANGLE.

The approximate velocities at various angles of incidence are found by using the total weight of the machine, the total wing area, and the corrected lift coefficient. The corresponding slip stream factor is found from the curves given in note 10. That part of the wing area which lies inside the slip stream is multiplied by this factor and is added to the wing area outside the slip stream to give the corrected area. The lift of the tail and body is subtracted from the total weight of machine to give the corrected lift which varies with angle.

HORSEPOWER AVAILABLE AND HORSEPOWER REQUIRED.

HORSEPOWER AVAILABLE.

The brake horsepower at the desired engine revolutions per minute multiplied by the propeller efficiency for this revolution per minute at any speed of advance gives the horsepower available at this speed. Horsepower available is plotted as ordinates and speeds of advance as abscissae.

HORSEPOWER REQUIRED.

Any angle of incidence gives a corresponding speed of advance. The total resistance of the machine is found at these speeds.

Then required horsepower

$$=\frac{R_t V}{375}.$$

Where

R_t = total resistance of machine. (lbs.).

V = speed of advance. (m. p. h.)

Iorsepower required by hull in the water at a velocity of V m. p. h.:

$$\text{Hull HP}=\frac{R_t V}{375}$$

$$R=\frac{1}{\frac{\triangle}{R}}\triangle$$

$\triangle$ = displacement of hull. (lbs.)

From towing basin tests $\frac{\triangle}{R}=\frac{\triangle_m}{R_m}$

Where

$$\frac{\triangle_m}{R_m} = \frac{\text{Displacement of model in lbs.}}{\text{Resistance of model in lbs.}}$$

$$\triangle = W - L.$$

Where

W = Weight of machine. (lbs.)

L = Lift of wings. (lbs.)

$\triangle$ is determined by the angle of incidence of the wings which is a function of the angle of trim of the hull since the angle of the wings is fixed relative to the hull.

Horsepower required plotted with the curve of horsepower available gives the reserve (by subtracting the horsepower required from the horsepower available ordinate) or excess horsepower which may be used for climb.

CLIMB AND VARIATION OF CLIMB WITH ALTITUDE.

The rate of climb for any machine depends upon the excess horsepower, i. e., the difference between the horsepower available and required.

$$\text{Climb in feet per second} = \frac{\text{Excess H. P.} \times 550.}{\text{Wt. of machine (lbs.).}}$$

The variation of excess horsepower and of climb with altitude is treated in note 6–IV.

CEI ING.

The "ceiling" of an airplane is the height at which the rate of climb becomes zero; the "service ceiling" is the height at which the rate of climb is 100 feet per minute. The method of finding ceilings is given in note 6.

MAXIMUM RADIUS OF ACTION.

The following formulæ are applications of Breguet's solutions:

Let

W = weight, fully loaded (pounds).

We = weight, less fuel and oil (pounds).

a = pounds of fuel, per effective horsepower per hour.

= fuel consumption divided by propeller efficiency.

$\frac{L}{D}$ = Maximum $\frac{\text{lift}}{\text{drag}}$ of machine.

S = Total flight distance (miles).

T = Time (hours) to fly distance S.

Then

$$S = \frac{860}{a} \cdot \frac{L}{D} \cdot \log_{10}\left(\frac{W}{We}\right)$$

$$T = 10600\left(\frac{1}{\sqrt{We}} - \frac{1}{\sqrt{W}}\right)$$

INSTRUCTIONS FOR CALCULATING PERFORMANCE.

Lifts, drags and pitching moments of the parts of the machine can be added up to give with considerable accuracy the forces and moments on a complete machine (since the interferences are small). The increase in propeller efficiency due to interference of the body is counterbalanced by the increased resistance of the body to the propeller (slip stream effects in the case of tractor and interference in the case of the pusher). However, as previously stated, the biplane or the triplane effect should be eliminated by a wind tunnel test of the particular combination used. (Ref. A. C. A.)

DATA ON MACHINE AND CORRECTIONS TO COEFFICIENTS.

Obtain data from specifications of the particular machine and fill out all items.

Column 1.—Take angle of incidence at every two degrees over the flying range (usually 0°–16°, inclusive).

Column 2.—Biplane or triplane correction from figure 1, Note 9–II–1. (P. 7).

Column 3.—Gap/Chord ratio correction from figure 4, Note 9–II–1. (P. 10).

Column 4.—Aspect ratio correction from figure 1, Note 9–II–3. (P. 15).

Column 5.—Stagger correction from figures 1 or 2, Note 9–II–2. (P. 12).

Column 6.—Scale correction from figure 1, Note 9–II–6. (P. 19).

Column 7.—Correction for plan form of wing tips from Note 9–II–4. (P. 17).

Column 8.—Correction for aileron leakage from Figure 1, Note 9–II–5. (P. 18).

Column 9.—The total correction to K_y is the product of the corrections of columns 2 to 8, inclusive.

Column 10.—Monoplane K_y from wind tunnel tests on model.

Column 11.—Corrected biplane or triplane K'_y is the product of the monoplane K_y by the total correction factor (column 10 times column 9).

Column 12.—Biplane or triplane correction from figure 2, Note 9–II–1. (P. 8).

Column 13.—Aspect ratio correction from figure 2, Note 9–II–3. (P. 16).

Column 14.—Stagger correction from figures 3 or 4, Note 9–II–2. (P. 14).

Column 15.—Scale correction from figure 2, Note 9–II–6. (P. 20).

Column 16.—Aileron leakage correction from figure 1, Note 9–II–5. (P. 18).

Column 17.—The total correction to K_x is the product of the corrections of columns 12 to 16, inclusive.

Column 18.—Monoplane K_x from wind tunnel test on model.

Column 19.—Corrected biplane or triplane K'_x is the product of the monoplane K_x by the total correction factor (column 18 times column 17). This will be the final corrected K_x, except for wing sections having a very low minimum drag—in which case there is another correction (col. 20).

Column 20.—Reduction for the plan form of wing tips from Note 9–II–4. (P. 17).

Column 21.—Find corrected biplane or triplane K'_x.

Column 22.—Corrected $\frac{L}{D}=\frac{\text{Corrected } K'_y}{\text{Corrected } K'_x}$

Data on machine and corrections to coefficients.

Machine .... Type .... Wing section ....

AREAS IN SQUARE FEET.

Upper wing: Span.................ft. Chord................ft. Upper wing....... Middle wing....... Lower wing....... Engines: Type.................. Number..................

Middle wing: Span................ft. Chord................ft. Total wing area (including ailerons)........ Ailerons........ Normal B. H. P................ R. P. M..................

Lower wing: Span................ft. Chord................ft. Elevators.............. Stabilizer (horizontal).............. Weight of machine: Fully loaded.....................lbs.

Mean wing span...................ft. Mean wing chord.....ft. Rudders................. Fin (vertical)...................... Less fuel (gas and oil)..............lbs.

Mean gap................$\frac{\text{Gap}}{\text{Chord}}$ Ratio..................... Wing area: Inside slip stream.......... Outside.............. $\frac{\text{Lbs.}}{\text{Sq. ft.}}$: Loaded................. Less fuel.................

Tail area: Inside slip stream........... Outside............

Stagger, degrees...................... Aspect ratio............ Total area: Tail surfaces.................................... $\frac{\text{Lbs.}}{\text{HP.}}$: Loaded................. Less fuel.................

Percentage of total wing spans containing aileron joint.........

Angle of incidence.	K_y corrections.								Monoplane K_y.	Corrected biplane or triplane K'_y.	K_x corrections.						Monoplane K_x.	Corrected biplane or triplane K_x.	Reduction for plan form of wing tips.	Final biplane or triplane K'_x.	Corrected $\frac{L}{D}$.
	Biplane or triplane.	$\frac{\text{Gap}}{\text{Chord}}$ ratio.	Aspect ratio.	Stagger.	Scale.	Plan form of wing tips.	Aileron leakage.	Total correction.			Biplane or triplane.	Aspect ratio.	Stagger.	Scale.	Aileron leakage.	Total correction.					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)

Fill out data at top of sheet. Calculate the parasite resistance.

Column 1.—Take angle of incidence at every 2 degrees over the flying range (usually 0°–16°, inclusive).

Column 2.—Corrected K'_y from sheet I.

Column 3.—Corrected K'_x from sheet I.

Column 4.—Approximate velocity, $V=\sqrt{\frac{W}{K'_y A}}$

Column 5.—Slip stream factor, $\left(\frac{V_s}{V}\right)^2$, Note 10. (P. 2)..

Column 6.—Angle of incidence of tail, $i_t=0.5\,i-\theta-1°$, Note 6. (P. 20).

Column 7.—K_y for section used in tail surfaces (to be corrected in cases of large machine).

Column 8.—Equivalent tail area inside slip stream, m = area inside slip stream times slip stream factor (column 5).

Column 9.—Total equivalent area of tail surfaces, A'_t = equivalent area inside slip stream plus area outside slip stream.

Column 10.—Lift of tail, $L_t=K_y\ (\text{tail})\cdot A'_t\cdot V^2$.

Column 11.—Lift of body from wind tunnel tests, make reduction for interference.

Column 12.—Lift of wings = (weight of machine) − (lift of tail) − (lift of body), i e, $Lw=W-(Lt+Lb)$.

Column 13.—Equivalent area of wings inside slip stream = area of wings inside slip stream times slip stream factor (column 5).

Column 14.—Total equivalent area of wings, A' = equivalent area inside slip stream plus area outside slip stream.

Column 15.—Corrected velocity, $V'=\sqrt{\frac{L_w}{K'_y A'}}$

Column 16.—Wing drag, $D=K'_x\cdot A'\cdot(V')^2$.

Column 17.—Parasite drag inside slip stream at velocity V' = (parasite drag inside slip stream at 70 m. p. h.) times slip stream factor (column 5) times $\left(\frac{V'}{70}\right)^2$.

Column 18.—Parasite drag outside slip stream at velocity V' = (parasite drag outside slip stream at 70 m. p. h.) times $\left(\frac{V'}{70}\right)^2$.

Column 19.—Total parasite drag = (column 17) + (column 18).

Column 20.—Total drag = wing drag + parasite drag = (column 16) + (column 18).

Column 21.—$\frac{L}{D}$ for machine $=\frac{\text{weight of machine}}{\text{total drag}}$.

Column 22.—Horsepower required $=\frac{\text{total drag (lbs.) times velocity (m. p. h.)}}{375}$.

Column 23.—Slip function $=\frac{88\ V}{ND}$. V = velocity (m. p. h.); N = engine (r. p. m.); D = propeller diameter (feet).

Column 24.—Propeller efficiency from propeller efficiency curve.

Column 25.—Horsepower available = brake horsepower times propeller efficiency.

CALCULATIONS OF PARASITE RESISTANCE.

Using Note 10 (p. 2), find the resistance of each unit at 70 m. p. h. and tabulate as indicated. Note that the resistances of parts inside the slip stream are separated from those outside.

To find the total parasite resistance at any speed: Multiply the sum of the resistances outside the slip stream by $\left(\frac{V}{70}\right)^2$, and the sum of the resistances inside the slip stream by $\left(\frac{V}{70}\right)^2$ times the slip stream factor as given in Note 10 and add the two results. That is,

$$R_p = (R_i + R_o \cdot S. S. F.) \left(\frac{V}{70}\right)^2.$$

Where

R_i = resistance inside slip stream.
R_o = resistance outside slip stream.
S. S. F. = slip stream factor.

Parasite resistance.

	Pounds resistance at 70 m. p. h.	
	Inside slip stream.	Outside slip stream.
1. Body, or hull		
2. Landing gear:		
a. Floats, or wheels		
b. Struts		
c. Fittings, wires, etc		
3. Tail:		
a. Control surfaces		
b. Control cables		
c. Struts, braces		
d. Fittings, horns, etc		
4. Interplane bracing:		
a. Struts		
b. Flying and landing wires		
Aileron control wires		
c. Fittings, horns, etc		
5. Miscellaneous		

Performance for .... *machine.*

Wing area: Inside slip stream............................Sq. ft. Weight.......... Lbs. $\frac{\text{Lbs.}}{\text{Sq. ft.}}$.......... $\frac{\text{Lbs.}}{\text{H. P.}}$.......... Engines: Number .... Type ....

Outside slip stream..........................Sq. ft. Normal B. H. P............. R. P. M............

Total..................................Sq. ft. Propeller diameter........... Ft.

Tail area: Inside slip stream............................Sq. ft. Parasite drag at 70 M. P. H.—Inside slip stream.........Lbs.

Outside slip stream..........................Sq. ft. Outside slip stream.......Lbs.

Angle between wings and tail................................

Angle between wings and prop. axis.........................

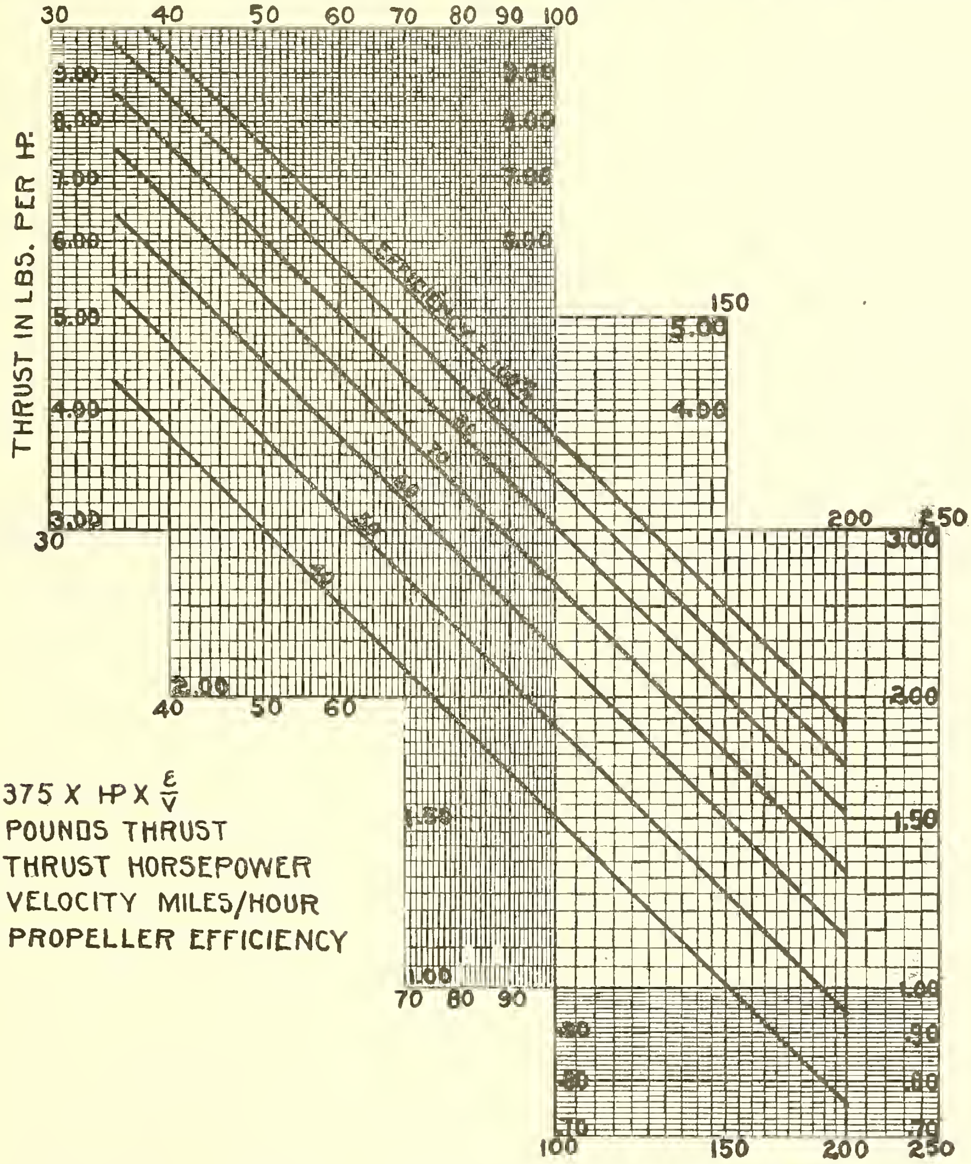
Angle of incidence wings, degrees.	Corrected coefficients.		Approximate velocity V M.P.H.	Slip stream factor	Tail surfaces.					Lift of body L_b lbs.	Wings.			Corrected velocity V′ M.P.H.	Wing drag, lbs.	Parasite drag.			Total drag, D′ lbs.	$\frac{L}{D'}$ machine	Horsepower required.	Slip function $\frac{88V'}{ND}$	Propeller efficiency, per cent.	Horsepower available.
	K'_x	K'_y			Angle of incidence, degrees.	K_y	Equivalent area inside slip stream, sq. ft.	Total equivalent area, sq. ft.	Lift L_t lbs.		Lift L_w lbs.	Equivalent area inside slip stream, sq. ft.	Total equivalent area A′ sq. ft.			Inside slip stream, lbs.	Outside slip stream, lbs.	Total lbs.						
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)

REMARKS:

WASHINGTON : GOVERNMENT PRINTING OFFICE : 1919

AIRCRAFT DESIGN DATA. NOTE NO. 7.

THRUST AT VARIOUS SPEEDS AND EFFICIENCIES.



$T = 375 \times HP \times \frac{\varepsilon}{V}$

T = POUNDS THRUST

HP = THRUST HORSEPOWER

V = VELOCITY MILES/HOUR

ε = PROPELLER EFFICIENCY

THRUST IN POUNDS AT VARIOUS SPEEDS AND EFFICIENCIES.

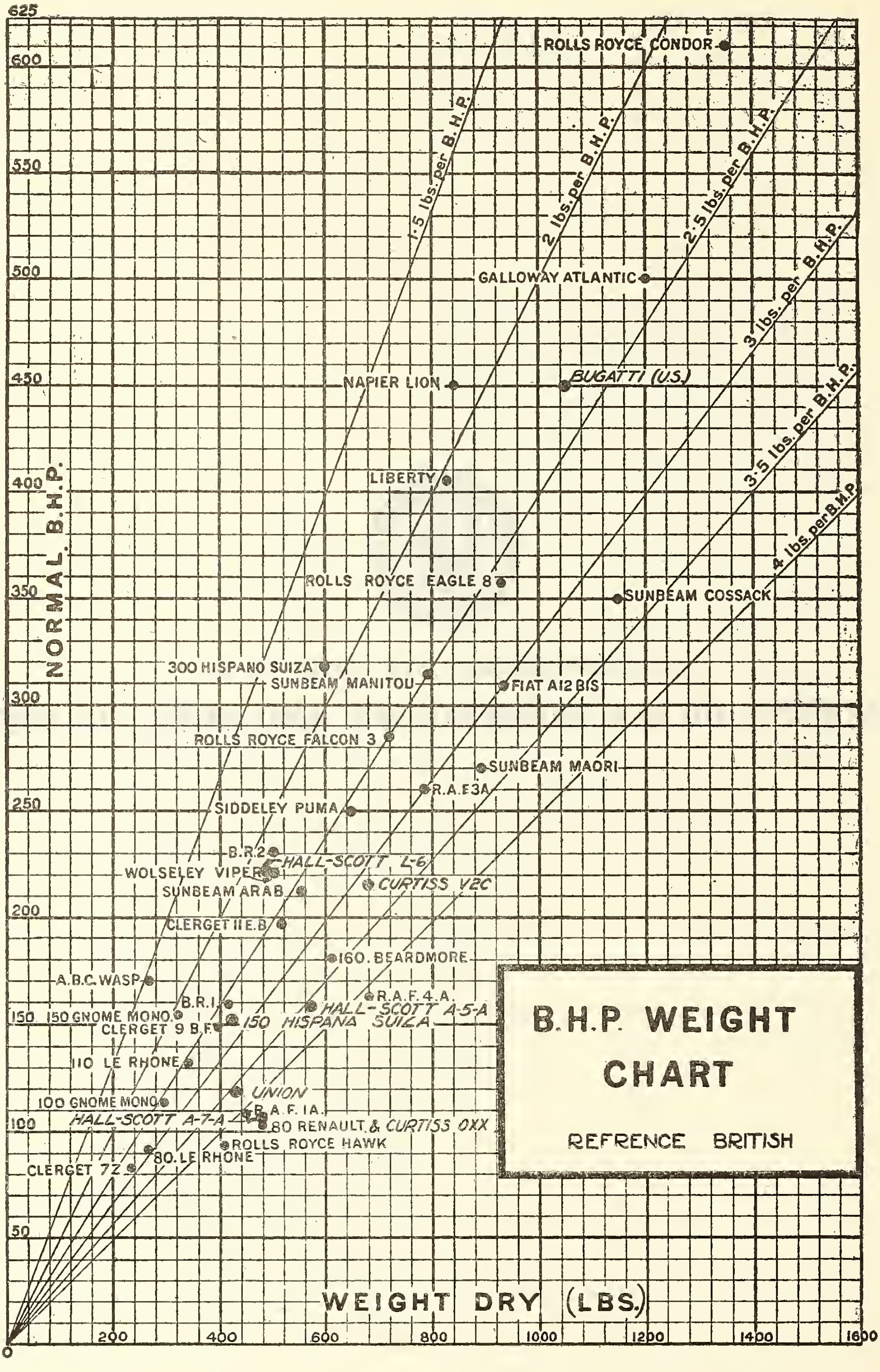
98257—19—No. 7

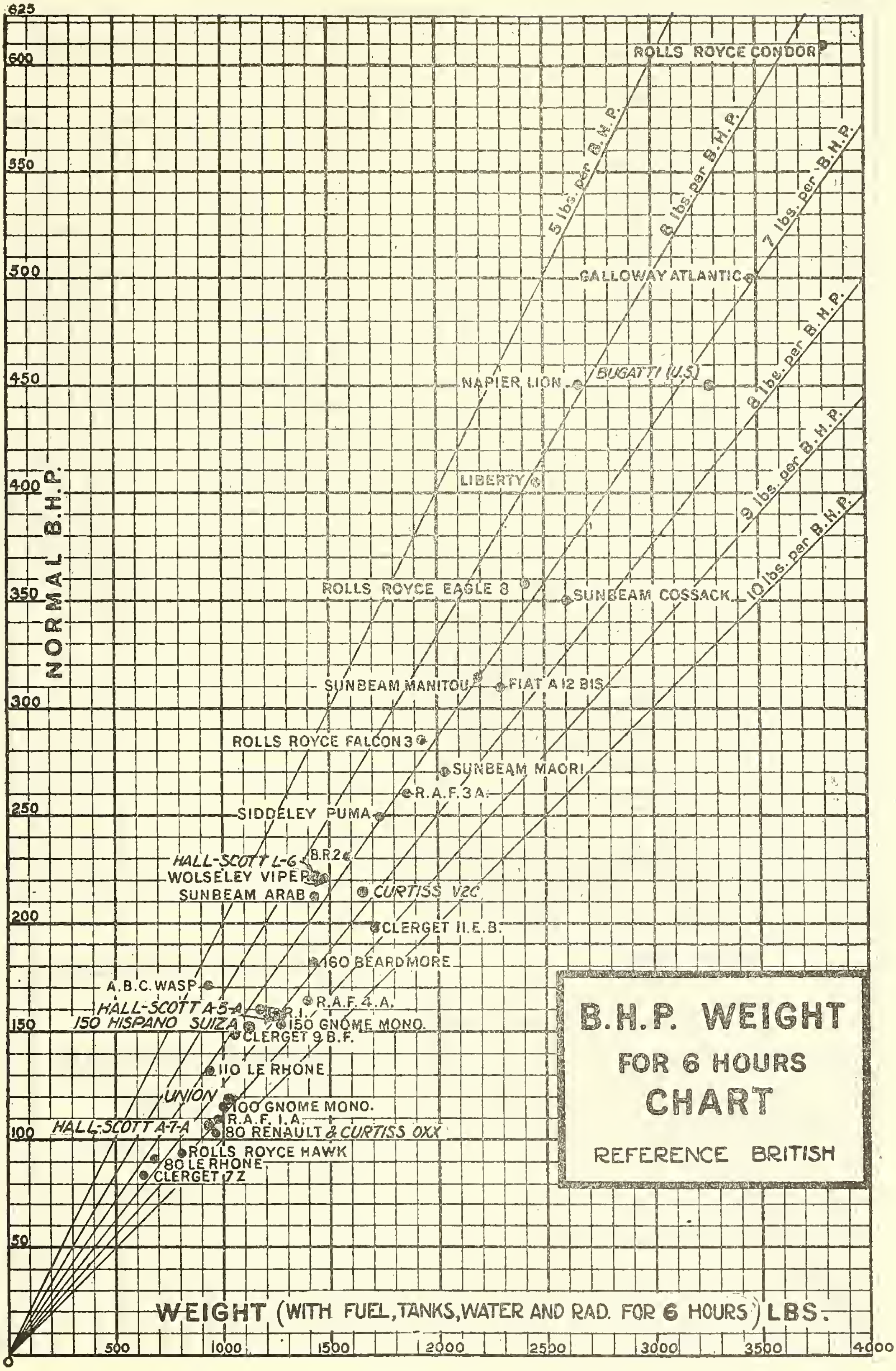
WASHINGTON : GOVERNMENT PRINTING OFFICE : 1919

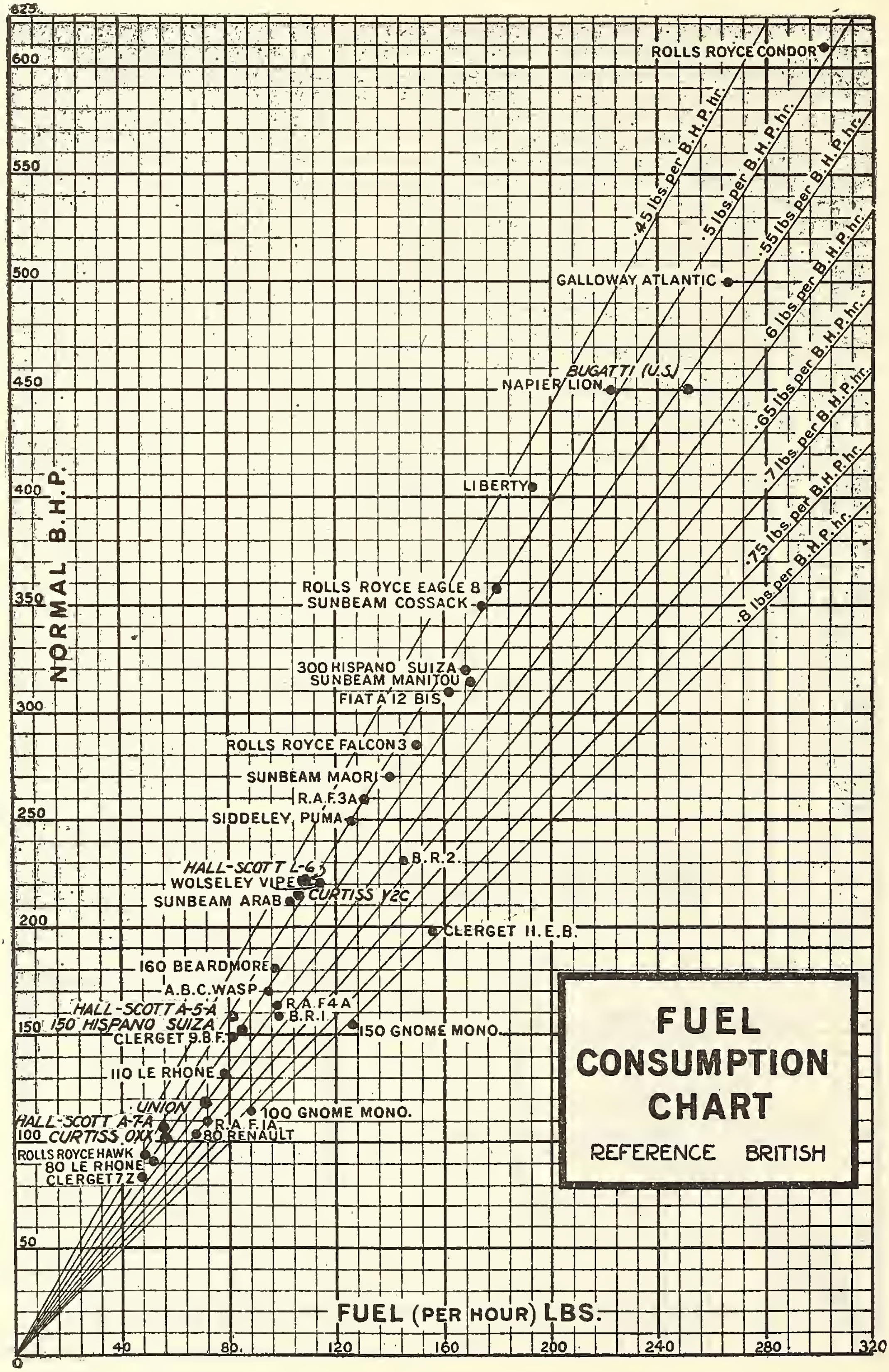
AIRCRAFT DESIGN DATA. NOTE NO. 8.

ENGINES—BRAKE HORSEPOWER AND FUEL CHARTS.

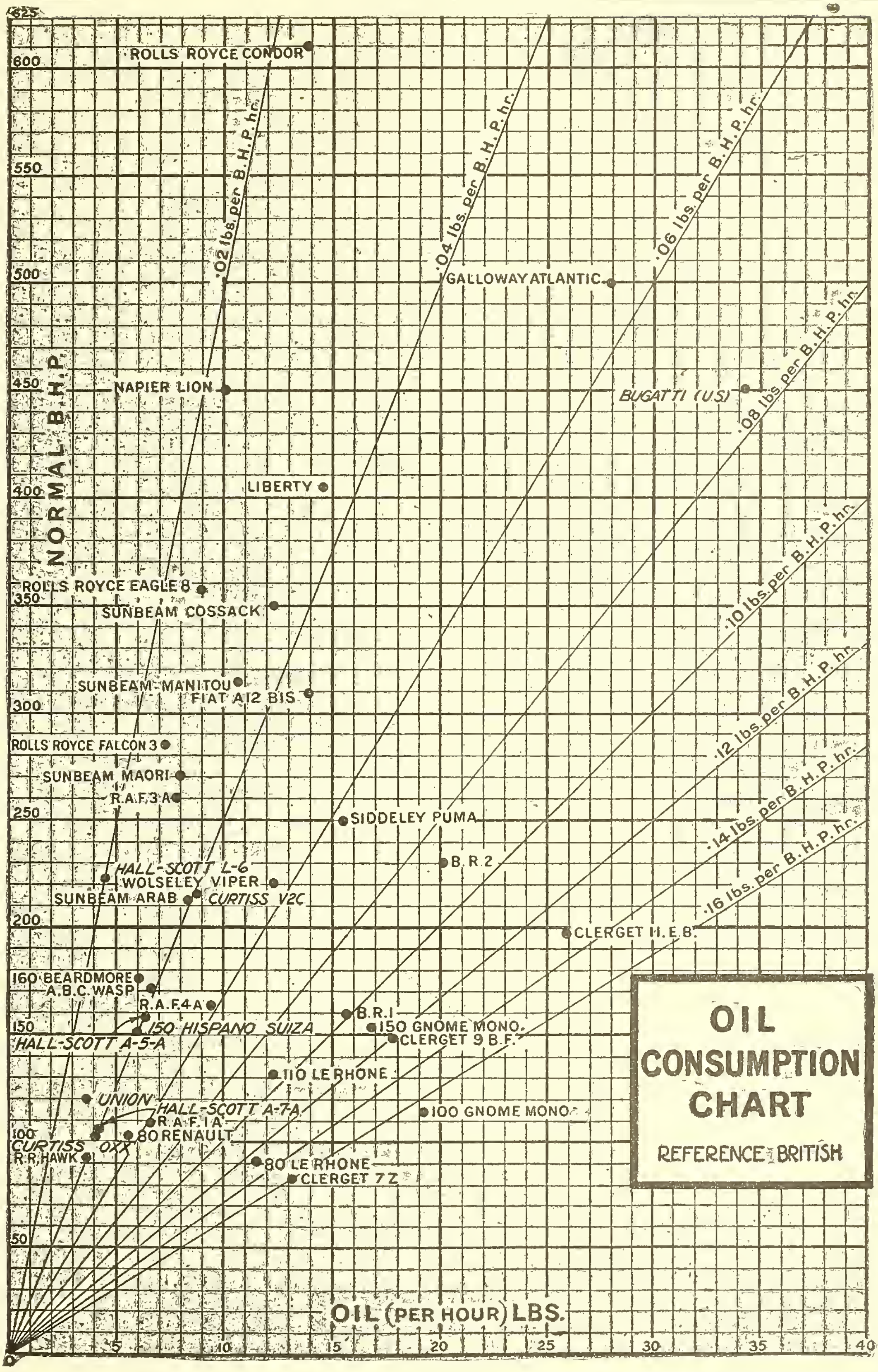
98257—19—No. 8


B.H.P. WEIGHT CHART
REFRENCE BRITISH
NORMAL B.H.P.
WEIGHT DRY (LBS.)
1·5 lbs. per B.H.P.
2 lbs. per B.H.P.
2·5 lbs. per B.H.P.
3 lbs. per B.H.P.
3·5 lbs. per B.H.P.
4 lbs. per B.H.P.
ROLLS ROYCE CONDOR
GALLOWAY ATLANTIC
NAPIER LION
BUGATTI (U.S.)
LIBERTY
ROLLS ROYCE EAGLE 8
SUNBEAM COSSACK
300 HISPANO SUIZA
SUNBEAM MANITOU
FIAT A12 BIS
ROLLS ROYCE FALCON 3
SUNBEAM MAORI
R.A.F.3A
SIDDELEY PUMA
B.R.2
HALL-SCOTT L-6
WOLSELEY VIPER
SUNBEAM ARAB
CURTISS V2C
CLERGET 11 E.B.
160. BEARDMORE
A.B.C. WASP
R.A.F. 4.A.
B.R.1
HALL-SCOTT A-5-A
150 GNOME MONO.
CLERGET 9 B.F.
150 HISPANA SUIZA
110 LE RHONE
UNION
100 GNOME MONO
R.A.F. 1A.
HALL-SCOTT A-7-A
80 RENAULT & CURTISS OXX
ROLLS ROYCE HAWK
80. LE RHONE
CLERGET 7Z
0
50
100
150
200
250
300
350
400
450
500
550
600
625
200
400
600
800
1000
1200
1400
1600


B.H.P. WEIGHT
FOR 6 HOURS
CHART
REFERENCE BRITISH
NORMAL B.H.P.
WEIGHT (WITH FUEL, TANKS, WATER AND RAD. FOR 6 HOURS) LBS.
5 lbs. per B.H.P.
6 lbs. per B.H.P.
7 lbs. per B.H.P.
8 lbs. per B.H.P.
9 lbs. per B.H.P.
10 lbs. per B.H.P.
ROLLS ROYCE CONDOR
GALLOWAY ATLANTIC
NAPIER LION
BUGATTI (U.S.)
LIBERTY
ROLLS ROYCE EAGLE 8
SUNBEAM COSSACK
SUNBEAM MANITOU
FIAT A 12 BIS
ROLLS ROYCE FALCON 3
SUNBEAM MAORI
R.A.F. 3 A.
SIDDELEY PUMA
B.R.2
HALL-SCOTT L-6
WOLSELEY VIPER
SUNBEAM ARAB
CURTISS V2C
CLERGET 11.E.B.
160 BEARDMORE
A.B.C. WASP
R.A.F. 4. A.
HALL-SCOTT A-5-A
B.R.1
150 HISPANO SUIZA
150 GNOME MONO.
CLERGET 9 B.F.
110 LE RHONE
UNION
100 GNOME MONO.
R.A.F. 1. A.
HALL-SCOTT A-7-A
80 RENAULT & CURTISS OXX
ROLLS ROYCE HAWK
80 LE RHONE
CLERGET 7 Z
625
600
550
500
450
400
350
300
250
200
150
100
50
0
500
1000
1500
2000
2500
3000
3500
4000



FUEL CONSUMPTION CHART
REFERENCE BRITISH
NORMAL B.H.P.
FUEL (PER HOUR) LBS.
625
600
550
500
450
400
350
300
250
200
150
100
50
0
40
80
120
160
200
240
280
320
·45 lbs. per B.H.P. hr.
·5 lbs. per B.H.P. hr.
·55 lbs. per B.H.P. hr.
·6 lbs. per B.H.P. hr.
·65 lbs. per B.H.P. hr.
·7 lbs. per B.H.P. hr.
·75 lbs. per B.H.P. hr.
·8 lbs. per B.H.P. hr.
ROLLS ROYCE CONDOR
GALLOWAY ATLANTIC
BUGATTI (U.S.)
NAPIER LION
LIBERTY
ROLLS ROYCE EAGLE 8
SUNBEAM COSSACK
300 HISPANO SUIZA
SUNBEAM MANITOU
FIAT A 12 BIS
ROLLS ROYCE FALCON 3
SUNBEAM MAORI
R.A.F.3A
SIDDELEY PUMA
B.R.2.
HALL-SCOTT L-6
WOLSELEY VIPER
CURTISS Y2C
SUNBEAM ARAB
CLERGET 11.E.B.
160 BEARDMORE
A.B.C. WASP
R.A.F.4A
B.R.1.
HALL-SCOTT A-5-A
150 HISPANO SUIZA
CLERGET 9.B.F.
150 GNOME MONO.
110 LE RHONE
UNION
100 GNOME MONO.
HALL-SCOTT A-7-A
R.A.F.1A
100 CURTISS OXX
80 RENAULT
ROLLS ROYCE HAWK
80 LE RHONE
CLERGET 7 Z



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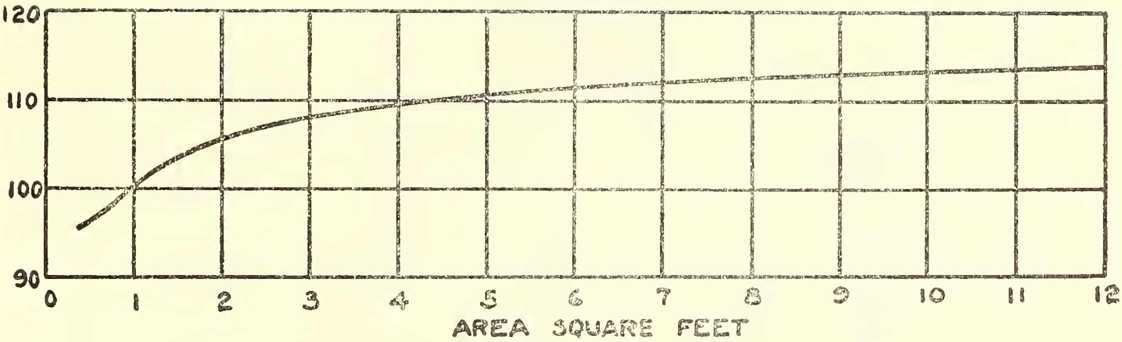
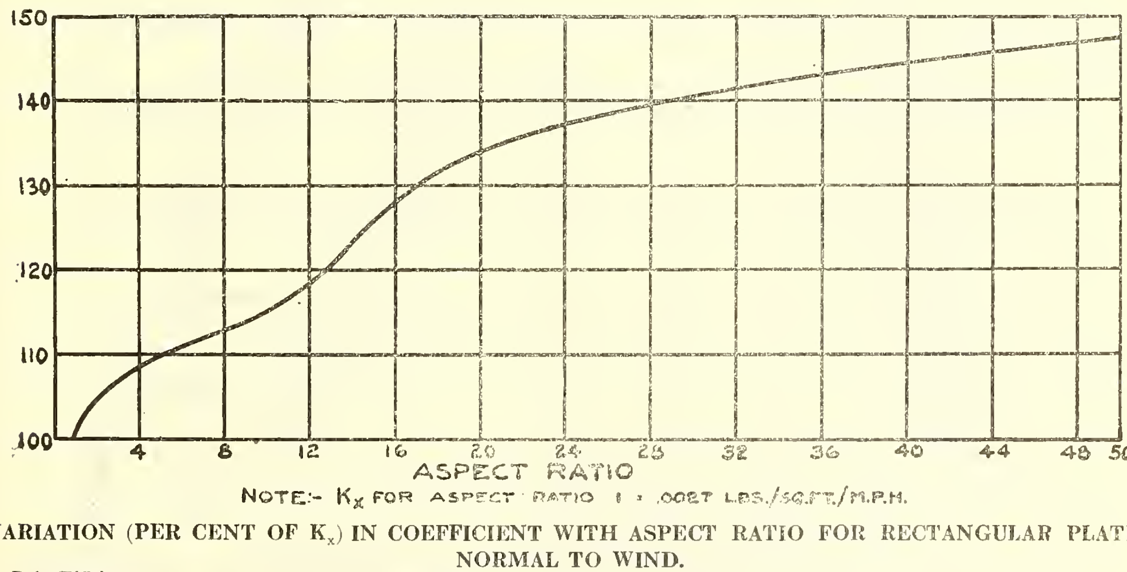
AIRCRAFT DESIGN DATA. NOTE NO. 9.

AEROFOILS.

I.—FLAT PLATES.



VARIATION (PER CENT OF K_x) IN COEFFICIENT WITH AREA FOR SQUARE PLATES NORMAL TO WIND



VARIATION (PER CENT OF K_x) IN COEFFICIENT WITH ASPECT RATIO FOR RECTANGULAR PLATES NORMAL TO WIND.

Ref.: Eiffel.

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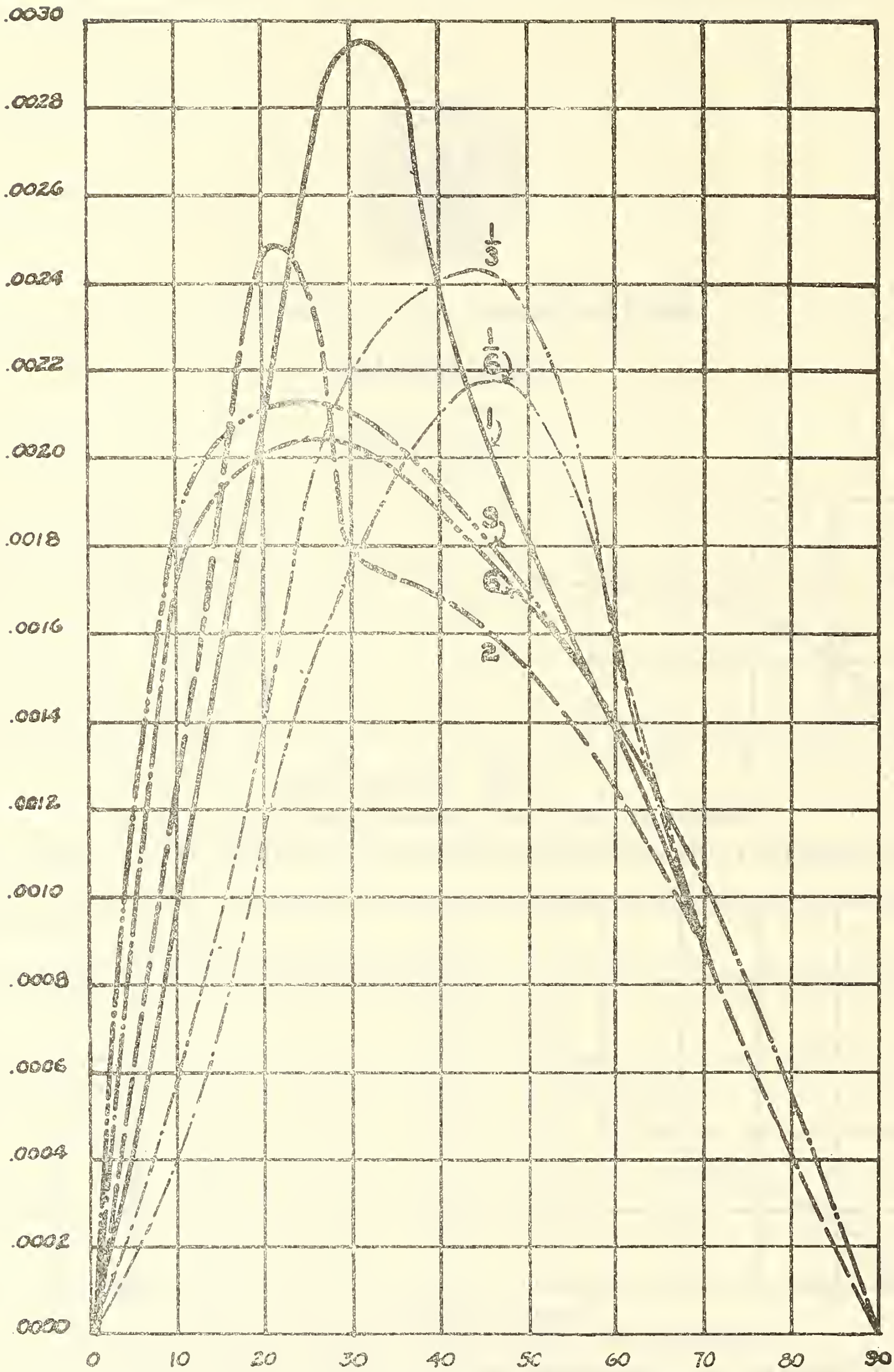
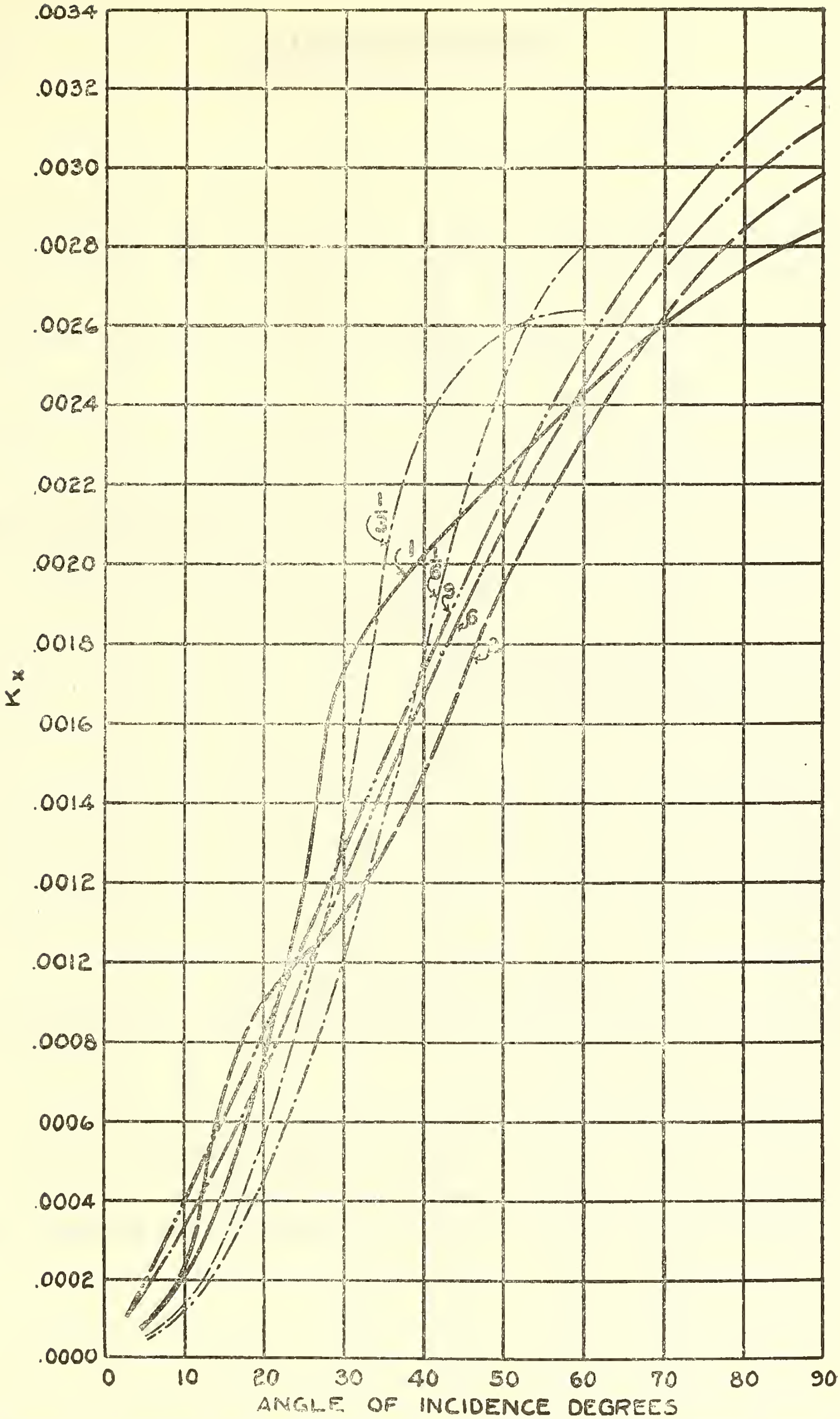
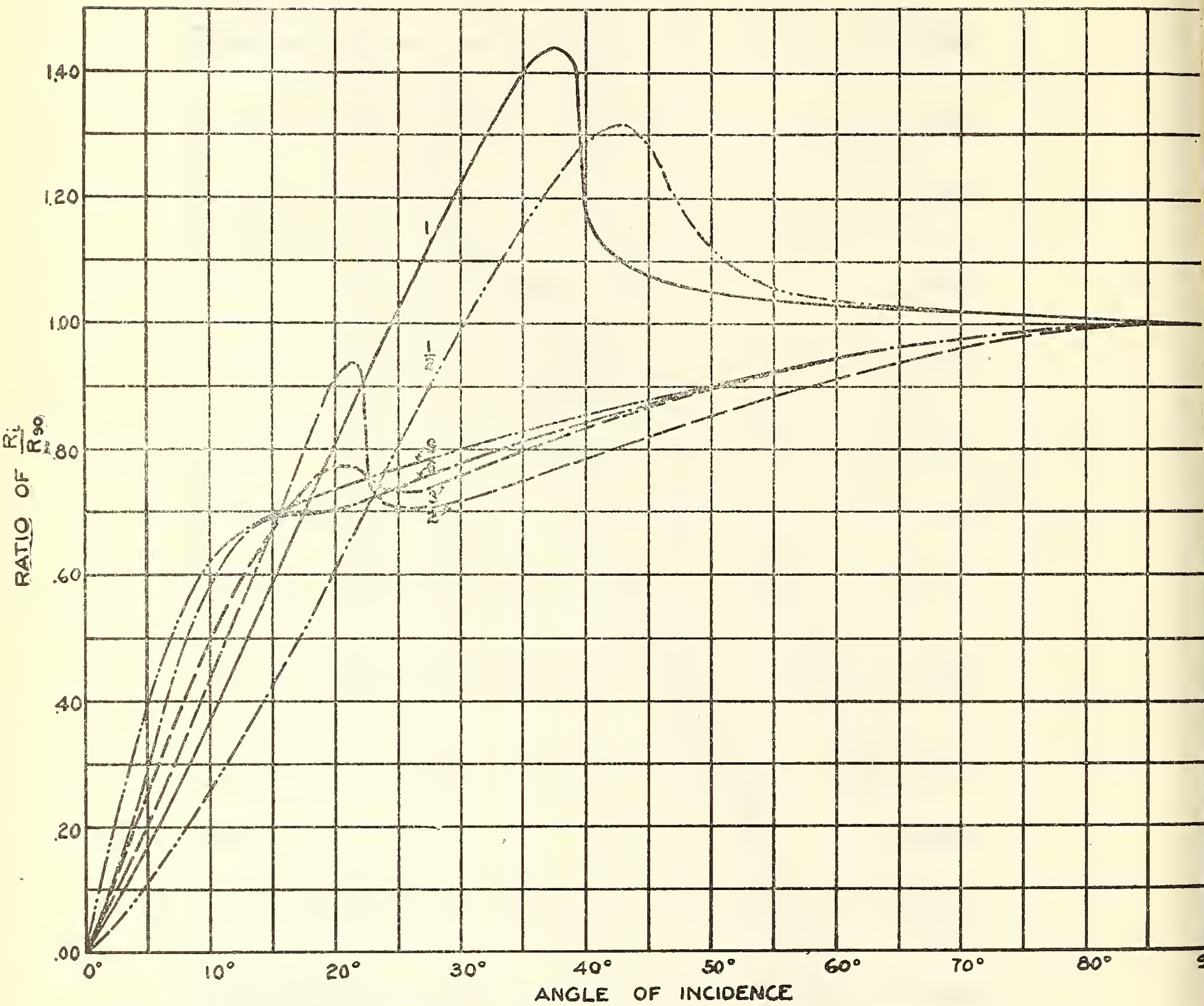


FLAT PLATES.—K_y—POUNDS PER FOOT PER M. P. H. FOR DIFFERENT ASPECT RATIOS.

Ref.: Eiffel.

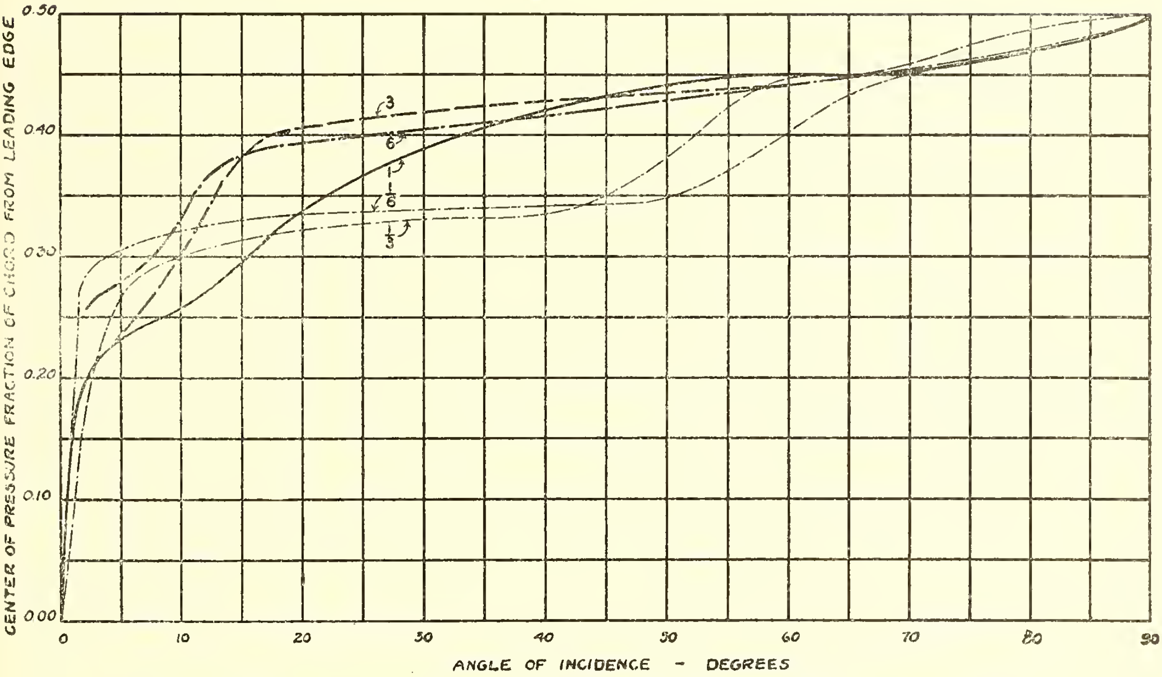


FLAT PLATES.—K_x—POUNDS PER SQUARE FOOT PER M. P. H. FOR DIFFERENT ASPECT RATIOS.
Ref.: Eiffel.



REACTION ON FLAT PLATES—PER CENT OF REACTION AT 90 DEGREES FOR DIFFERENT ASPECT RATIOS.

Ref.: Eiffel.



CENTER OF PRESSURE—FLAT PLATE—FOR DIFFERENT ASPECT RATIOS.

Ref.: Eiffel

II.—CORRECTIONS TO MODEL TESTS.

In carrying out performance calculations as in a preliminary selection incident to a new design, it is necessary to obtain values of lift and drag coefficients which apply to the particular wing combination used. In most cases the only available data is from wind-tunnel tests on monoplane models. To make the results of these tests apply to the full-size machine the following corrections are necessary:

1. Biplane or triplane and gap-chord ratio.
2. Stagger.
3. Aspect ratio and overhang.
4. Plan form of wing tips.
5. Aileron leakage.
6. Scale, or LV.

(It is to be noted that a majority of the tests from which the various correction coefficients have been deduced were made on one wing section—the R. A. F. 6. These corrections are therefore approximate, but are approximations based on all available experimental data)

Biplane or triplane and gap-chord ratio corrections.—The first set of curves, figures 1–3, converts the monoplane values into biplane or triplane values for a gap-chord ratio of 1.2. The second set, figures 4 and 5, makes the correction to the desired gap-chord ratio. These corrections are interconnected, since the interference is a function of the gap-chord ratio.

Stagger.—Positive stagger, i. e., the top wing projecting forward of the lower, is used chiefly on account of improved vision. The use of negative stagger is limited. The corrections for various degrees of stagger for various angles of incidence are given.

The effect of stagger, being connected with interference and down-wash, will depend upon the gap-chord ratio, but no tests have been made showing to what extent. However, the correction is never large and the curves may be considered reliable for use with any reasonable gap-chord ratio.

Aspect ratio.—This correction is from a twofold effect. Increasing the aspect ratio:

1. Decreases drag by decreasing the effect of end losses.
2. Increases lift at small angles by shifting the lift curve so that the maximum occurs at a smaller angle.

The effect of aspect ratio varies somewhat erratically with wing section and angle; therefore, for each ratio an average correction which applies to all angles is given.

In no case does any value vary more than a few per cent from these averages, which are plotted so that there is no correction for an aspect ratio of six—the standard value used in wind-tunnel tests.

For lack of accurate data on overhang in a biplane or triplane, it is considered that the mean aspect ratio takes into account any necessary correction for overhang.

Plan form of wing tips.—When rounded or raked tips are used, there is an aerodynamic improvement due to a reduction of end losses, since the lines of air flow are crowded toward the center of the span. This improvement will vary with the particular plan form or wing section used, but the most reliable tests show that for rounded tips the effect is to increase lift at all angles and to decrease drag at small angles.

Aileron leakage.—With the usual types of construction there is a leakage at the aileron hinge joint, causing a decrease in lift and an increase in drag over that portion of the wing containing the aileron. The correction amounts to about 3 per cent for lift-drag in most cases.

The curves show this correction plotted against the ratio of sum of aileron spans to sum of wing spans, so that, given the spans of ailerons and of wings, the correction may be read off directly.

Scale or LV correction.—Denoting the wing chord (in feet) by L and the wind velocity (in miles per hour) by V, model tests have a low value of LV, usually less than 10, while machines in flight have a high value, usually greater than 200. If the LV of the model test is less than 30, a correction must be made in applying the results to a full-size machine.

Within the limits yet tested, no correction is required beyond an LV of about 30. This gives a single correction to a model test sufficient to make the results hold true for all full-size machines.

The curves show this correction at various angles of incidence plotted against the LV of the model test. To use these curves, the size of the model and the wind velocity of the test must be known. The standard values of LV are indicated on the curves by light vertical lines.

1. BIPLANE AND TRIPLANE COEFFICIENTS.

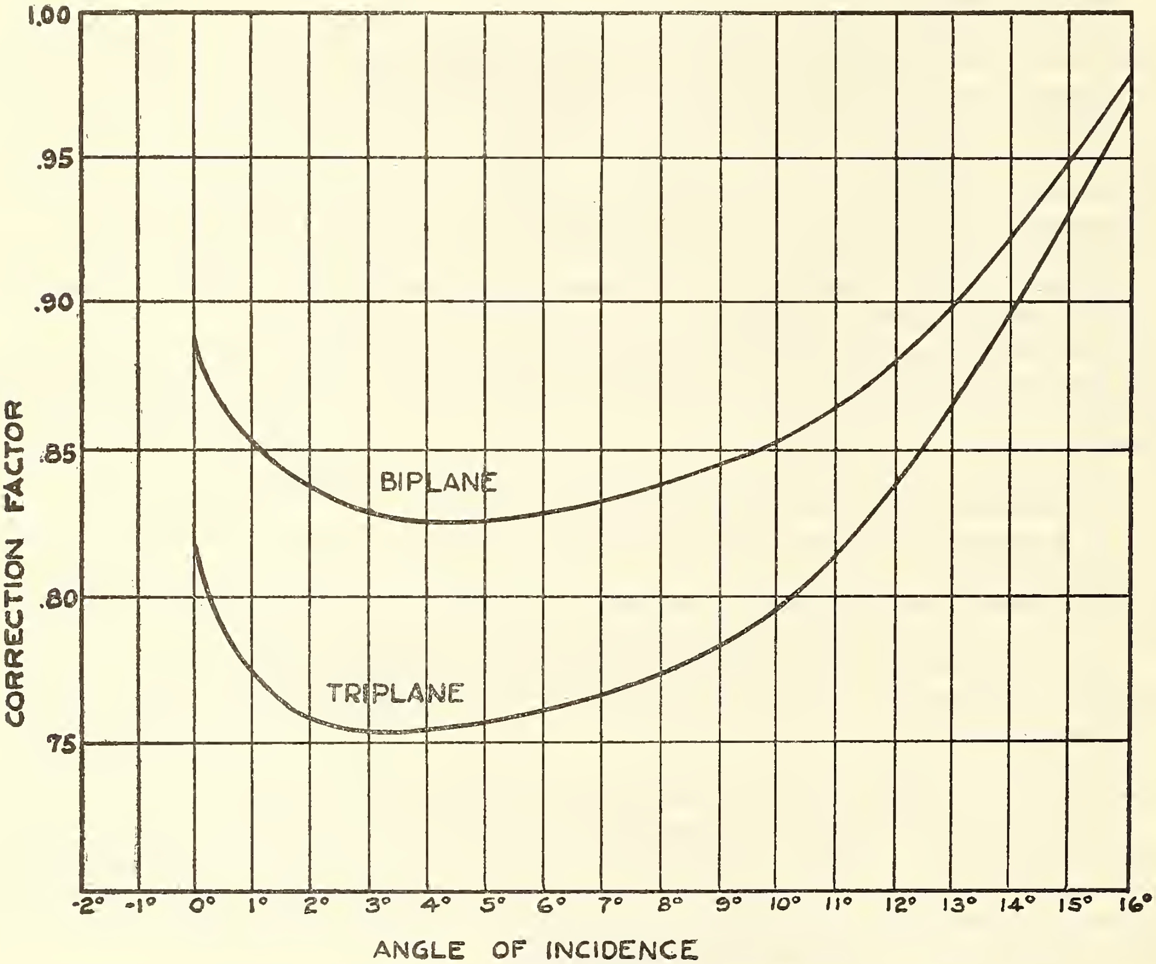


Fig. 1.—K_y correction factors to be used to obtain biplane and triplane values from monopl ne tests.

NOTE.—Curves are drawn for aspect ratio 6; for gap-chord ratio 1.2. Take corrections for any other G:C ratio from G:C ratio curves.

Ref.: Hunsaker; A. C. A.; Eiffel.

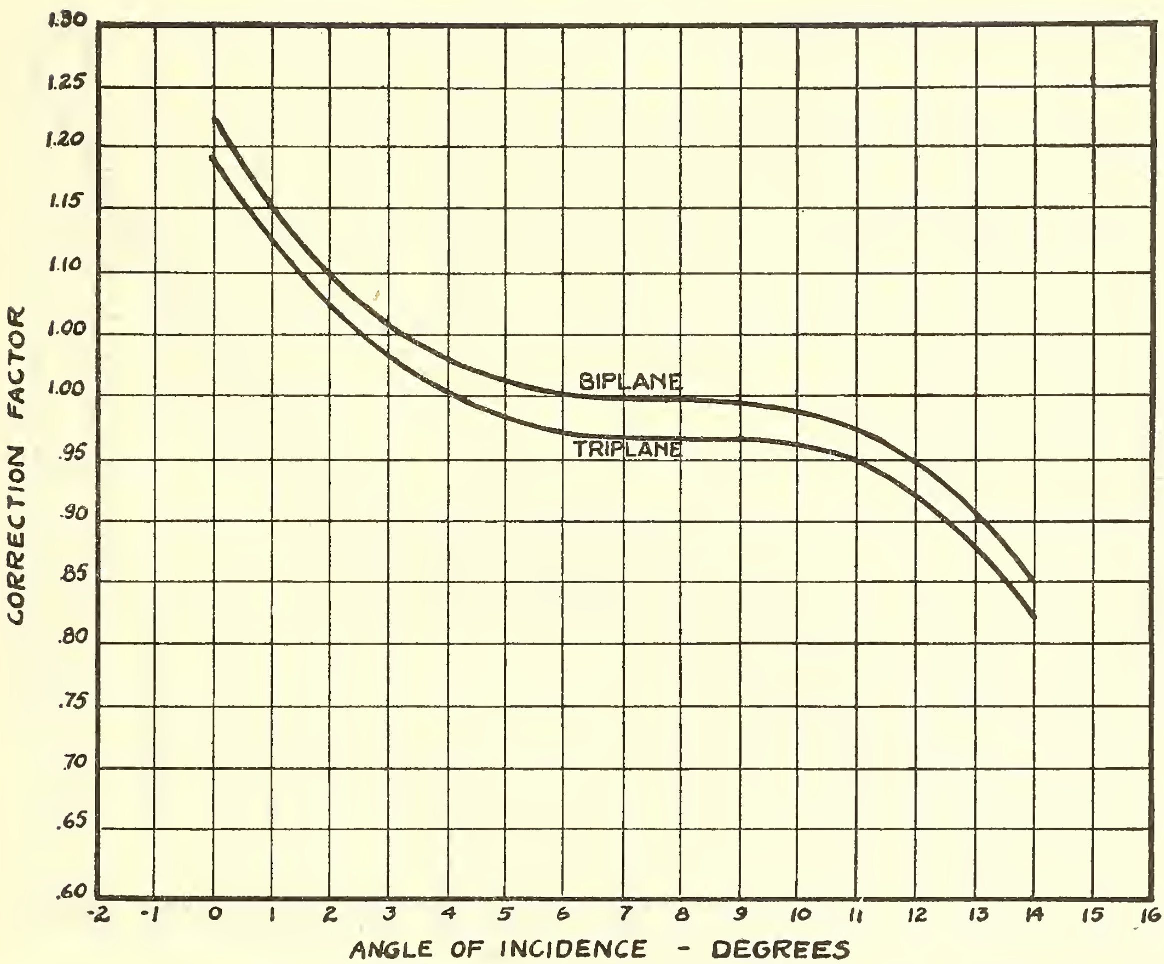


Fig. 2—K_x correction factors to be used to obtain biplane and triplane values from monoplane tests.

NOTE.—Curves are drawn for aspect ratio 6; for gap-chord ratio 1.2.

Ref.: Hunsaker; A. C. A.; Eiffel.

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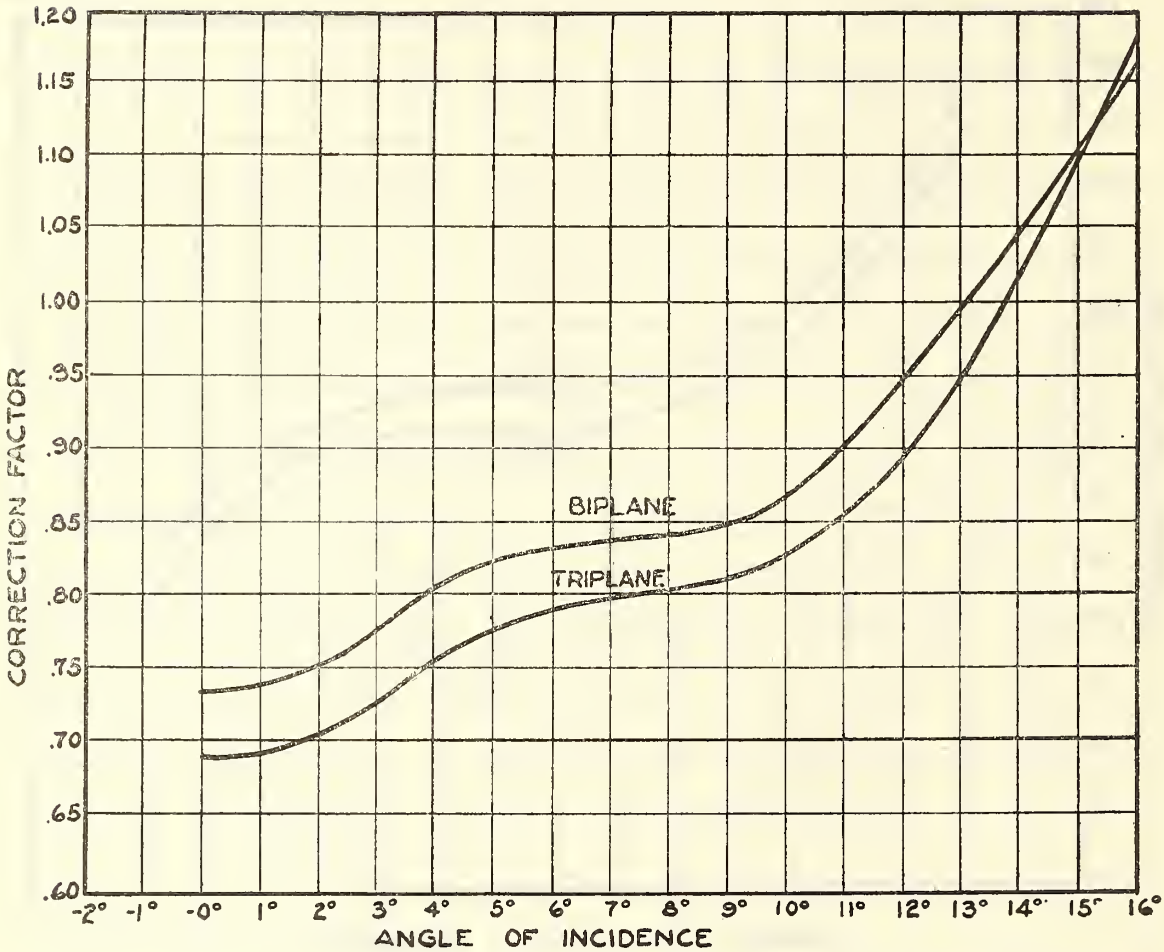


Fig. 3.—$\frac{L}{D}$ correction factors to be used to obtain biplane and triplane values from monoplane tests.

NOTE.—Curves are drawn for aspect ratio 6; for gap-chord ratio 1.2. Take correction for any other G:C ratio from G:C ratio curves.

Ref.: Hunsaker; A. C. A.; Eiffel.

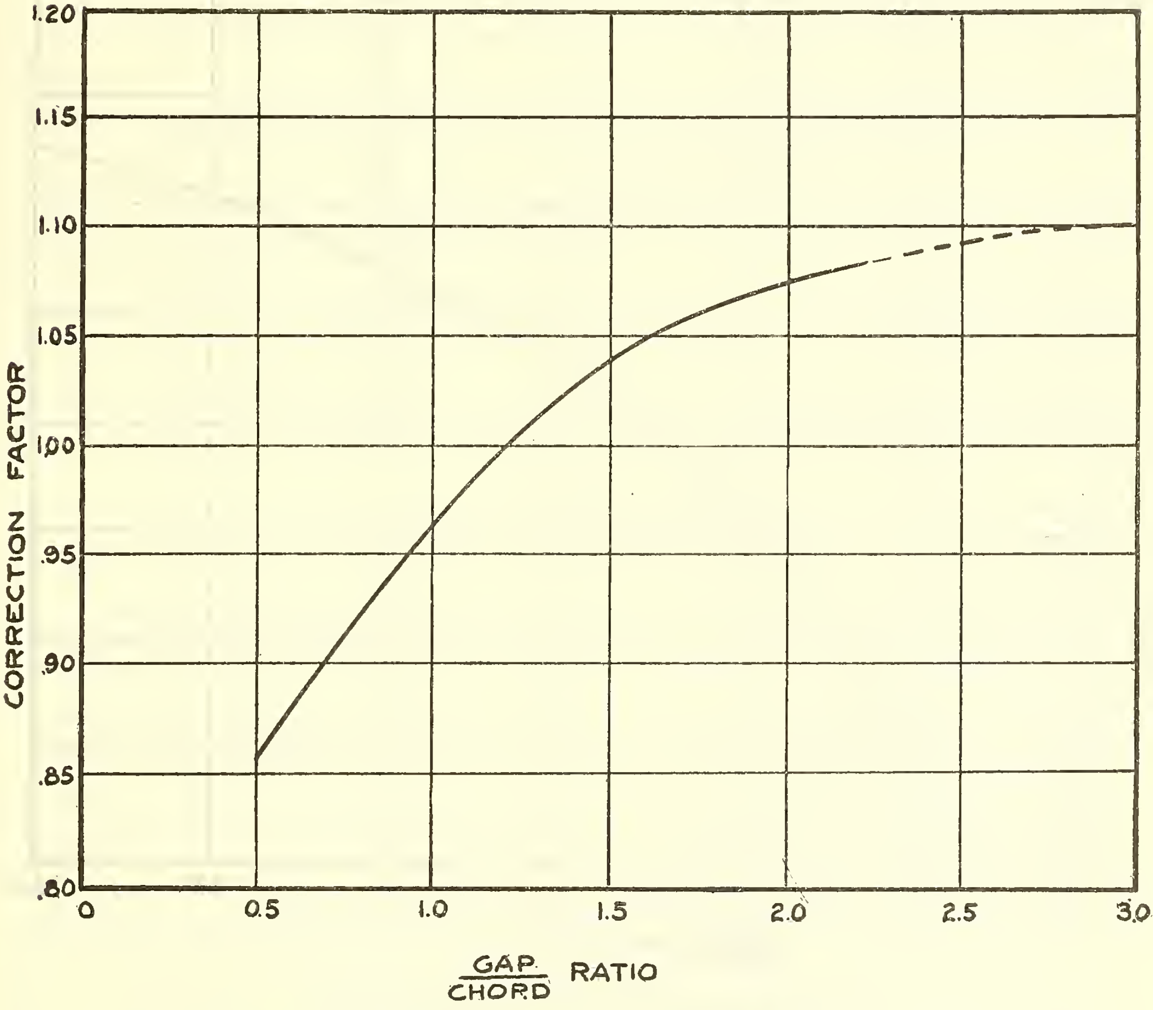


Fig. 4.—K_y correction factors for gap-chord ratio.

Ref.: A. C. A.

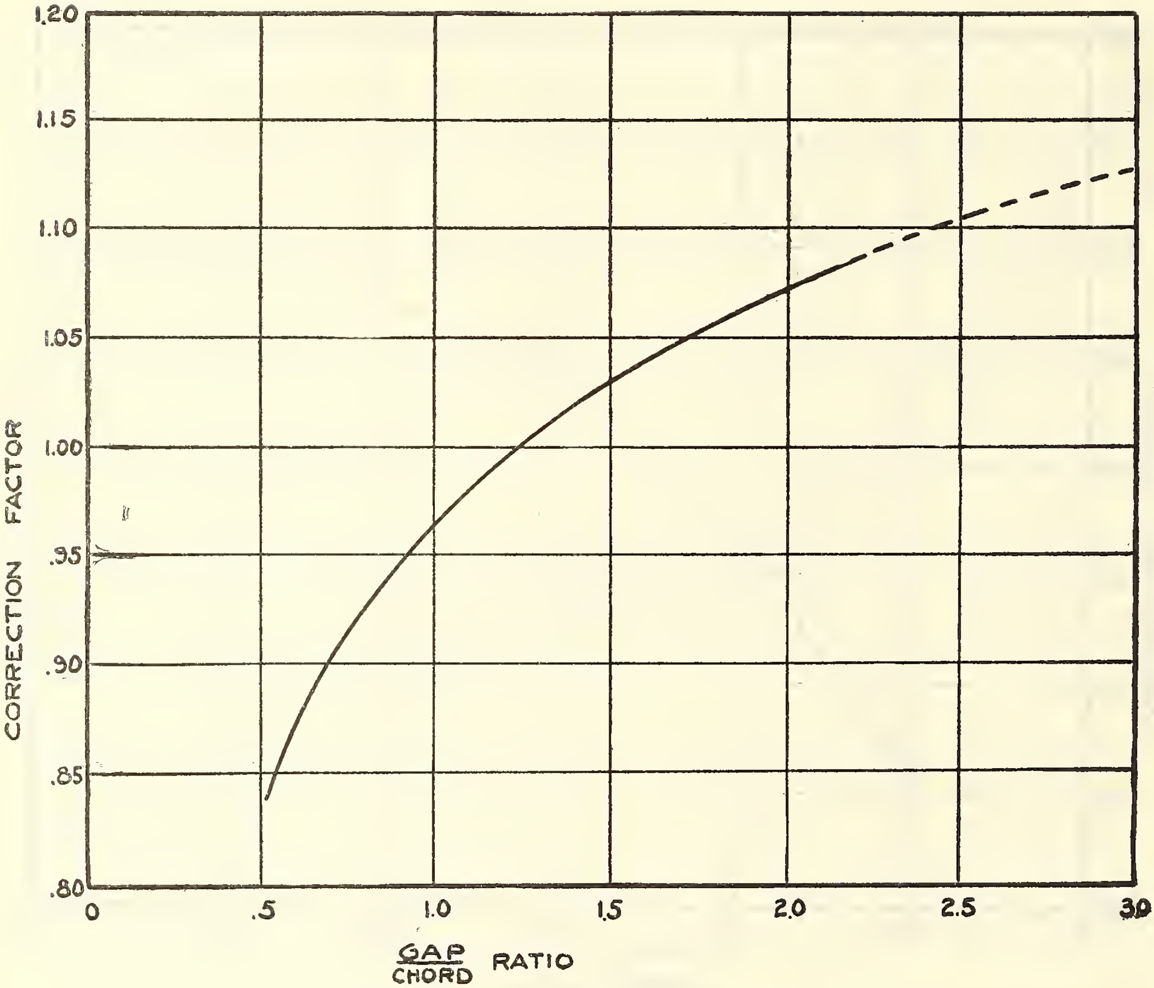


Fig. 5.—$\frac{L}{D}$ correction factors for gap-chord ratios.

Ref.: A. C. A.

2. STAGGER CORRECTION COEFFICIENTS.

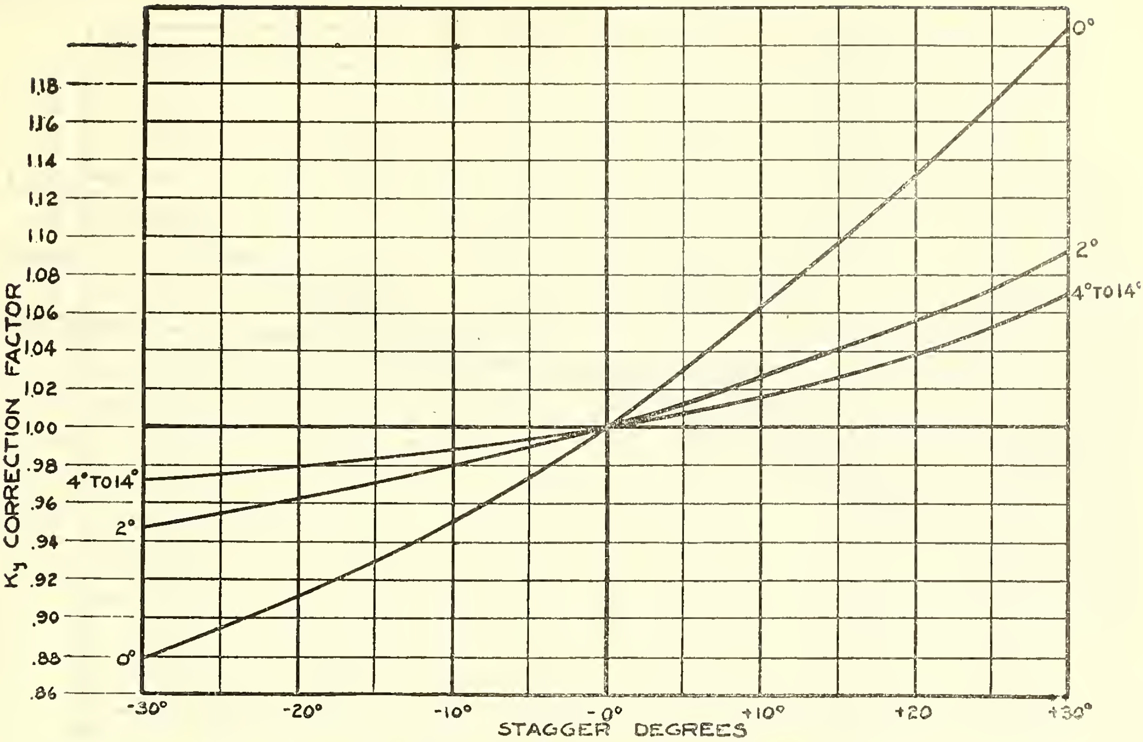


Fig. 1.—K_y correction factors for stagger biplane.

Ref.: A. C. A.; Eiffel.

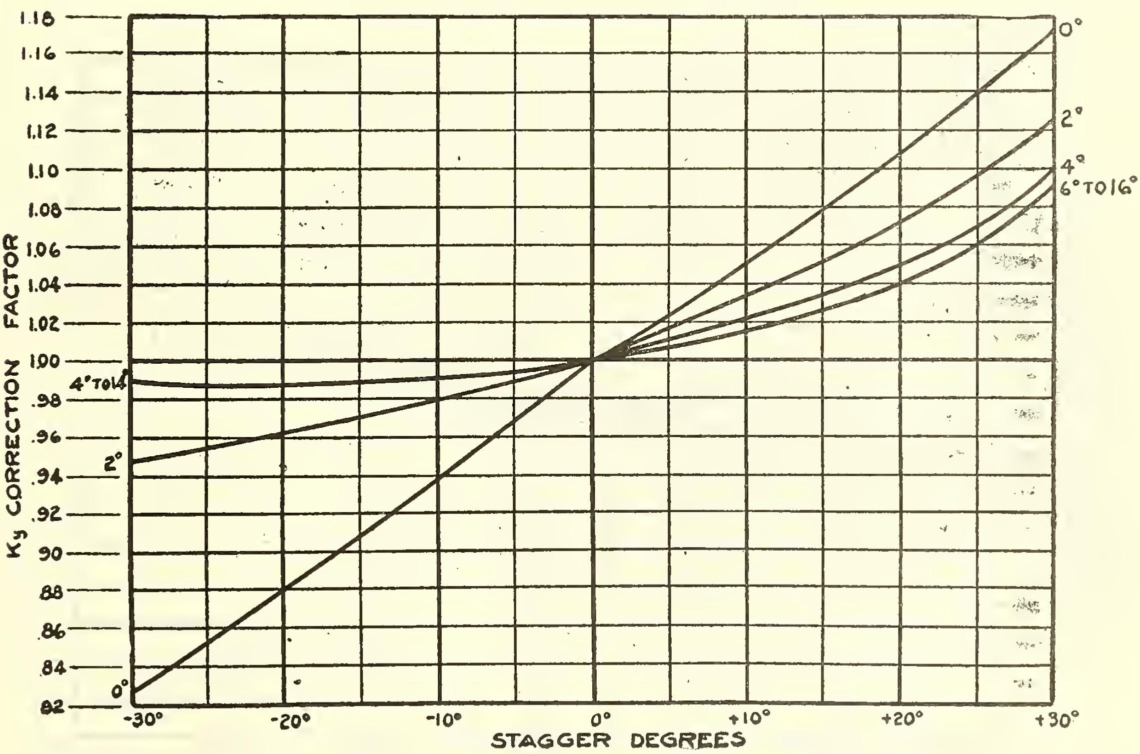


Fig. 2.—K_y correction factors for stagger triplane.

Ref.: A. C. A.; Eiffel.

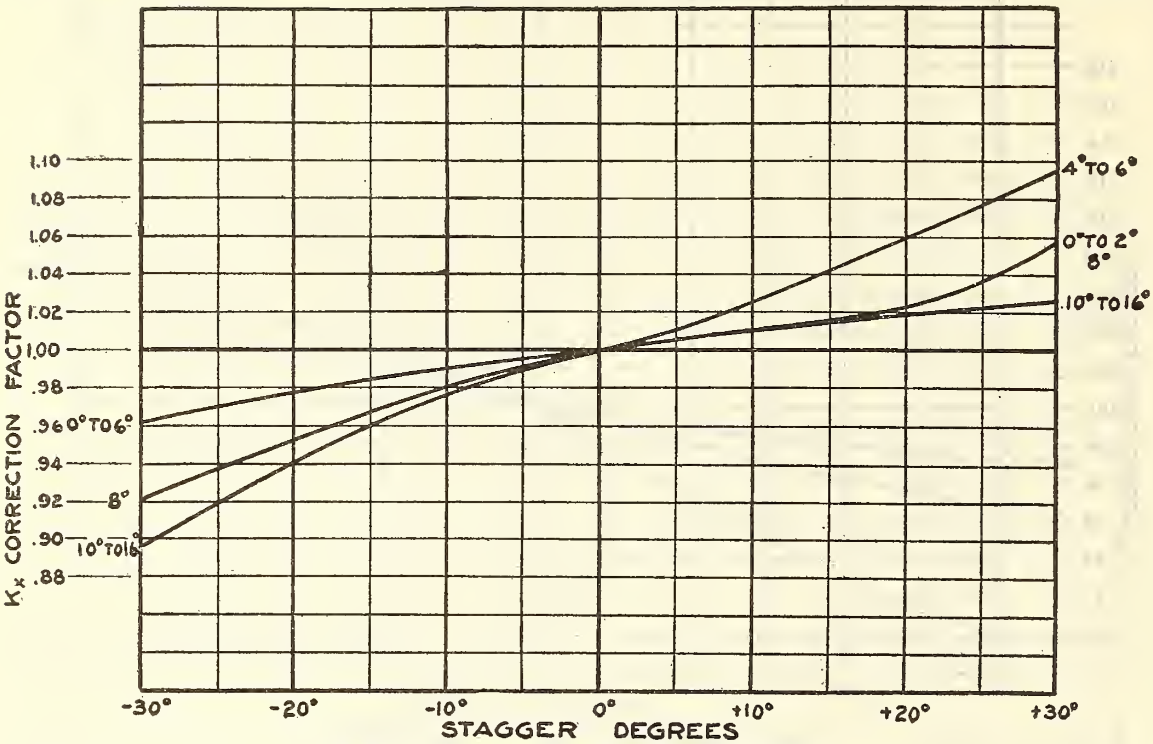


Fig. 3.—K_x correction factors for stagger biplane.

Ref.: Hunsaker; A.; C. A.; Eiffel.

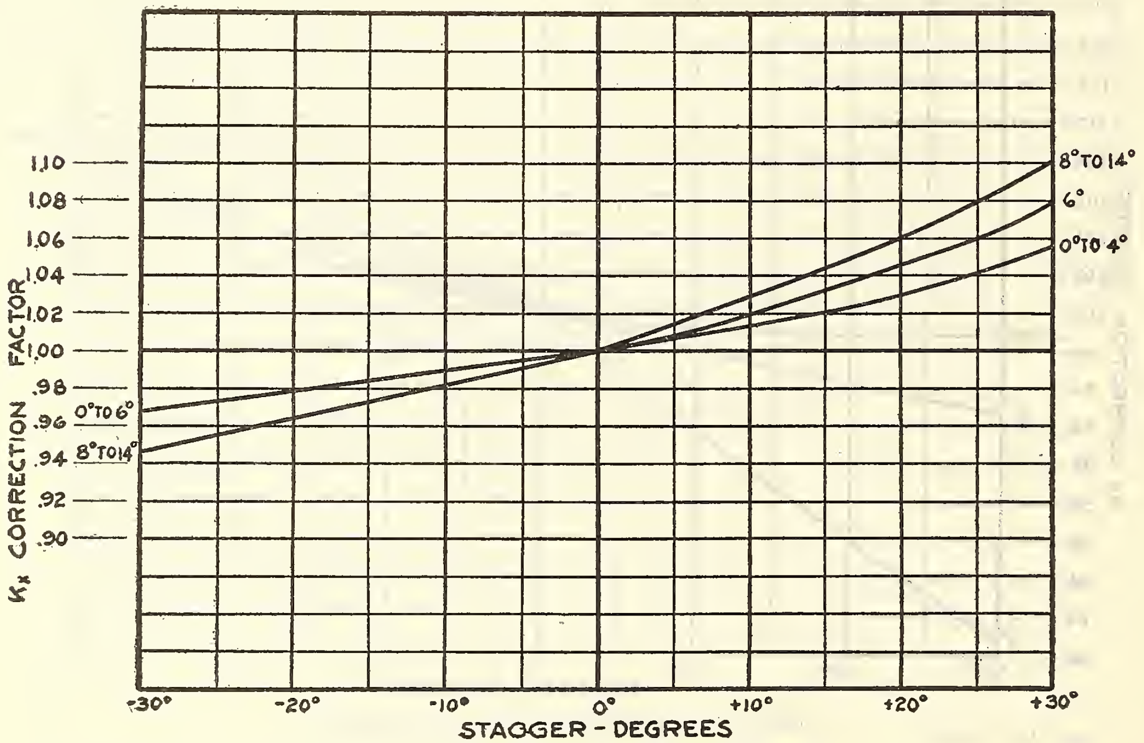


Fig. 4.—K_x correction factors for stagger triplane.

Ref.: Hunsaker; A. C. A.; Eiffel.

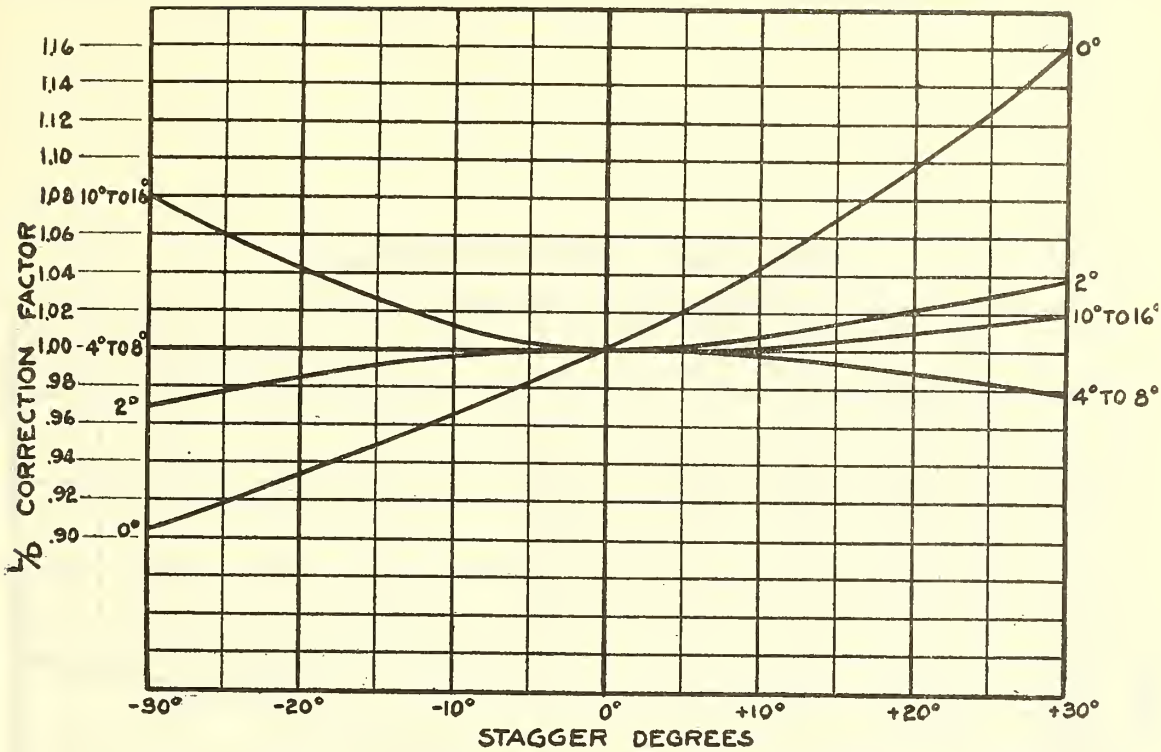


Fig. 5.—$\frac{L}{D}$ correction factors for stagger biplane.

Ref.: A. C. A.; Eiffel.

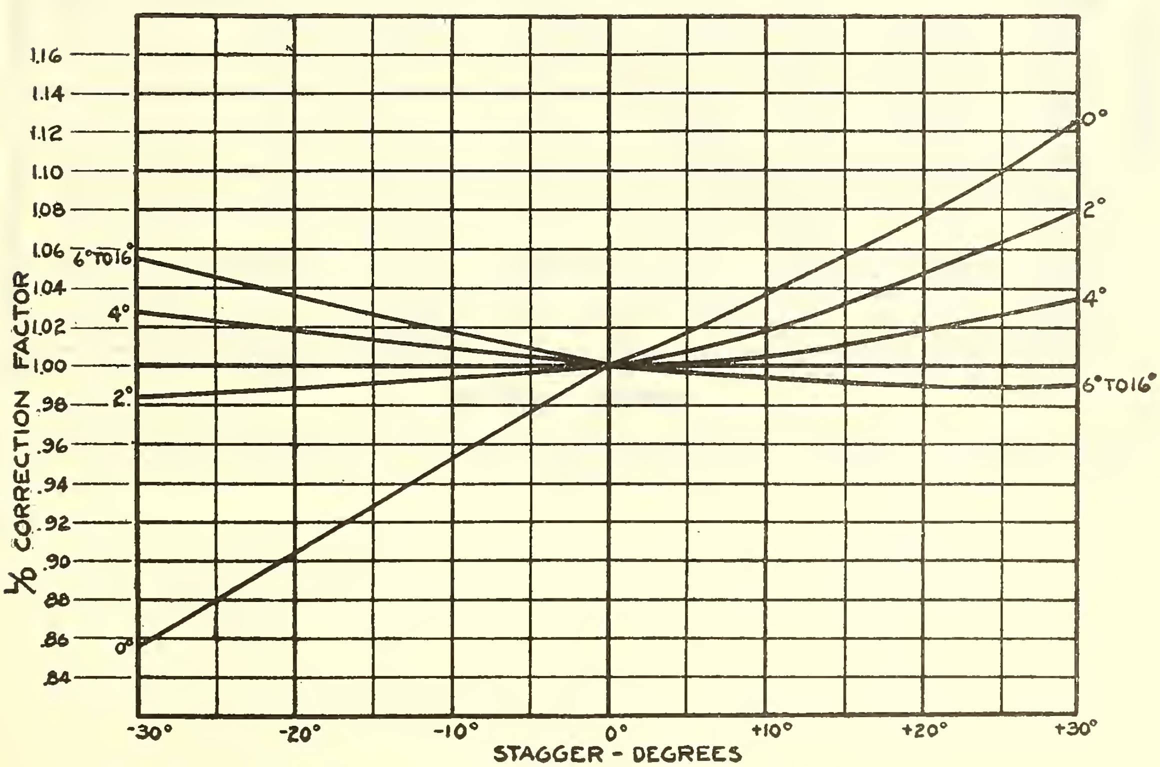


Fig. 6.—$\frac{L}{D}$ correction factors for stagger triplane.

Ref.: A. C. A.; Eiffel.

3. ASPECT RATIO CORRECTION COEFFICIENTS.

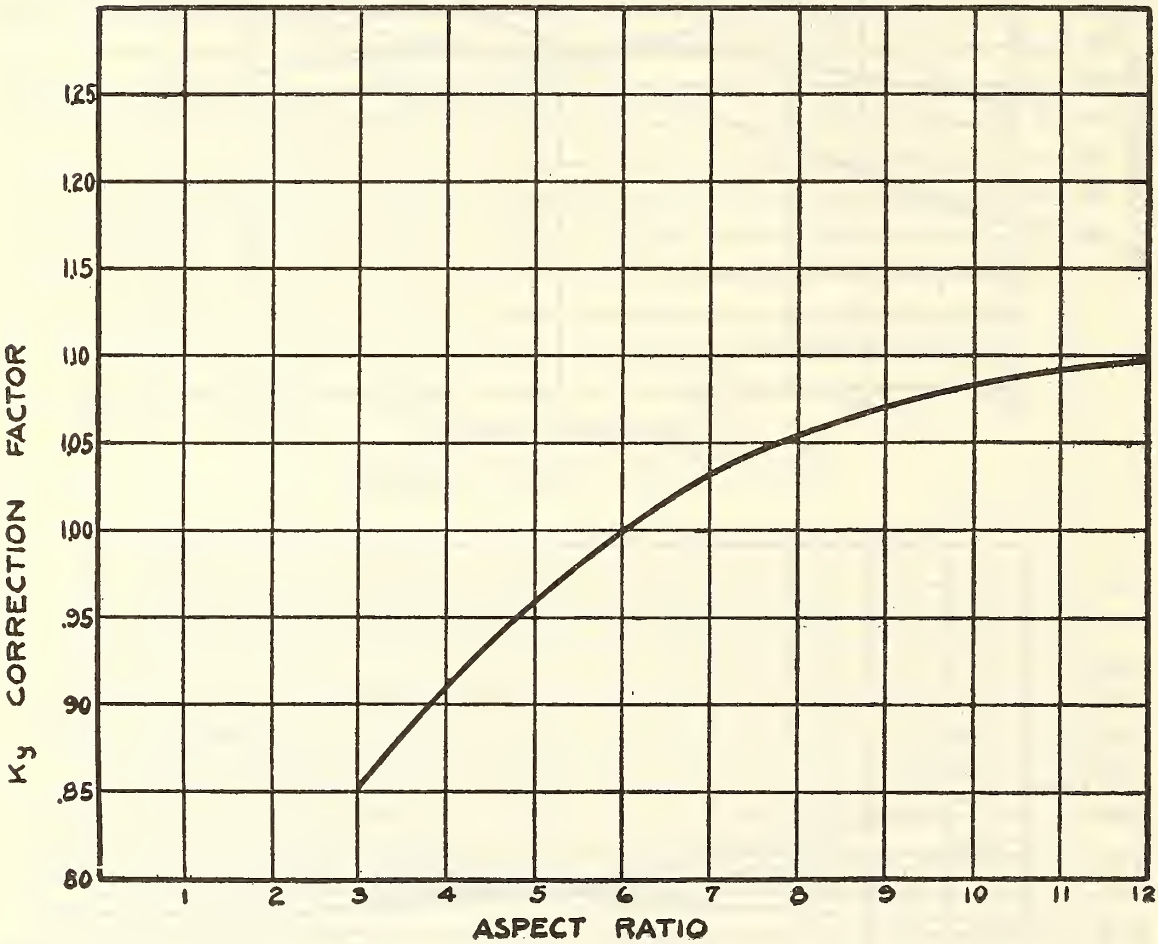


Fig. 1.—Aspect ratio correction factors, biplane and triplane, to be applied to monoplane tests with aspect ratio 6.

Ref.: A. C. A.

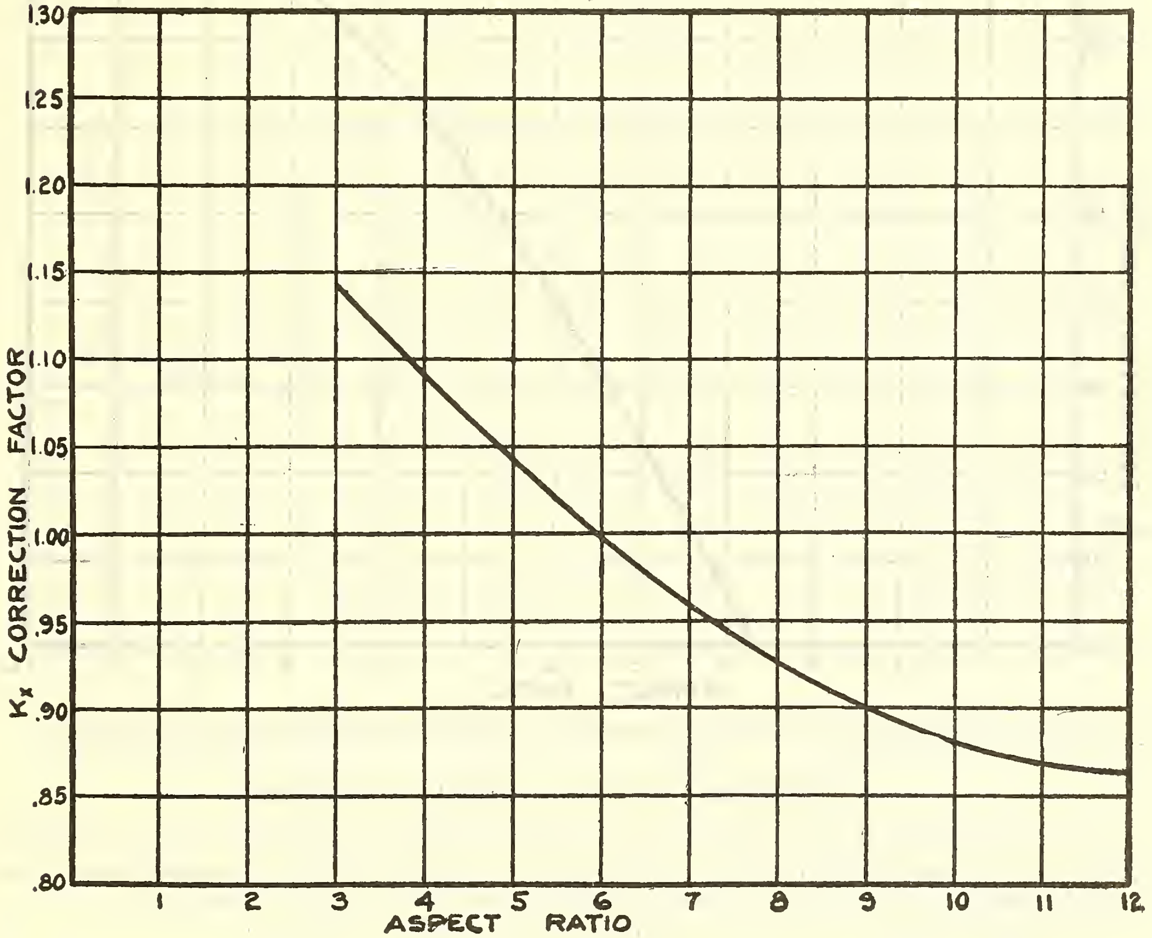


Fig. 2.—Aspect ratio correction factors, biplane and triplane, to be applied to monoplane tests with aspect ratio 6.

Ref.: A. C. A.

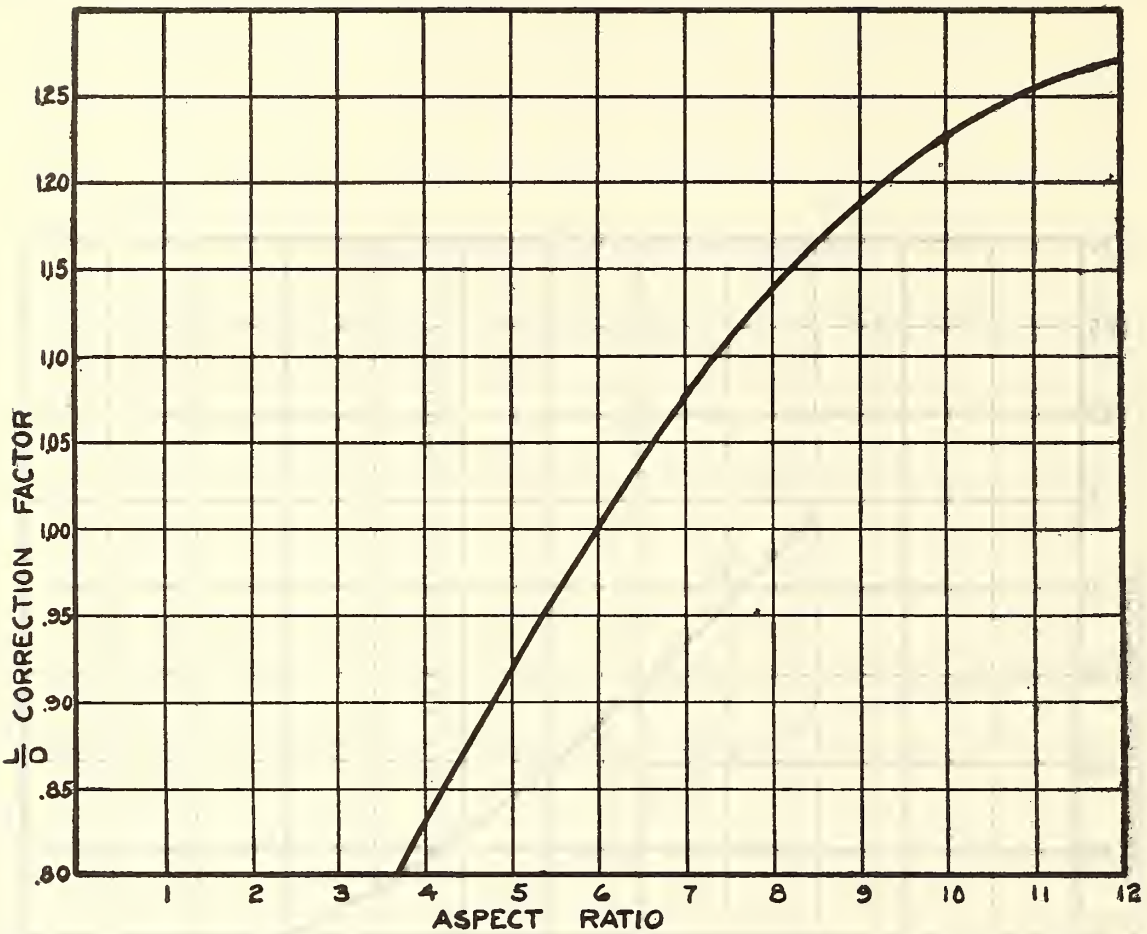


Fig. 3.—Aspect ratio correction factors, biplane and triplane, to be applied to monoplane tests with aspect ratio 6.

Ref.: A. C. A.

4. CORRECTIONS FOR PLAN FORM OF WING TIPS.

The most reliable data on this subject is from a series of comparative tests on models, having rounded tips of elliptical form. The results show that the lift is increased about 4 per cent at all angles, but that the drag is not affected appreciably except at small angles.

Denoting the aspect ratio by A. R., the corrections may be expressed:

$$K'_y = \left(1 + \frac{0.24}{A.R.}\right) K_y$$

$$K'_x = K_x - \frac{.000009}{A.R.}$$

where the primes indicate the corrected values of the coefficients. The corrected lift drag ratio may be obtained from corrected lift and drag.

The values given refer to a R. A. F. 6 section with rounded tips, but are to be used for any similar section having either rounded or raked tips.

Ref.: A. C. A.; Eiffel.

5. AILERON LEAKAGE CORRECTION COEFFICIENTS.

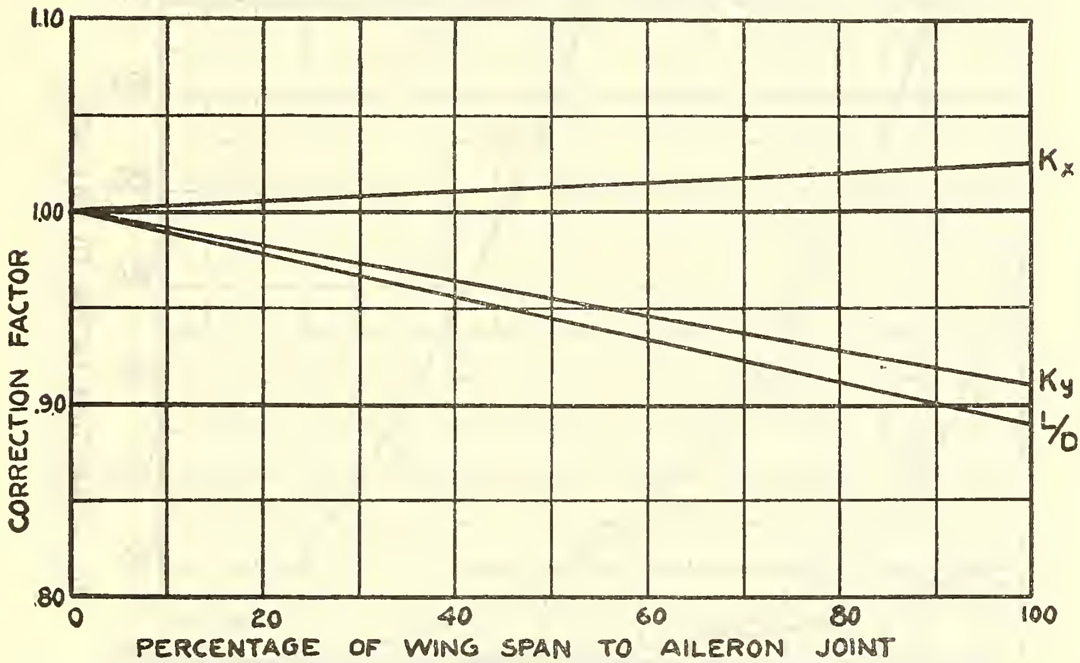


Fig. 1.—Correction for losses due to leakage through aileron hinges (of usual construction) at all flying angles.

NOTE.—For biplanes and triplanes use sum of wing spans and sum of aileron lengths to find percentage.

Ref.: A. C. A.

6. SCALE CORRECTION COEFFICIENTS.

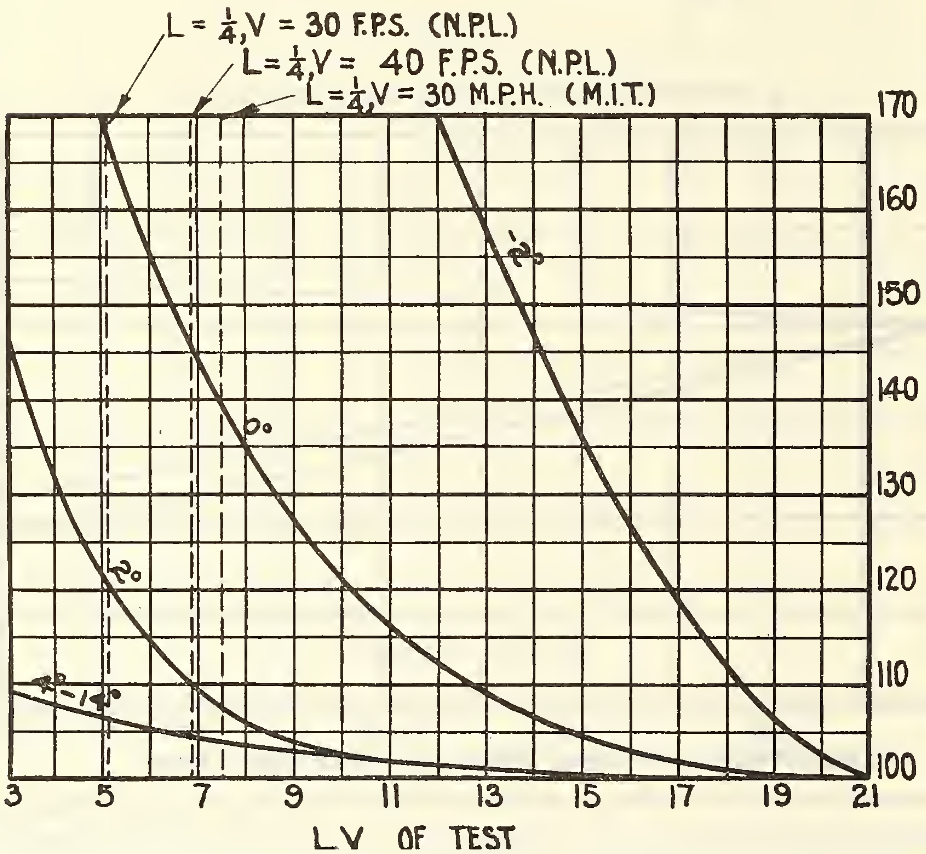


Fig. 1.—Correction for scale effect to be used in applying results of wind tunnel tests on models to full-size aerofoils.

L=Chord of model in feet.
V=Velocity of wind in test, M. P. H.

Ref.: A. C. A.

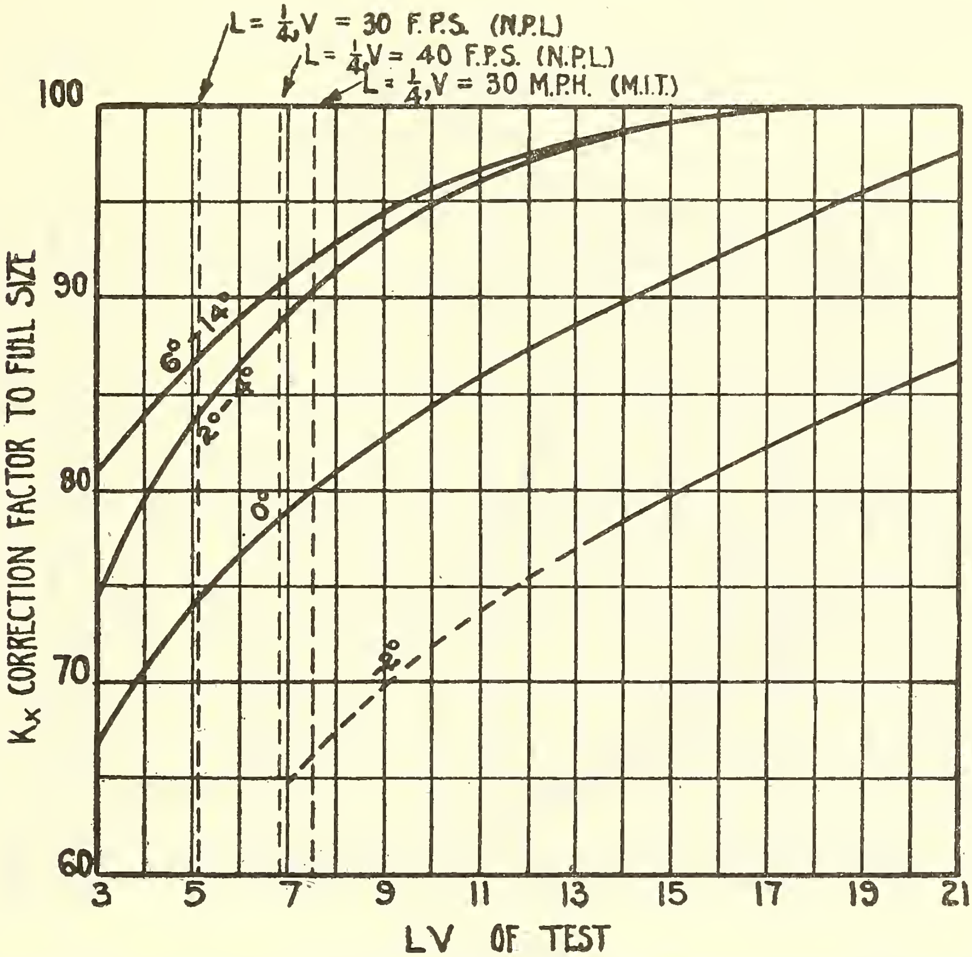


Fig. 2.—Correction for scale effect to be used in applying results of wind tunnel tests on models to full-size areofoils.

L=Chord of model in feet.
V=Velocity of wind in test, M. P. H.

Ref.: A. C. A.

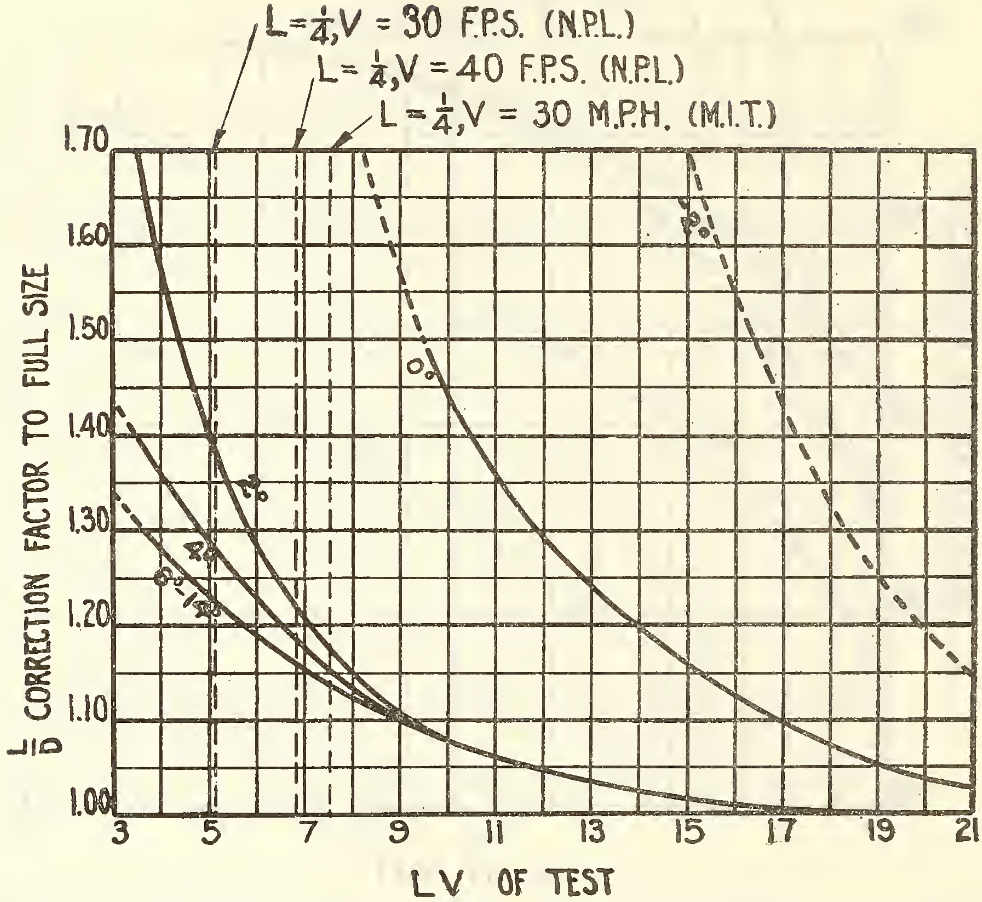


Fig. 3.—Correction of scale effect to be used in applying results of wind tunnel tests on models to full-size aerofoils.

L=Chord of model in feet.
V=Velocity of wind in test, M. P. H.

Ref.: A. C. A.

III.—LOAD DISTRIBUTION ALONG THE CHORD.

The lift load taken by the wings depends upon the weight of the airplane. The load normal to the chord varies as the angle of incidence, and is equal to the weight of the airplane divided by the cosine of the angle of incidence. This normal load then is greater at high angles of incidence corresponding to slow speed than at low angles corresponding to high speed, and in addition the distribution of this load along the chord varies with the angle of incidence.

At high speed the location of the center of pressure is such that the rear spar takes the greater share of the load, while at low speed the front spar takes the greater share.

On plate No. 1 is shown the distribution for R. A. F. 6 and R. A. F. 15 wing sections. The mean curve of both is shown by the heavy line. The areas under the curves represent total load. Areas above the chord line are positive and those below are negative. For high speed the actual net load is equal to the area above the chord line minus the area below the chord line.

On Plate No. 2 is shown the load, shear, and bending moment curves for load distribution at high speed. The negative load at the leading edge would make a strength test of the wing rib tedious and difficult. A new straight line load curve shown by the dotted line omits this negative load on the nose. As shown by the shear and bending moment curves the maximum moments between spars and also those at the rear spar are practically equal for both loads. The location of the maximum bending moment between spars is not identical for both loads, but the difference is so small that it is not appreciable in the results of a wing rib test. The area of the approximate loading is greater than that of the actual loading, the ratio being 1.31 to 1. The only appreciable difference in the resultant curves is that the approximate load gives a greater maximum shear at the front spar. However, at low speed the distribution gives a shear at the front spar much greater than this given by the approximate load. Since the rib must also be designed to take the low speed load the additional shear on the front spar imposed by the approximate loading will not affect the strength determination of the rib.

Plate No. 2 is for the average spar spacing, i. e., front spar 12.5 per cent of chord length from leading edge and rear spar 70 per cent of chord length from leading edge.

The low-speed angle in plate No. 1 is given as 12°. The low-speed angle is very often higher than this, but the distribution of pressure is so nearly the same for all low speeds that for rib testing the distribution for 12° may be used for all low speeds and corresponding angles of incidence. The same holds true with respect to high speeds for the distribution at 0°.

EXAMPLE OF METHOD OF USING LOADING CURVES IN MAKING A STRENGTH TEST OF A WING RIB.

I. LOW SPEED TEST.

Given

Chord $=C$ (ft.).

Lift load per square foot $=w$ (lbs.).

Maximum rib spacing $=s$ (ft.).

Angle of incidence $=i$ (°).

Then

Area supported by one rib $=C\times s$ (sq. ft.).

Normal load on rib per square foot $=\frac{w}{\cos i^\circ}$.

Total normal load on rib $= \frac{w}{\cos i^\circ} \times Cs.$

Factor of safety $= \text{breaking load} \div \frac{w}{\cos i^\circ} \times Cs$

Chord C is laid off to any convenient scale. From Plate No. 3 the load curve is plotted (plate No. 4). The ordinates are drawn to any scale. The area A under the load curve is found by a planimeter or if plotted on cross section paper by counting the squares. This area will represent the normal unit load

$$= \frac{wCs}{\cos i^\circ}$$

The points of application of the load having been previously decided upon the share of the load taken at each point is determined by the areas *a*, *b*, *c*, *d*, etc. The load at point

(1) $= \frac{a}{A} \times \frac{wCs}{\cos i^\circ}$

(2) $= \frac{b}{A} \times \frac{wCs}{\cos i^\circ}$

etc.

II. HIGH SPEED TEST.

(*a*) If the actual load distribution is used, the procedure is exactly as shown for case 1. Area A equals the area above the chord minus the area below the chord. If the negative area at the nose equals minus *a* $(-a)$, then the load which must be applied to represent this area

$$= -\frac{a}{A} \times \frac{wCs}{\cos i^\circ}.$$

(*b*) If the approximate load distribution is used the procedure is as above except that the total normal load applied to the rib is equal to

$$\frac{w}{\cos i^\circ} \times Cs \times 1.31.$$

The factor of safety by the approximate loading

$$= \text{breaking load} \div \frac{w}{\cos i^\circ} \times Cs \times 1.31.$$

Ref.: A. C. A.

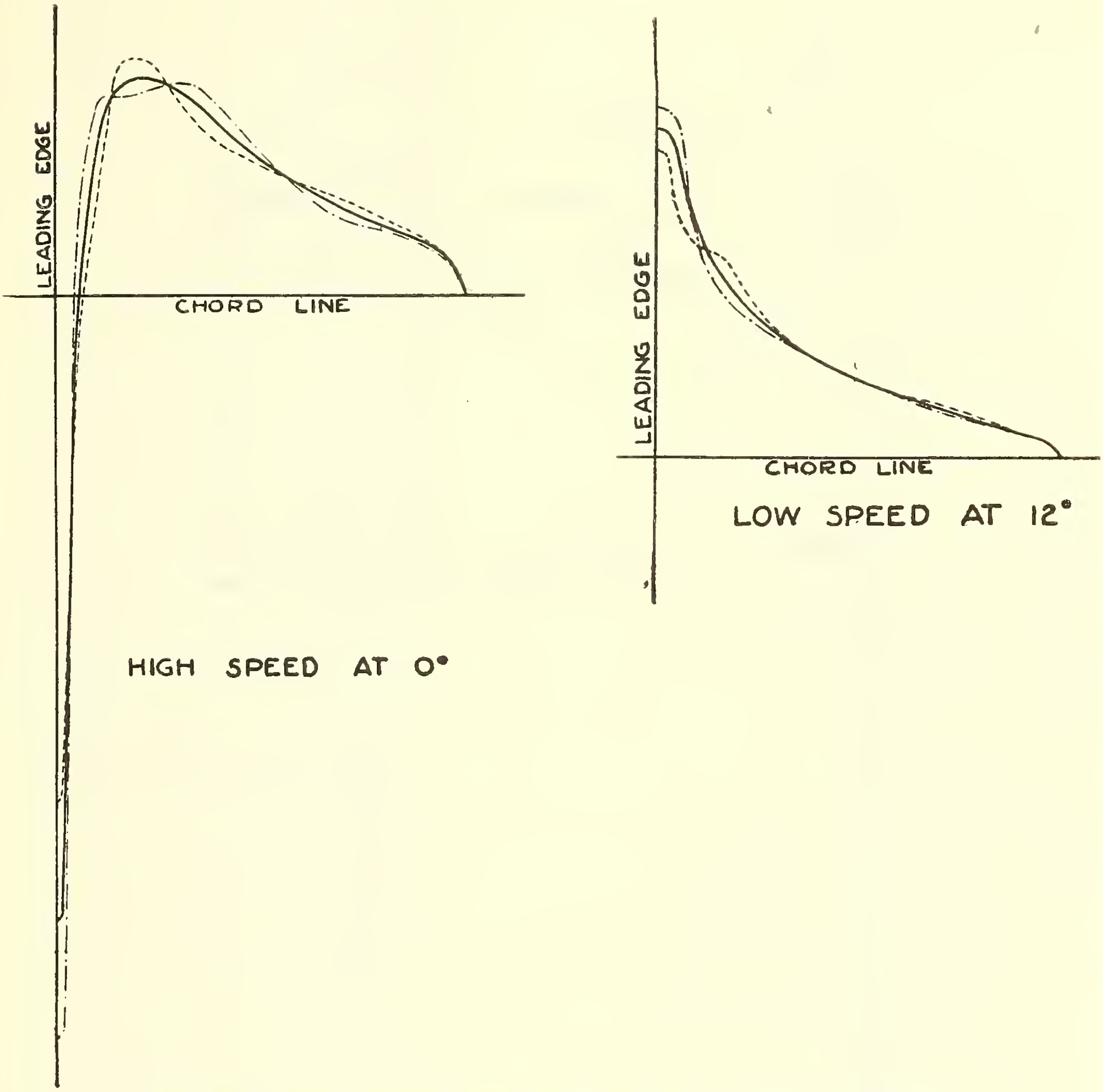


Plate No. 1.—Curves showing normal pressure distribution along the wing chord.

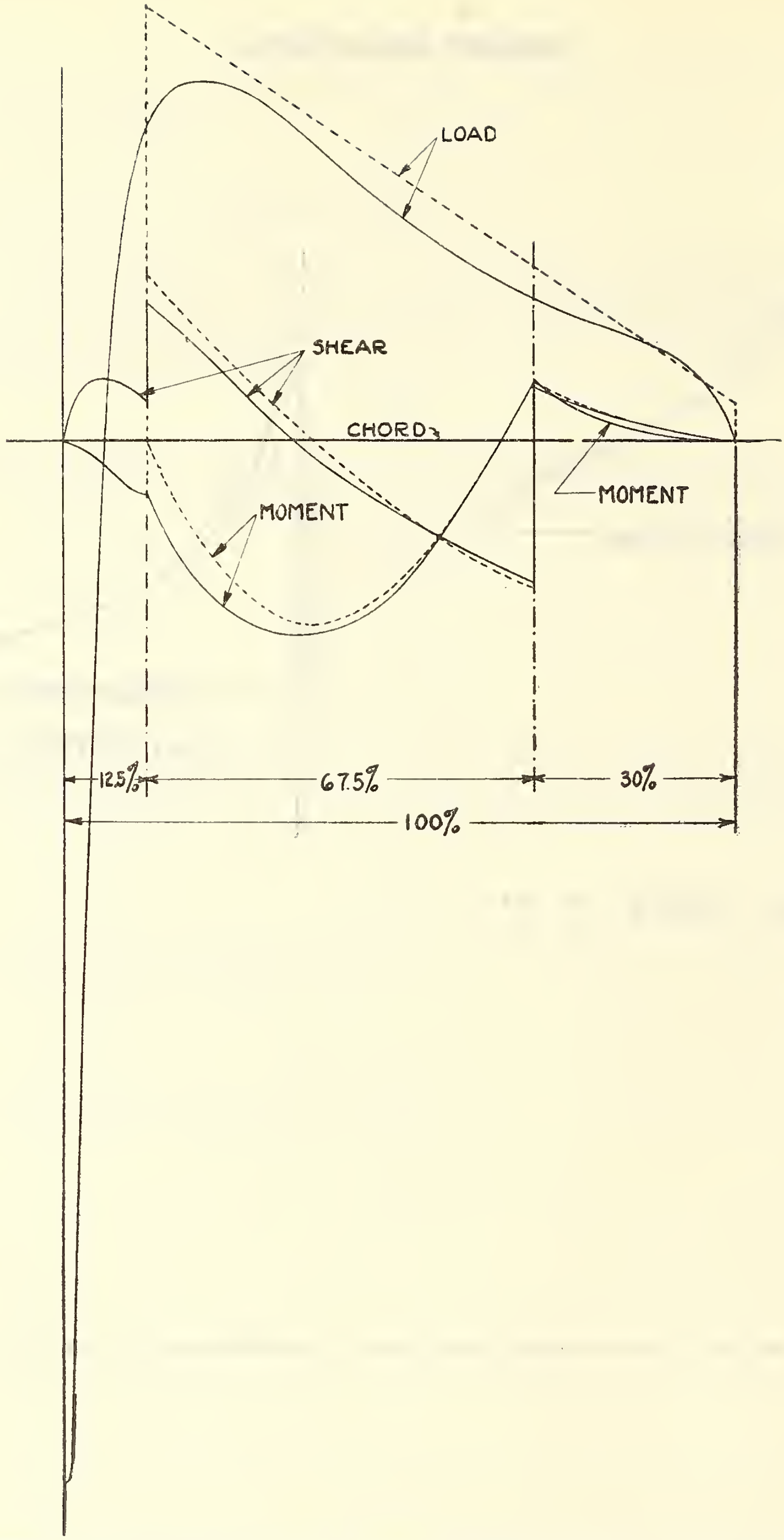


Plate No. 2.—Load, shear and moment curves of pressure distribution along the wing chord at high speed.

Full lines show curves of actual distribution. Dotted lines show curves of approximate distribution obtained by a straight line loading.

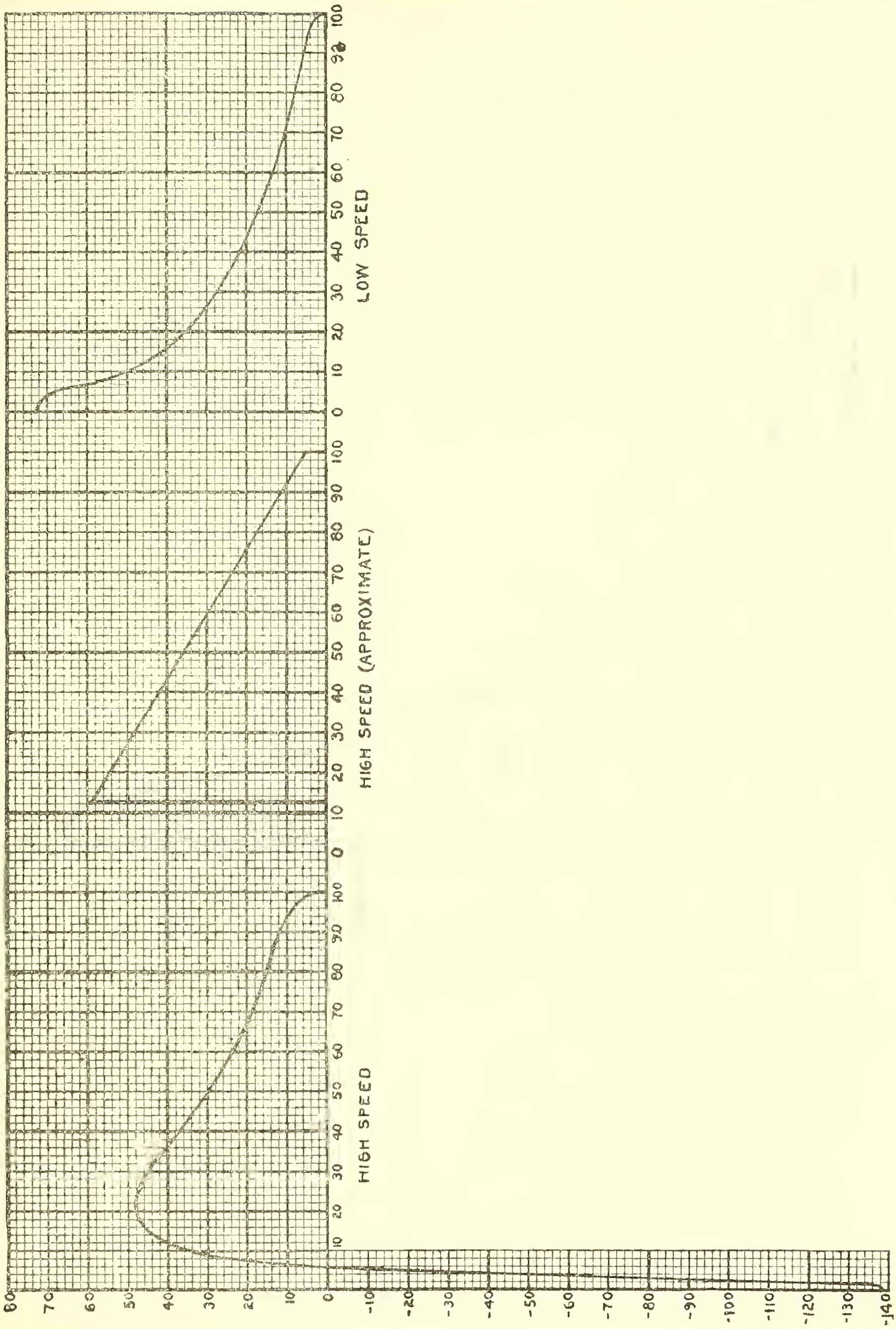


Plate No. 3.—Load distribution curves, plotted to scale.

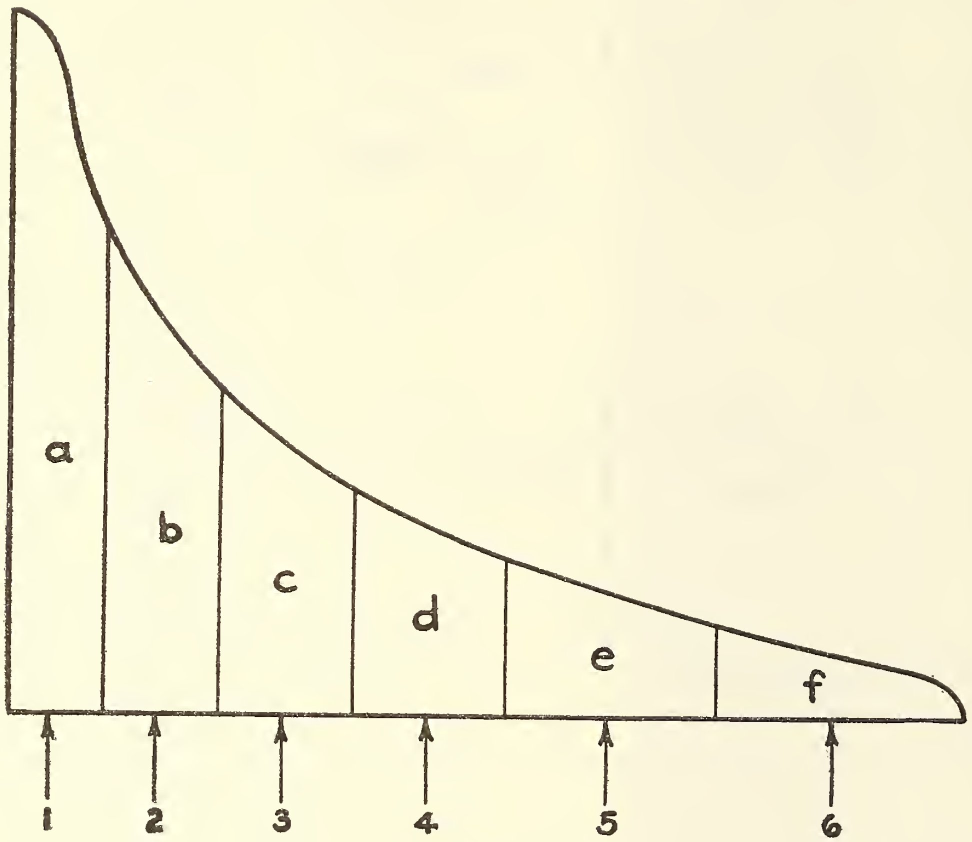


Plate No. 1.

IV.—MEAN CHORD AND CENTER OF PRESSURE.

Owing to interference, the air forces, per unit area, are not the same for each wing of a biplane or of a triplane. This effect is shown by curves which give the relative loading per unit area.

An important application of these curves is in finding the mean chord, that is, a chord to which the resultant of the lift and drag forces may be referred. This chord is used in calculations for performance and stability; e. g., to find the line of action of wing drag. The curves may also be used to obtain wing loadings for stress analysis.

In order to calculate moments the center of pressure movement must be known. The exact variation of this movement with aspect ratio, gap-chord ratio, and stagger can not be given. Where great accuracy is desired tests must be made on a model of the particular design, but, in general, the assumption is made that the center of pressure movement on the mean chord of a biplane or a triplane is the same as the movement on a monoplane. The curves of figure 3, which compare the center of pressure travel of a biplane to a monoplane, show how closely this assumption holds true.

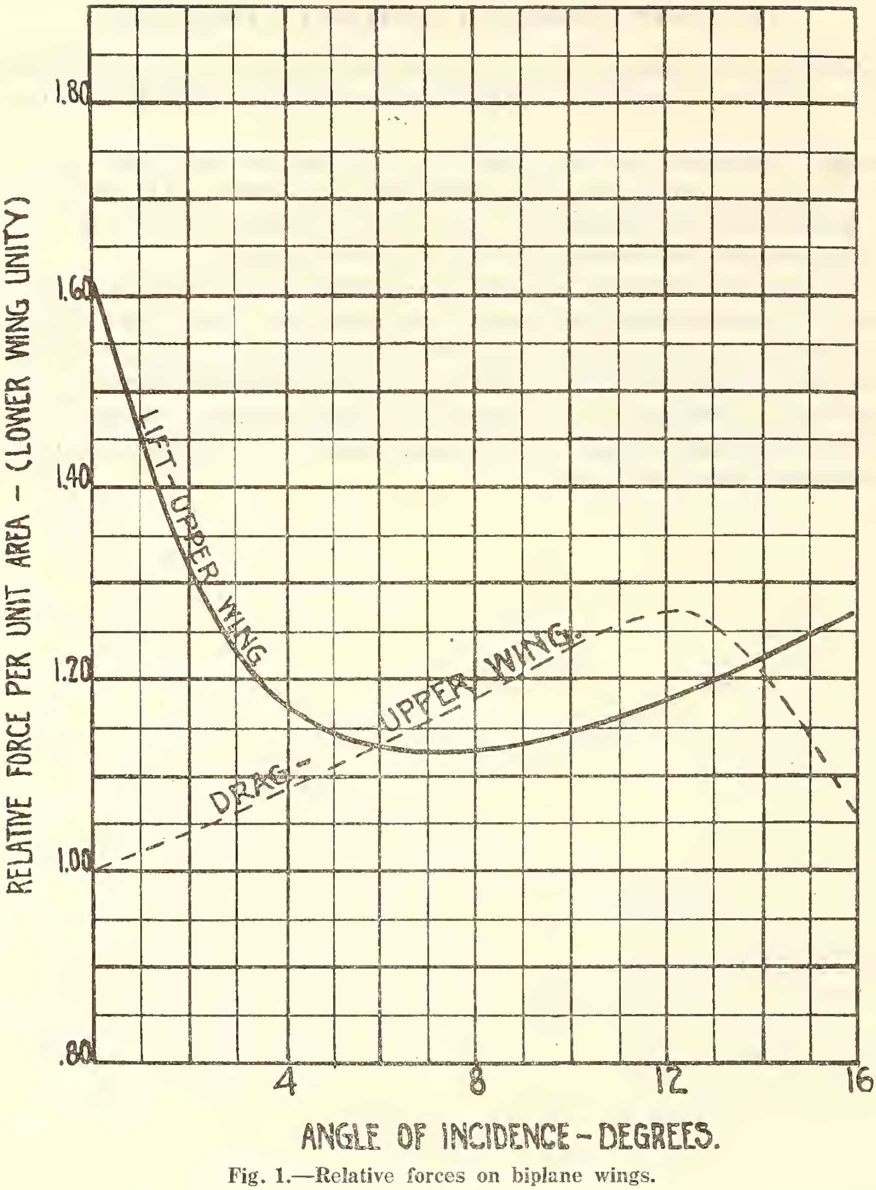


Fig. 1.—Relative forces on biplane wings.

Mean chord.—To find location of mean chord in a biplane at any angle, let A_u and A_l=areas of upper and lower wings.

L_u (or D_u)=relative force on upper wing.

G=gap.

Then

$$\frac{(A_u\, L_u)}{A_u+A_l}\, G=X \text{ or } \frac{(A_u\, D_u)}{A_u+A_l}\, G=X.$$

X=the distance of mean chord from lower wing.

NOTE.—It varies with angle and is not the same for lift as for drag forces.

Ref.: A. C. A.

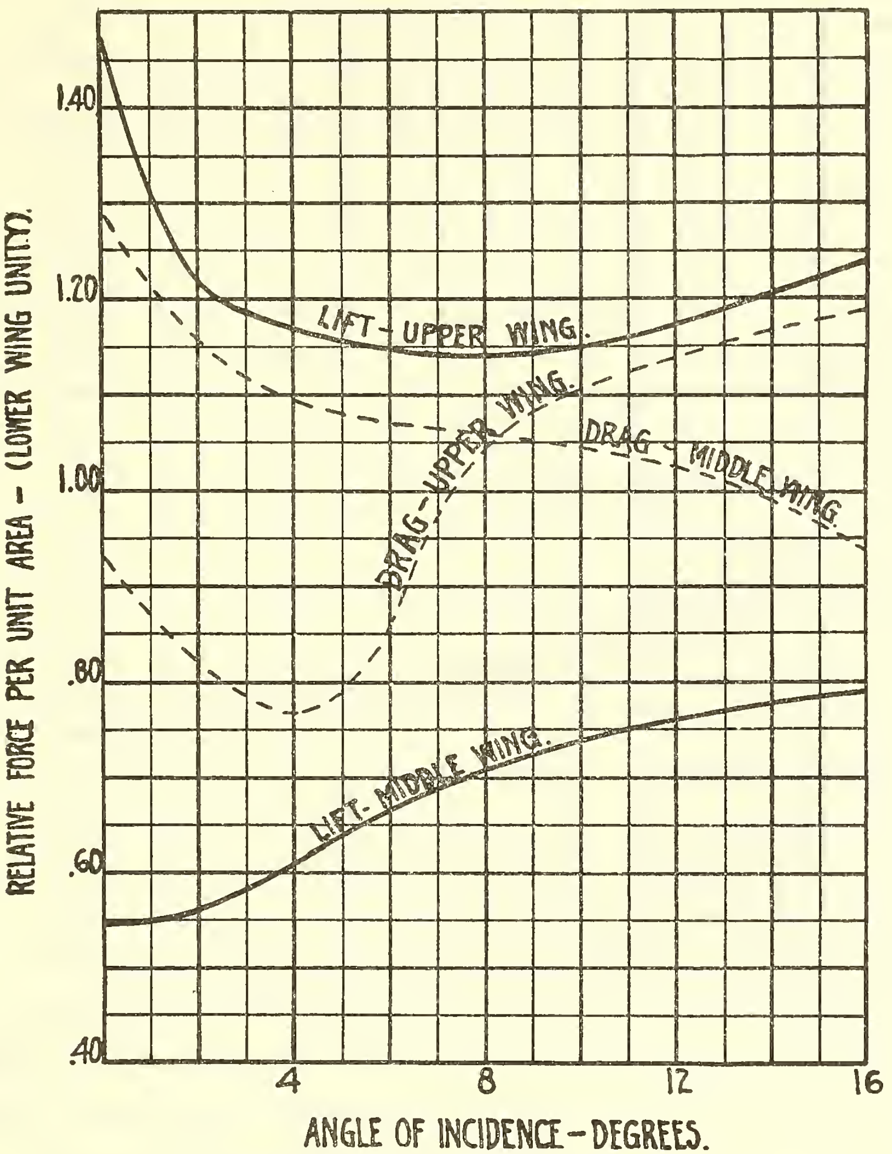


Fig. 2.—Relative forces on triplane wings.

Mean chord.—To find location of mean chord in a triplane, take moments about lower wing. Thus (at any angle) let

A_u, A_m, and A_l=areas of upper, middle, and lower wings.

L_u and L_m (or D_u and D_m)=relative forces on upper and middle wings.

G_u and G_m=distance of upper and middle wings from lower.

Then

$$\frac{(A_u\ L_m)\ G_u+(A_m\ L_m)\ G_m}{A_u+A_m+A_l}=X$$

X=the distance of mean chord from lower wing.

Note.—It varies with angle and is not the same for lift as for drag forces.

Ref.: Hunsaker; A. C. A.

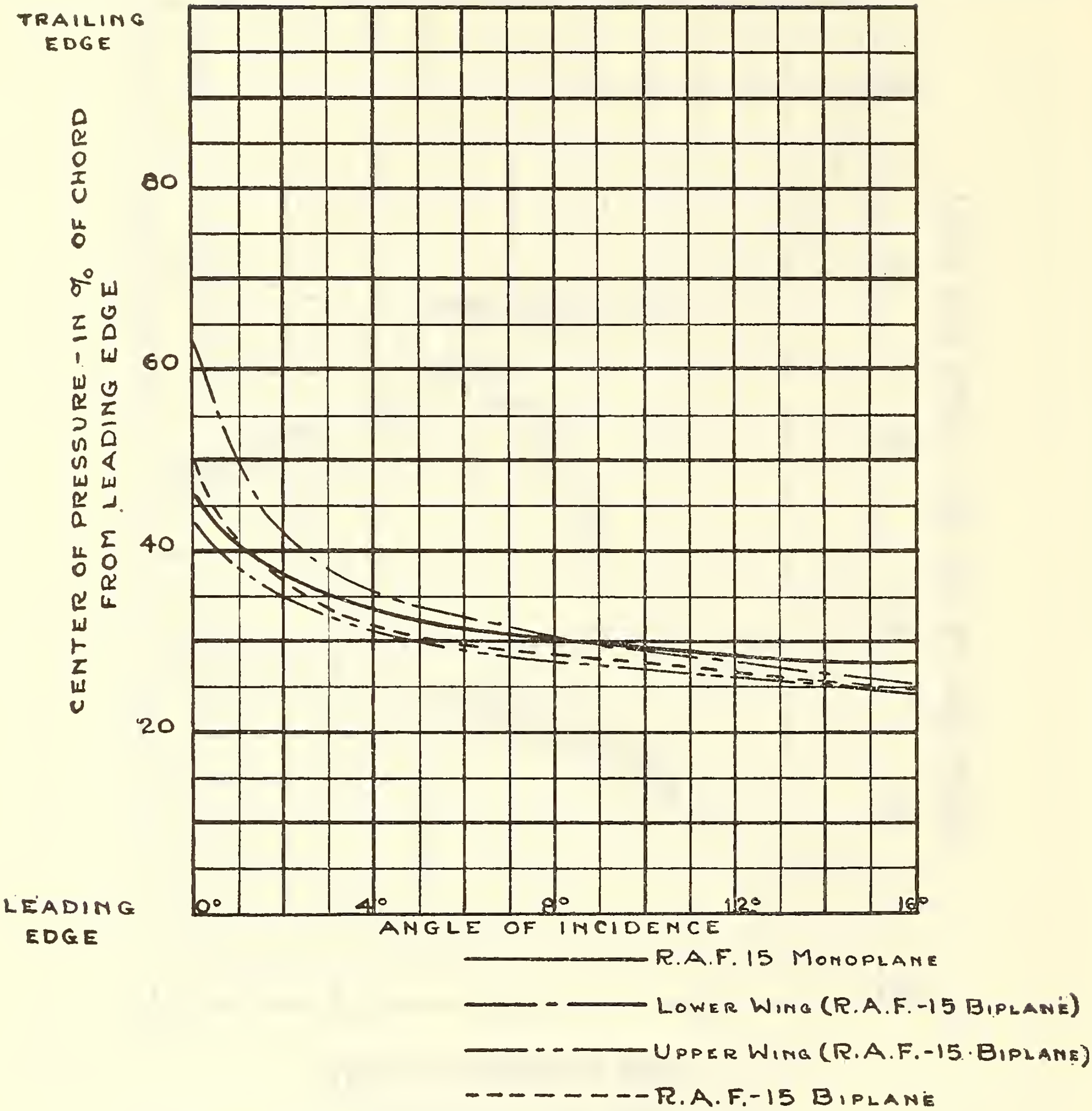


Fig. 3.—Center of pressure travel, monoplane and biplane (R. A. F. 15—wing section).

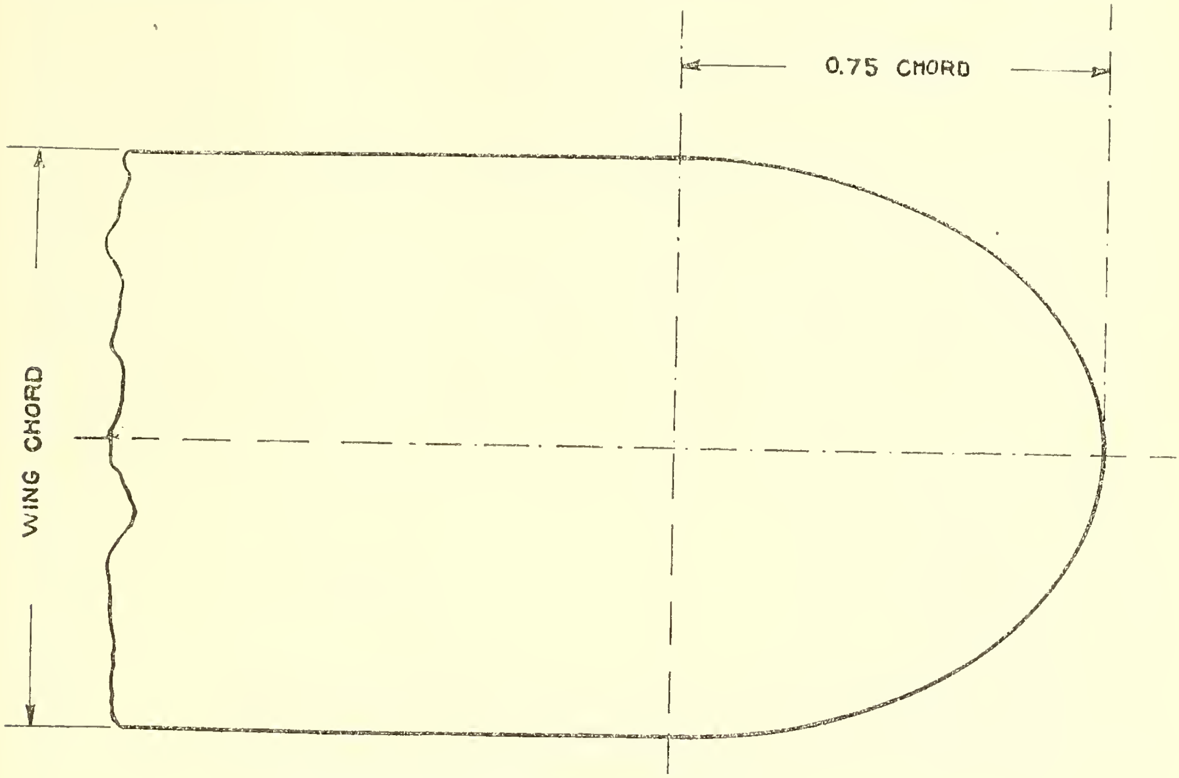
Model test data.

Aspect ratio	5.6
$\frac{G}{C}$ ratio	.88
Stagger	23°
Chord	6″
Wind velocity	50 f. p. s.

Ref.: A. C. A.

V.—STANDARD WING TIP FORM.

The wing tip form below is the best compromise aerodynamically and structurally of a series of tips. Where the ailerons are a part of the upper surface only, it may be necessary to use a raked tip to obtain the required amount of aileron surface.



PLAN FORM FOR STANDARD WING TIP.

Curvature is semi-elliptical, major axis=1.5 minor axis (chord). Any vertical section through tip parallel to chord is similar to inboard section. Lower camber is continuous out to extreme tip, i. e., has no upward or downward curvature.

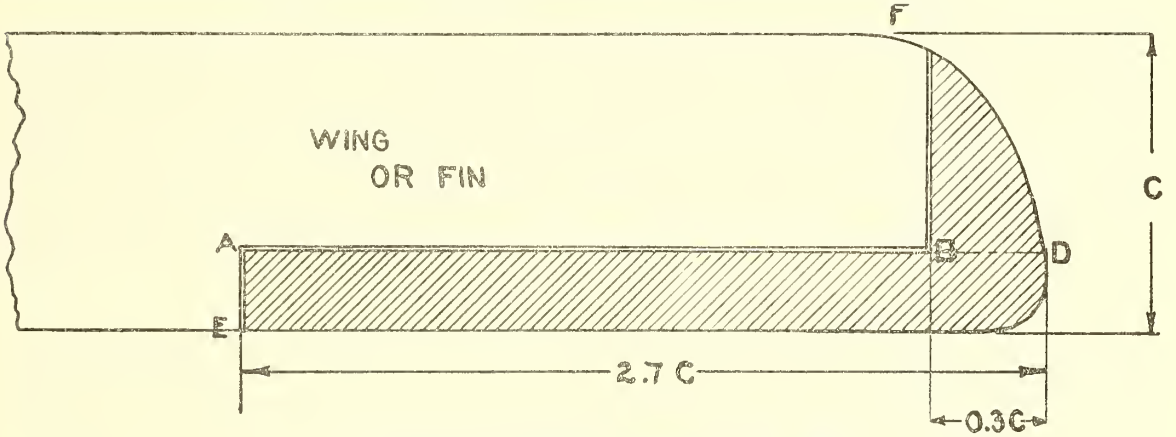
Ref.: A. C. A.

VI.—BALANCED CONTROLS.

An empirical method of balancing, which has been used with good results, is as follows: Divide the control surface into a number of strips parallel to the wind, assume the center of pressure of each strip to be 20 per cent of the chord from the leading edge and find the moment of the area of each strip using the distance of the centers of pressure from a base line. The sum of all these moments divided by the area of the control will give the location of the resultant center of pressure. The hinge is then placed forward of this resultant by an amount which depends upon the allowable force to be used in operating the controls. The distance will usually be from 3 inches for small machines to 5 inches for large machines.

Wind tunnel tests on various arrangements of areas have shown that while the resultant moment can be made zero at only one angle, it is possible to secure very good balance at all practical angles of flight. The figure shows the best arrangement yet tested. When this form is used the area forward of the hinge must not be more than 18 per cent of the total area of the control, and owing to variations in wind flow for various designs it is not advisable to use more than 15 per cent unless wind-tunnel tests show no overbalance. In case it is desired to use any other arrangement than that shown in figure 1, wind-tunnel tests should be made.

It should be noted that the tail surfaces are not, in general, entirely within the slip stream, and this factor must be accounted for in the design of rudders and elevators.



BALANCE CONTROLS FORM TO BE USED.

$$\text{Balance ratio } \frac{\text{area B F D}}{\text{area A B F D E}} = 0.15$$

Ref.: A. C. A.

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AIRCRAFT DESIGN DATA. NOTE 10.

PARASITE DRAG COEFFICIENTS.

THE RESISTANCE OF VARIOUS PARTS OF AN AIRPLANE.

The parts of an airplane excluding the actual wing surfaces are divided into two sections, those within the slip stream of the propeller and those outside of it. The total resistance of the airplane is the sum of the tabulated resistance of parts outside the slip stream plus the tabulated resistances of the parts in the slip stream times a slip stream factor.

The resistance of certain parts has been given only at 70 miles per hour. Where the resistance at some other speed "V" is desired, multiply the value for 70 miles per hour by (V in miles per hour)^2 and divide by 4900.

PROPELLER SLIP STREAM FACTOR.

The slip stream of the propeller is taken to be a cylindrical shell whose outer diameter is 0.8 times and inner diameter 0.2 times the diameter of the propeller. Values of the slip stream factor (i. e., the ratio of the resistance of a part in the slip stream to the resistance of the same part outside) are given in the following chart:

Where no allowance has been made for the interference of the tractor body on the air screw no allowance is to be made for the slip stream on the body.

(Ref.: A. C. A.)

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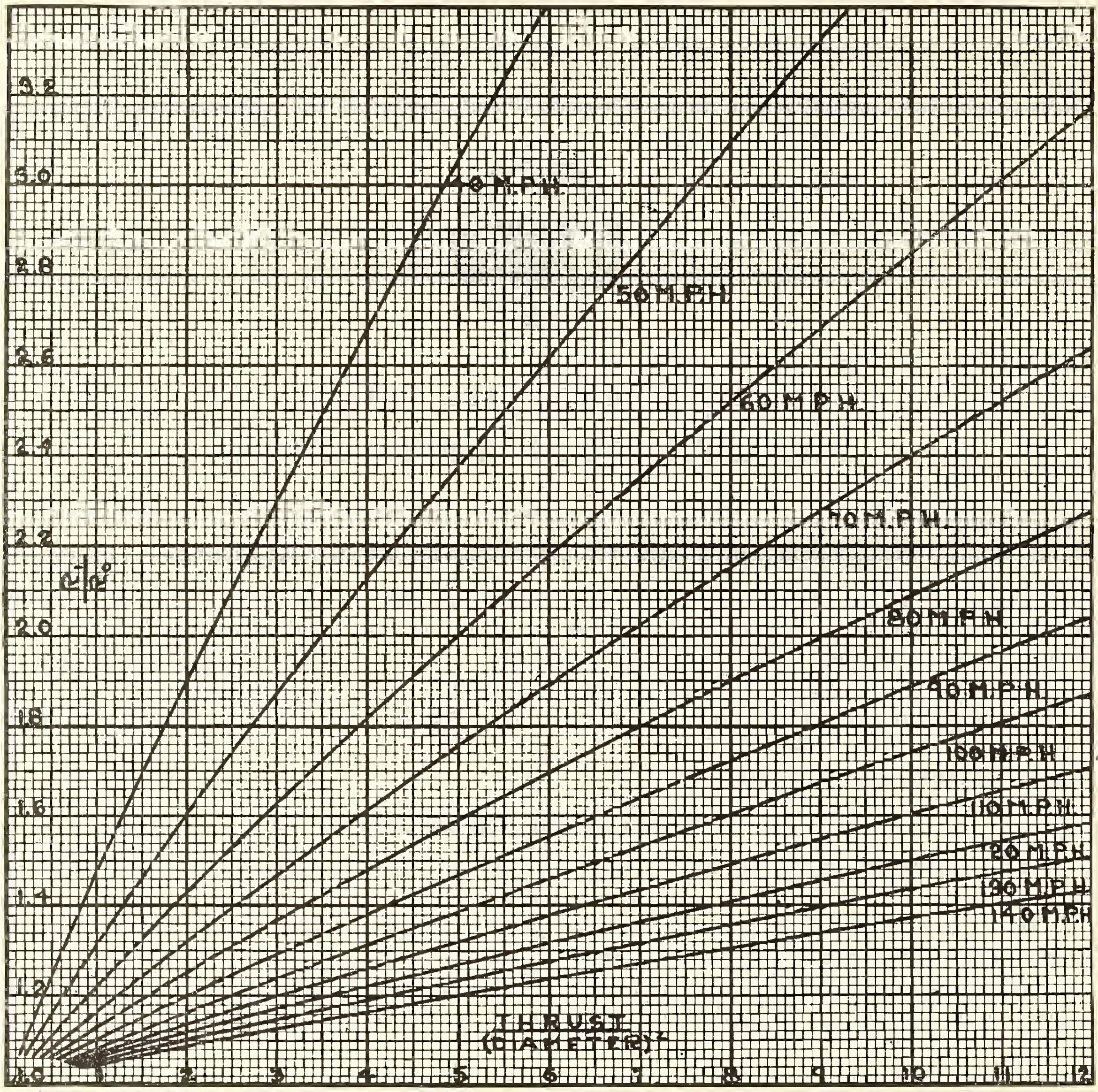


CURVES SHOWING RATIO OF RESISTANCE IN SLIP STREAM TO RESISTANCE OUT OF SLIP STREAM AT DIFFERENT THRUSTS.

The speeds marked on the curves are the forward speeds of the machine.

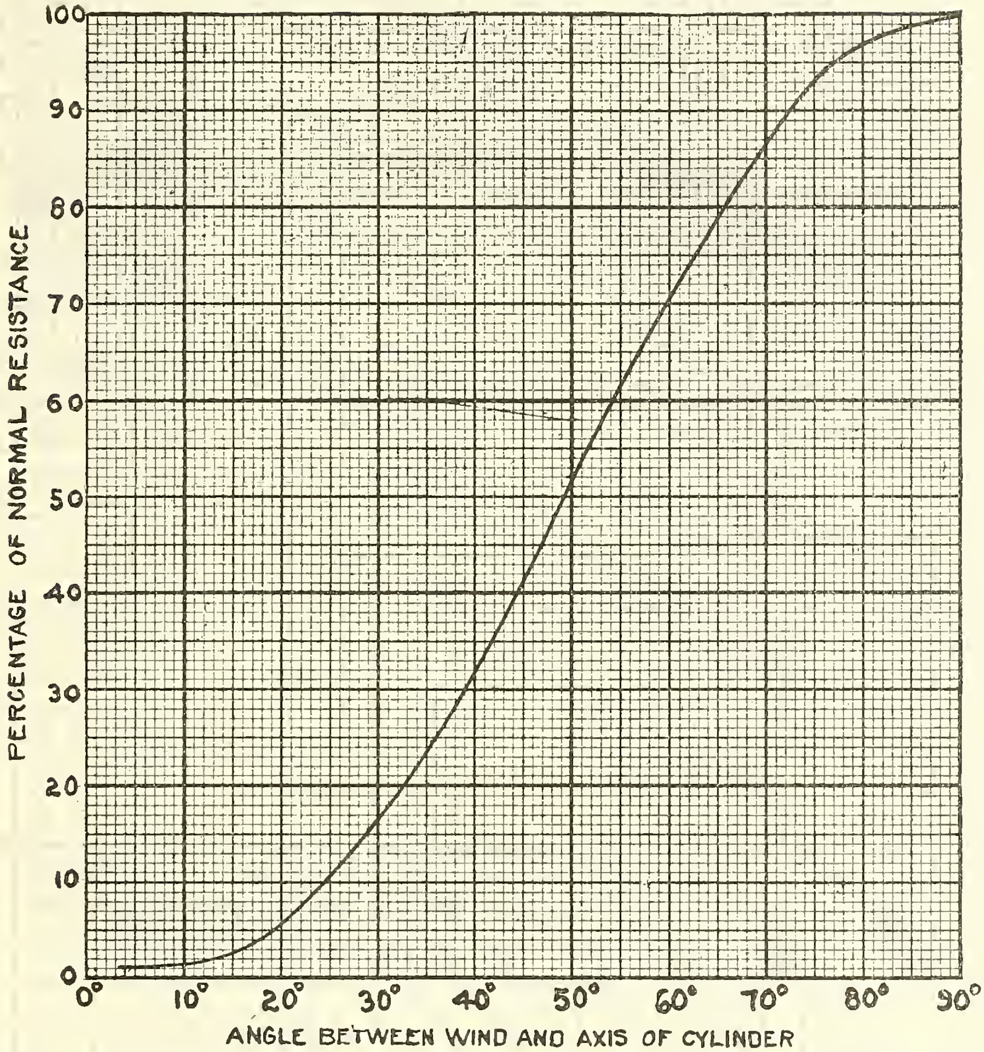
T=Thrust in pounds and D=diameter of propeller in feet.

R_1=Resistance in slip stream.

R_0=Resistance out of slip stream.

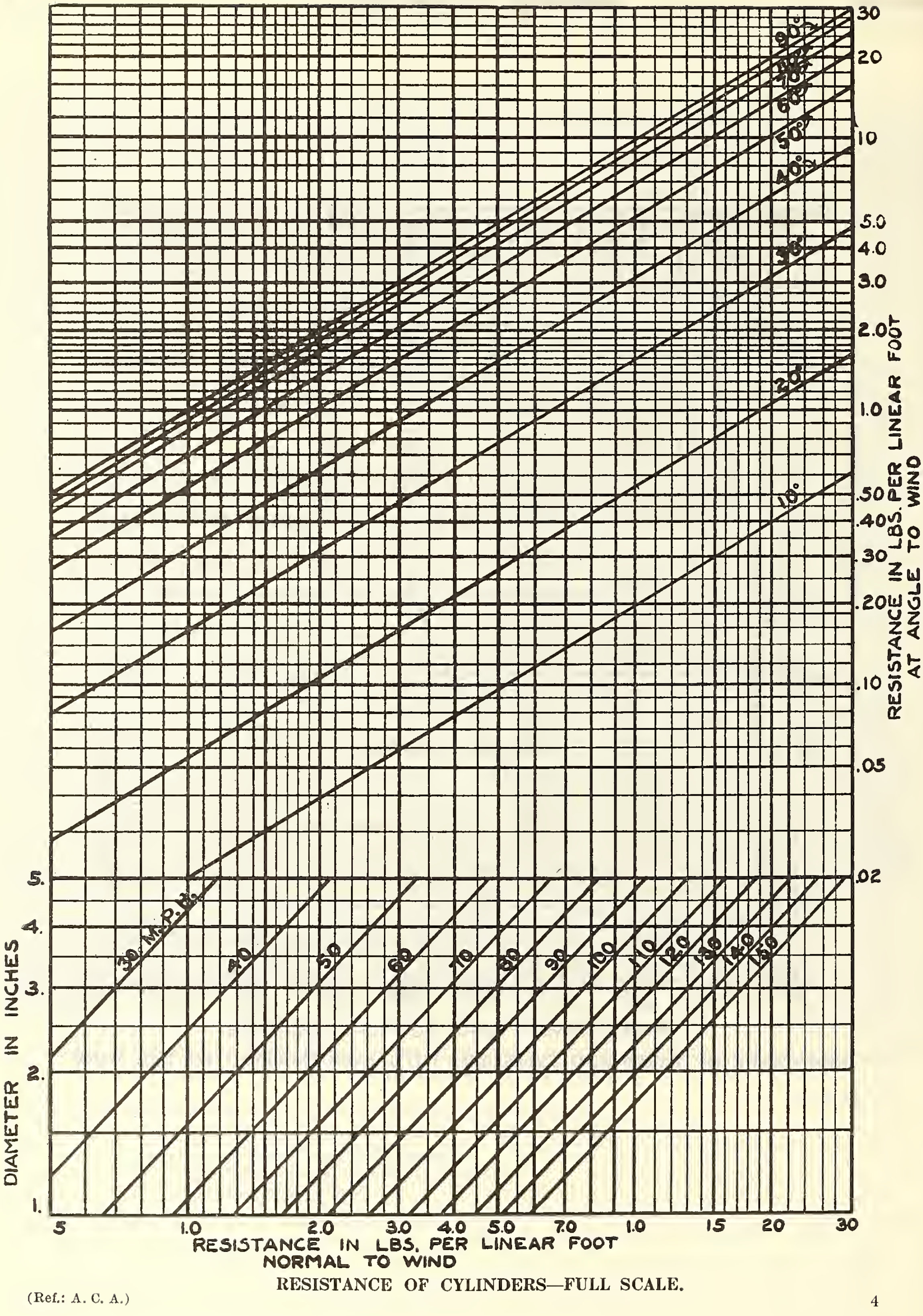
For Thrust see Note 7.

(Ref.: A. C. A.)



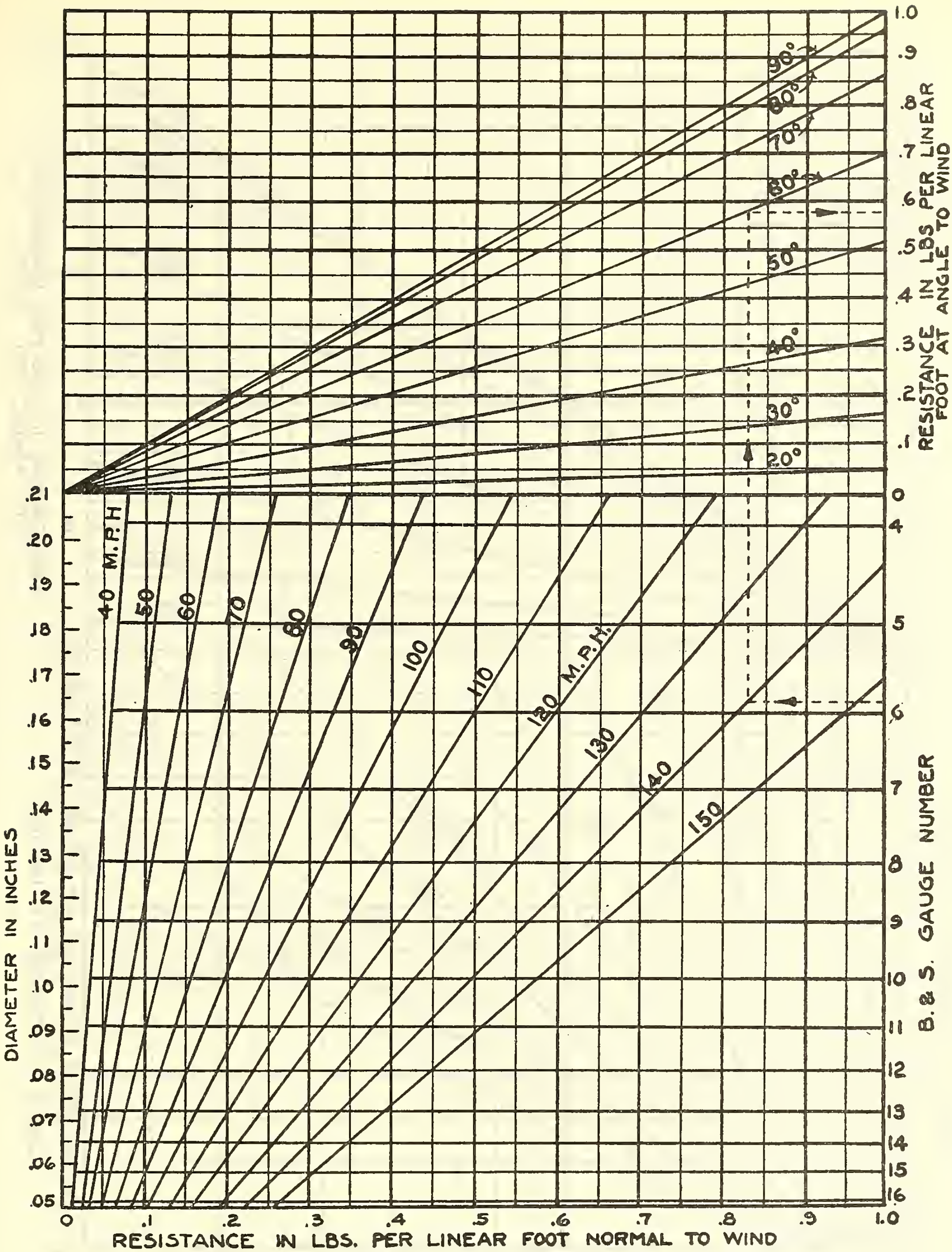
RESISTANCE OF WIRES AND CYLINDERS WITH AXES INCLINED TO THE WIND.

(Ref.: A. C. A.)



RESISTANCE OF CYLINDERS—FULL SCALE.

(Ref.: A. C. A.)



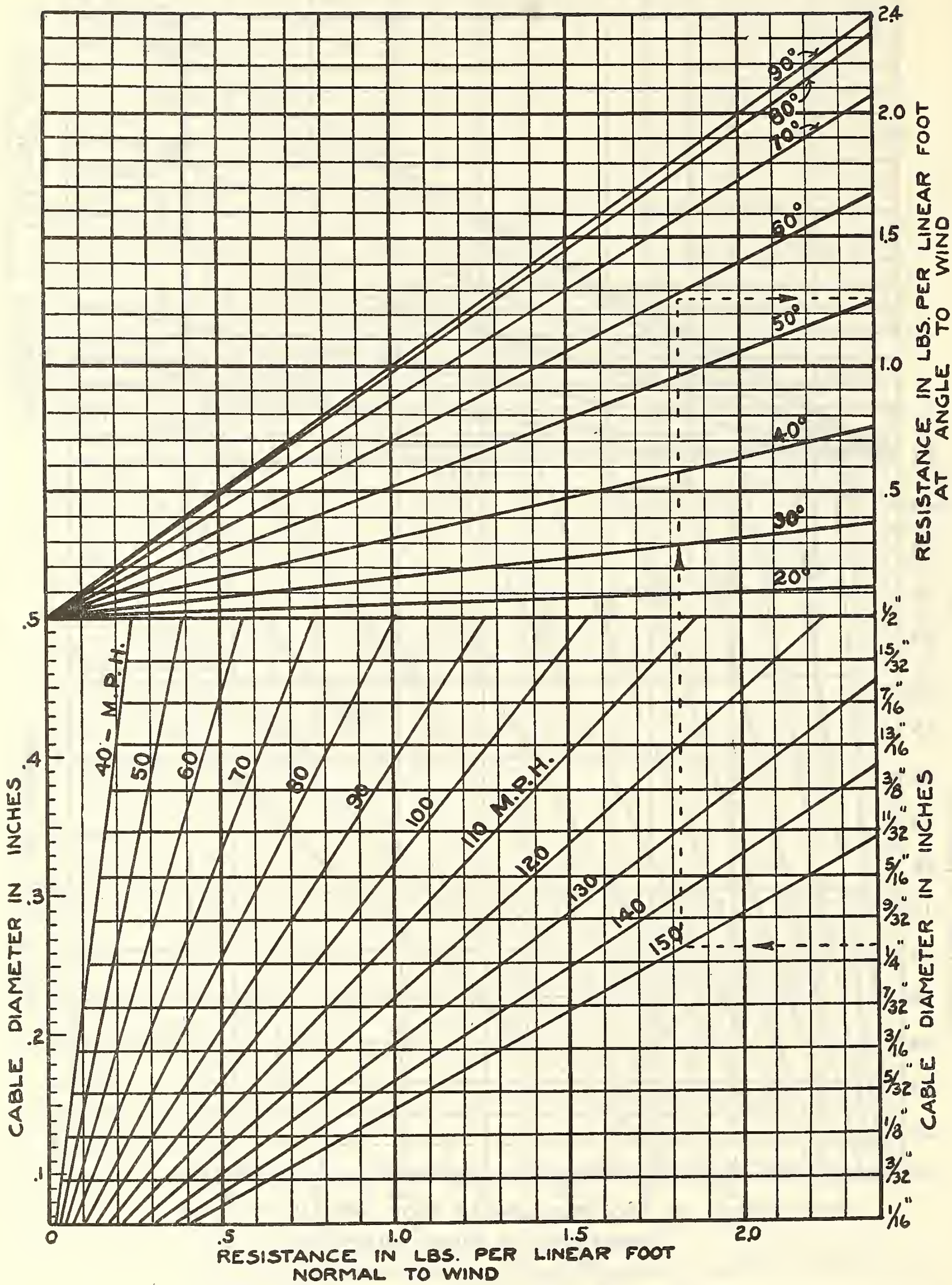
RESISTANCE OF WIRES, FULL SCALE.

NOTE.—Add 1 foot to length of wire for turnbuckle and 1 foot for eye and fitting.

When wire is at angle to wind, use actual length of wire.

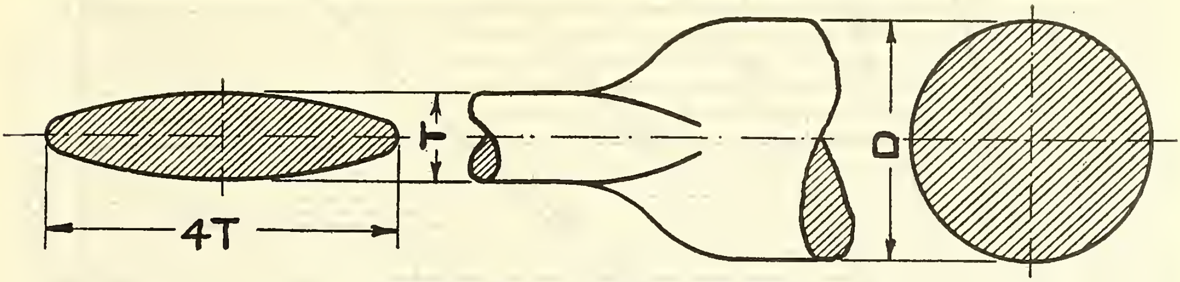
The resistance of wires parallel to wind is 0.05 pounds per square foot of wire surface at 70 M. P. H.

(Ref.: A. C. A.)



RESISTANCE OF CABLES, FULL SCALE.

NOTE.—Add 1 foot to length of wire for turnbuckle and 1 foot for eye and fitting. When cable is at angle to wind, use actual length of cable. The resistance of cables parallel to wind is 0.05 pounds per square foot of cable surface at 70 M. P. H.



RESISTANCE OF STREAM LINE WIRE.

Resistance of R. A. F. standard stream line wire at 70 m. p. h.

D.	T.	Resistance per foot run.	D.	T.	Resistance per foot run.
Inches.	*Inches.*	*Pounds.*	*Inches.*	*Inches.*	*Pounds.*
$\frac{3}{16}$	0.048	0.027	$\frac{3}{8}$	0.135	0.055
$\frac{9}{64}$	.064	.029	$\frac{13}{32}$	.149	.063
$\frac{1}{4}$	.087	.036	$\frac{7}{16}$	.159	.068
$\frac{9}{32}$	.101	.039	$\frac{15}{32}$	.173	.071
$\frac{5}{16}$	.11	.044	$\frac{1}{2}$	.183	.075
$\frac{11}{32}$	.124	.052			

Resistance of end fittings and wing plates for R. A. F. standard stream line wire at 70 m. p. h.

D.	Pounds.	Increase of resistance per wire.	
		D.	Pounds.
Inches.		*Inches.*	
$\frac{3}{16}$	0.223	$\frac{3}{8}$	0.950
$\frac{9}{64}$	.321	$\frac{13}{32}$	1.123
$\frac{1}{4}$	.468	$\frac{7}{16}$	1.242
$\frac{9}{32}$	.562	$\frac{15}{32}$	1.404
$\frac{5}{16}$	.716	$\frac{1}{2}$	1.920
$\frac{11}{32}$	.810		

NOTE.—For duplicate wires take half the above per wire. If simple fork ends are used, half these figures. When incidence wires are attached direct to strut socket, neglect the resistance of their end fittings.

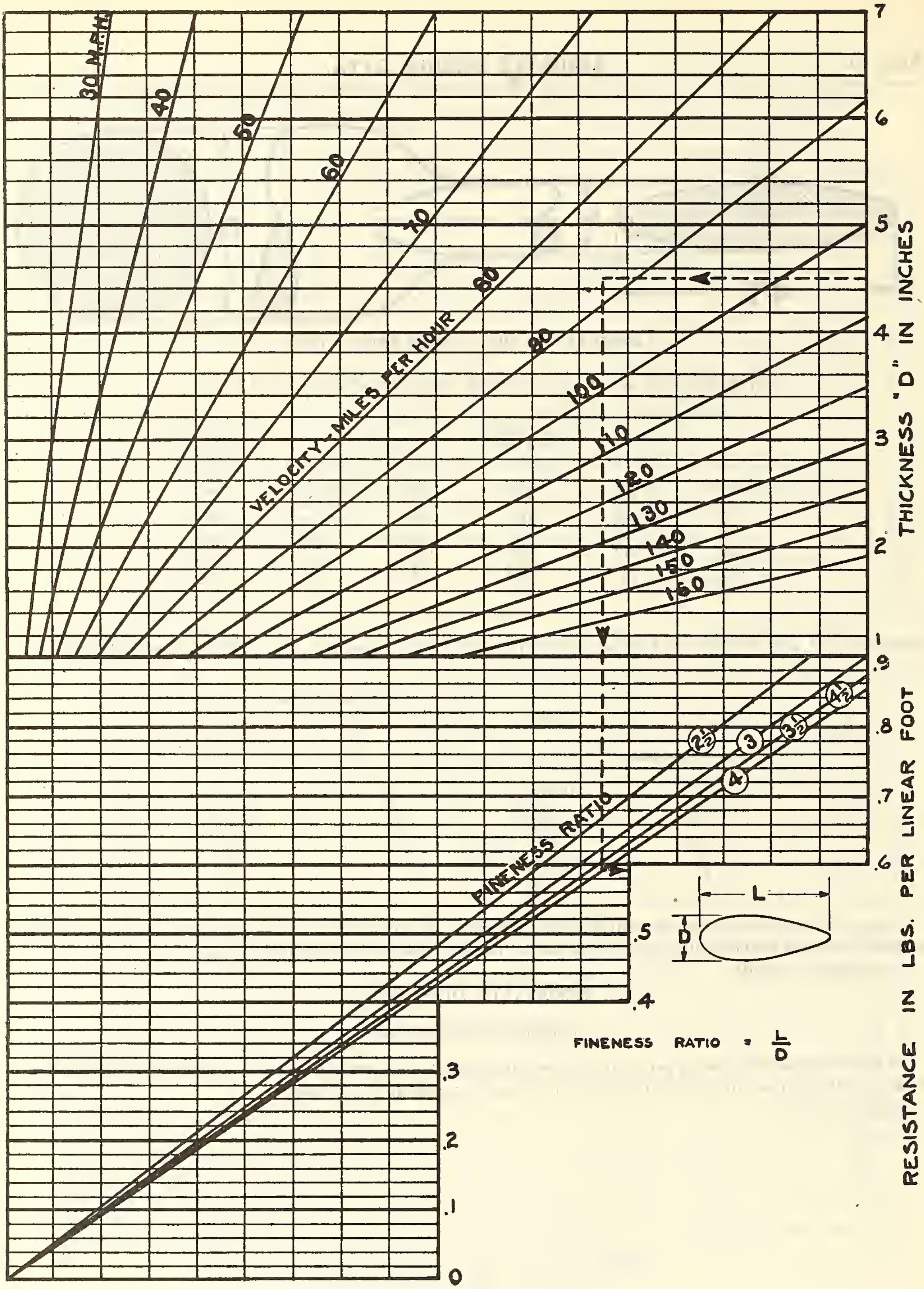
Use projected length.

RESISTANCE OF STRUTS.

Folding wing machines.

It is not desirable to place a pair of struts closer together than 4 strut thicknesses,—separated at a distance of 4 thicknesses the increase in drag is 10 per cent and is much greater at less distances.

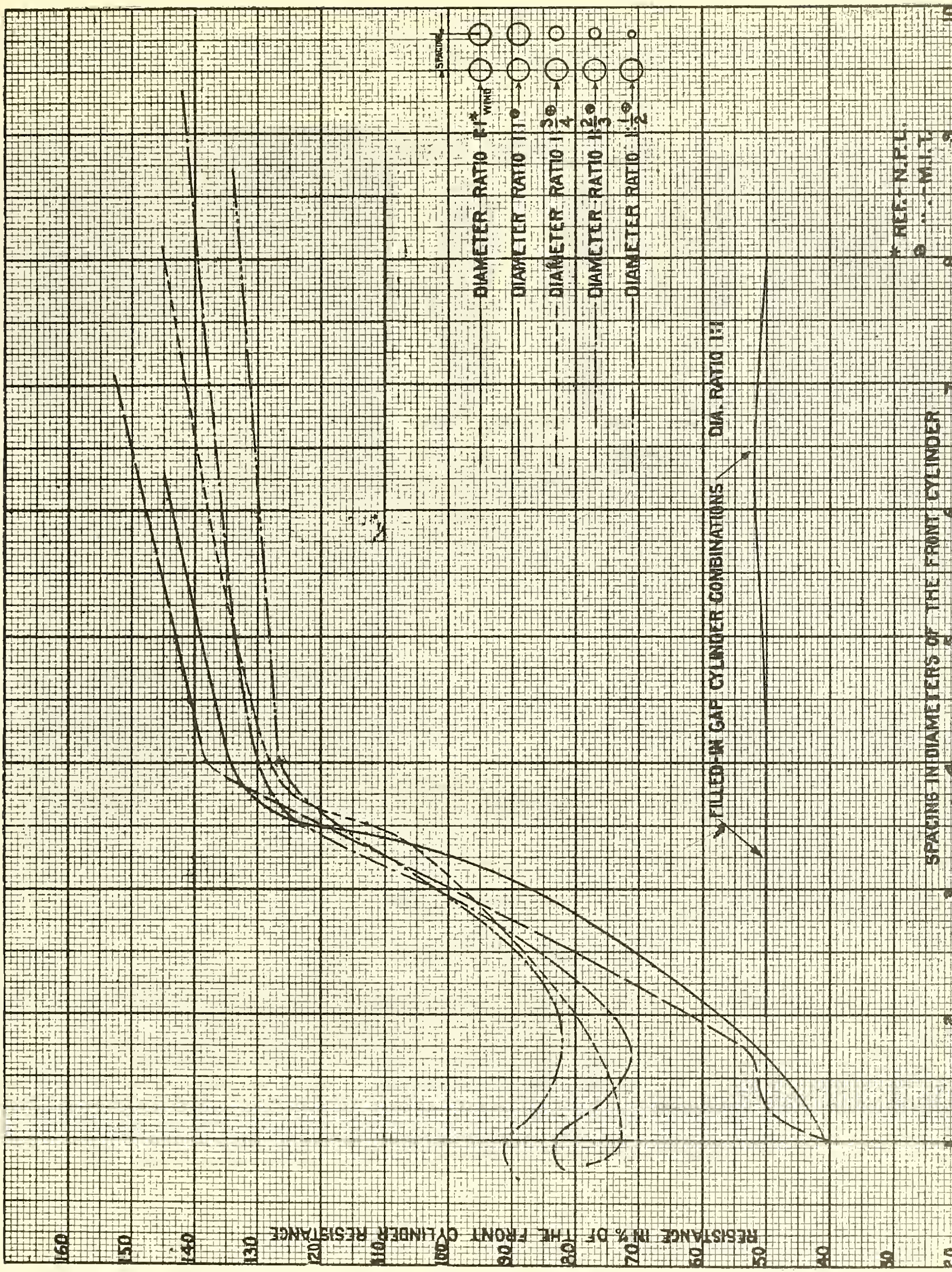
(Ref.: A. C. A.)



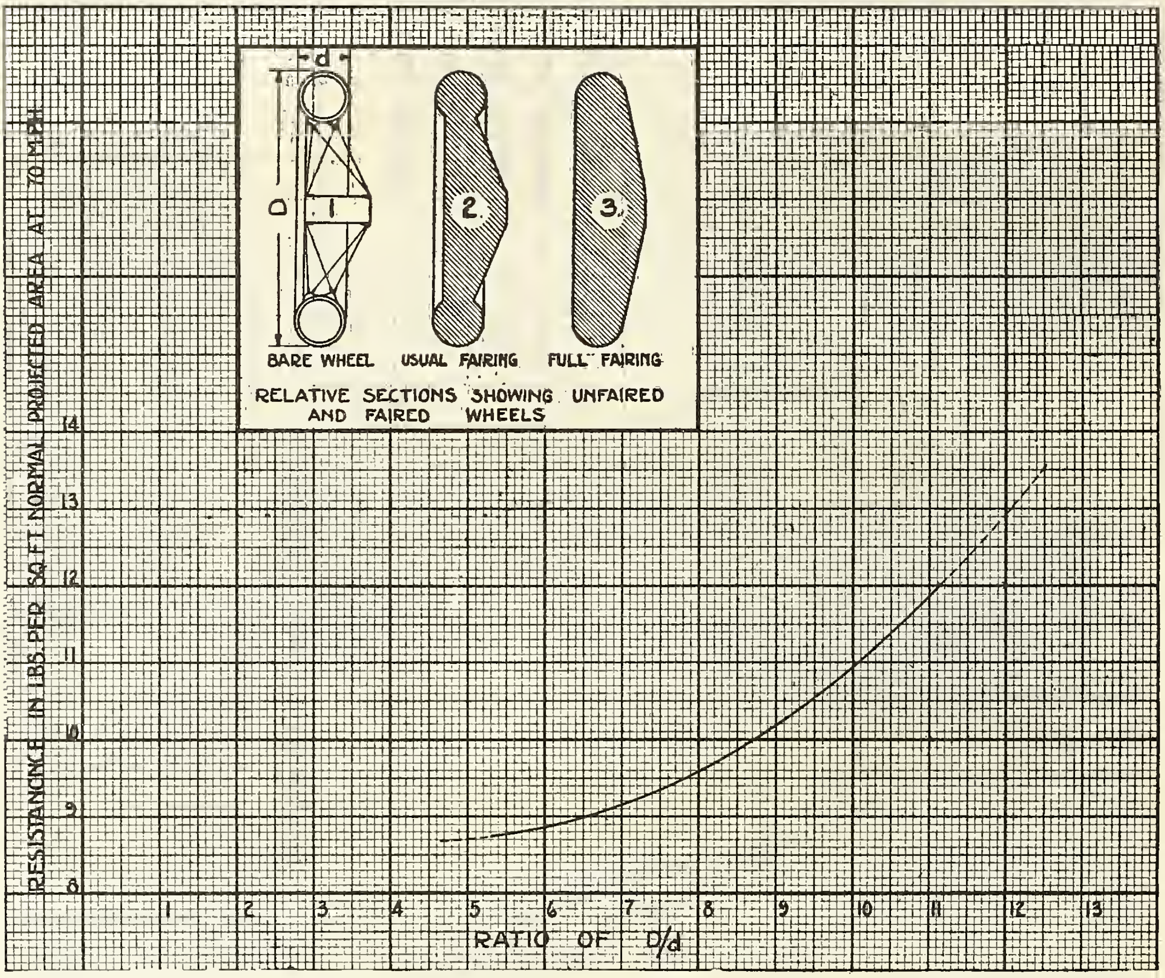
RESISTANCE OF STREAM LINE STRUTS. FULL SCALE.

NOTE.—For total resistance of strut use total length including space occupied by sockets and fittings and add three feet per strut for the additional resistance of the two end fittings.

For inclined struts use projected length and fineness ratio in plane of wind.



COMPARATIVE RESISTANCE OF CYLINDER COMBINATIONS.



RESISTANCE OF LANDING WHEELS.

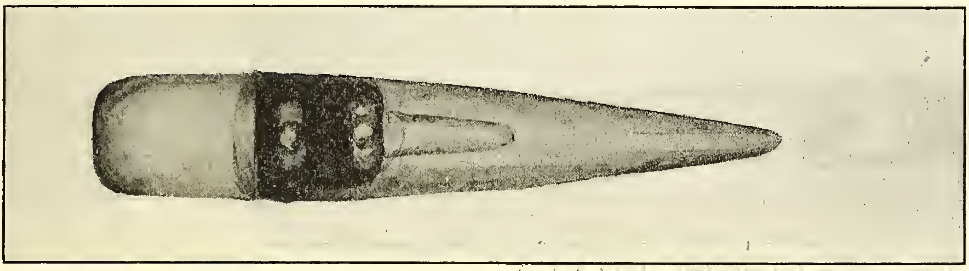
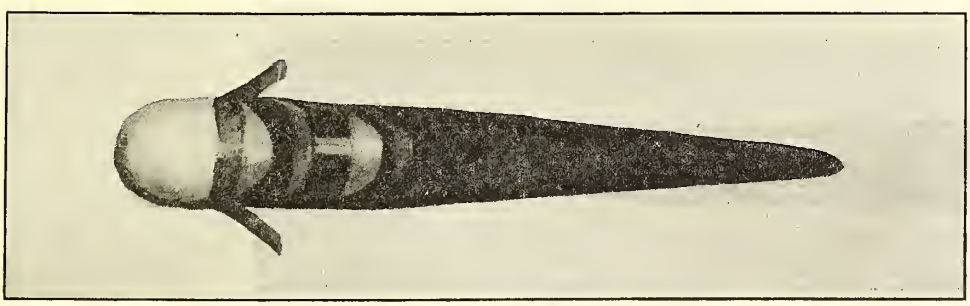
Curve showing resistance of bare wheel No. 1 per square foot of projected tire area normal to the wind for different ratios of $\frac{D}{d}$.

Resistance of No. 2 $=\frac{2}{3}$ of No. 1.
Resistance of No. 3 $=\frac{1}{3}$ of No. 1.

(Ref.: A. C. A. Eiffel.

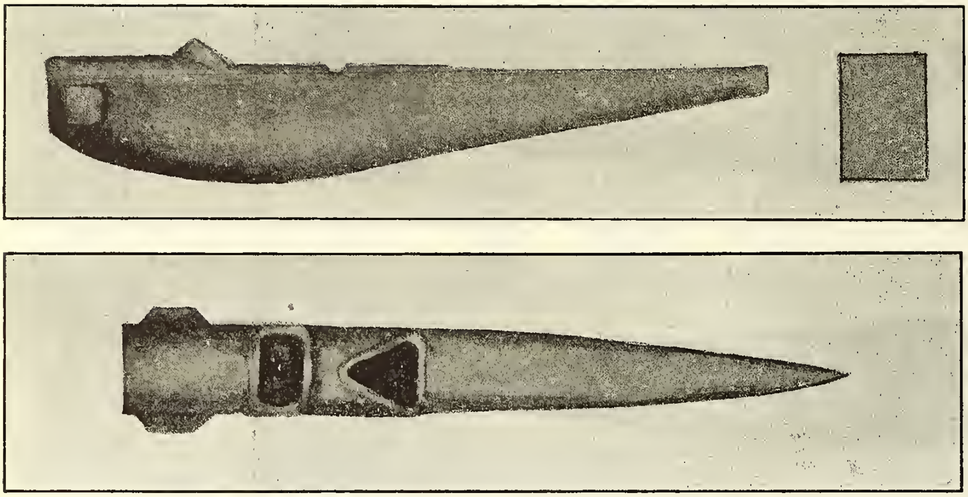
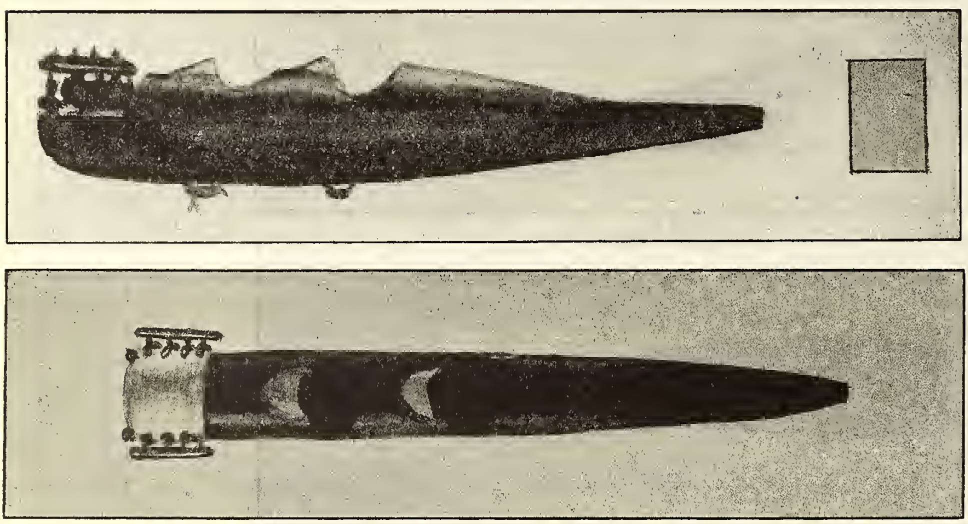
RESISTANCE OF BODIES.

R=resistance in pounds per square foot of maximum cross section at 70 miles per hour and angle of incidence of 0°. A scale correction factor of 90 per cent has been applied to wind tunnel results.

	R.	Length: Max. depth.
	Lbs.	
  BE–3 (no radiator). Ref.: A. C. A.	2. 8	7. 8
  RE–7 (no radiator). Ref.: A. C. A. (No scale correction factor has been applied.)	4. 2	6. 2

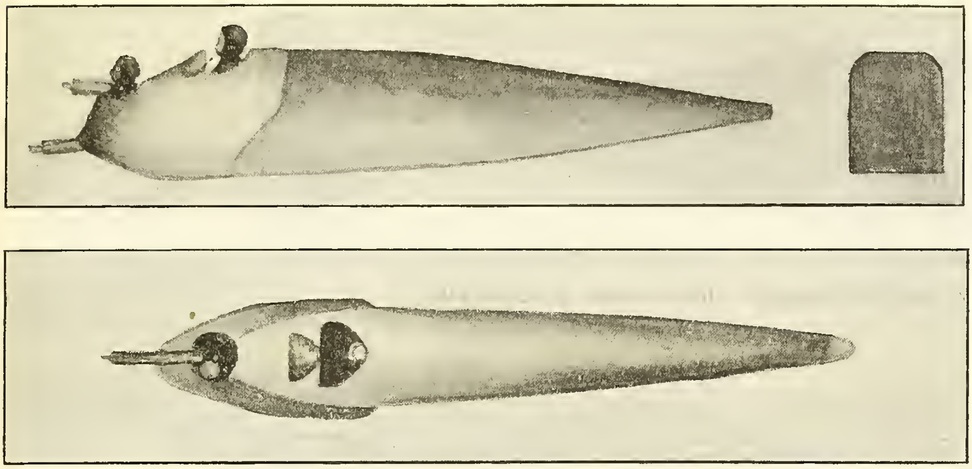
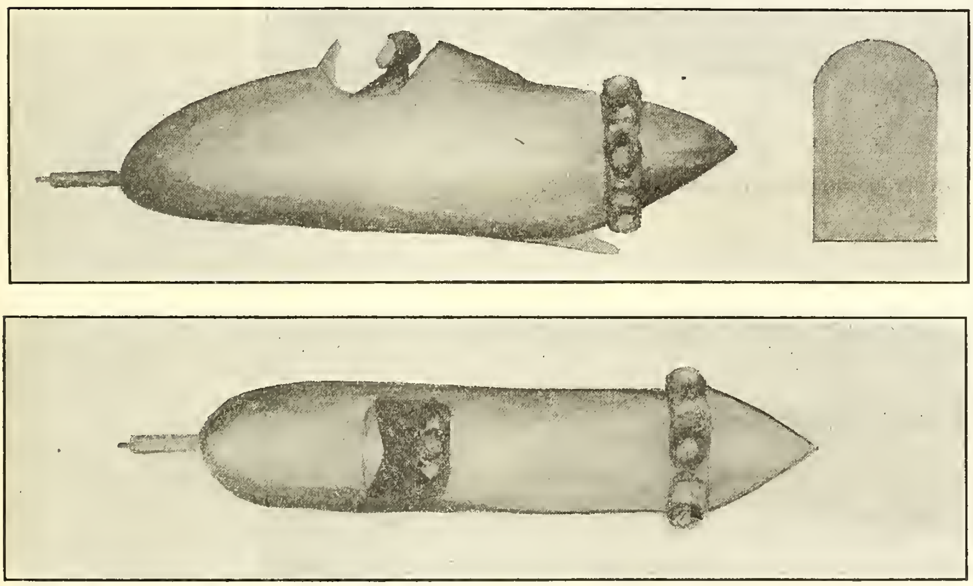
RESISTANCE OF BODIES—Continued.

R=resistance per square foot of maximum cross section at 70 miles per hour and angle of incidence of 0°. A scale correction factor of 90 per cent has been applied to wind tunnel results.

	R.	Length: Max. depth.
 Avro seaplane (no radiator). Ref.: A. C. A.	5.4	6.8
 BE–2c (no radiator). Ref.: A. C. A. (No scale correction factor has been applied.)	7.7	7.2

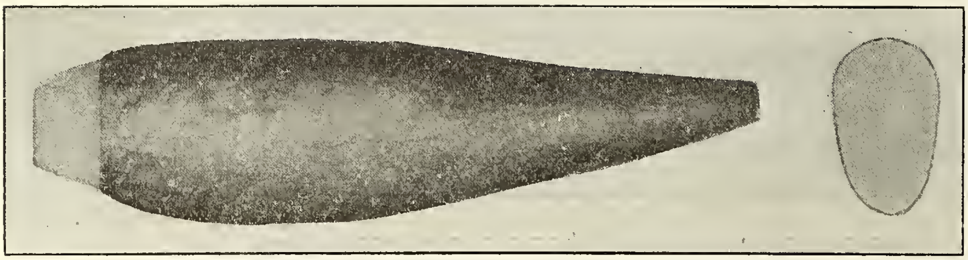
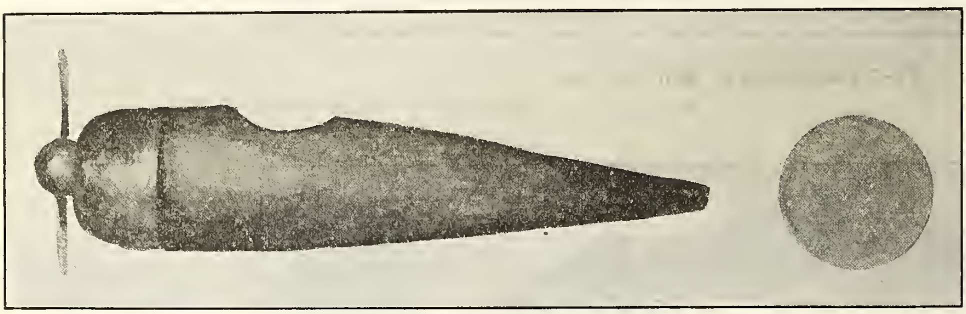
RESISTANCE OF BODIES—Continued.

R=resistance per square foot of maximum cross section at 70 miles per hour and angle of incidence of 0°.
A scale correction factor of 90 per cent has been applied to wind tunnel results.

	R.	Length: Max. depth.
 FE–7 (no radiator). Ref.: A. C. A.	2.9	7.6
 FE–8 (no radiator, 7-cylinder engine). Ref.: A. C. A.	4.9	3.1

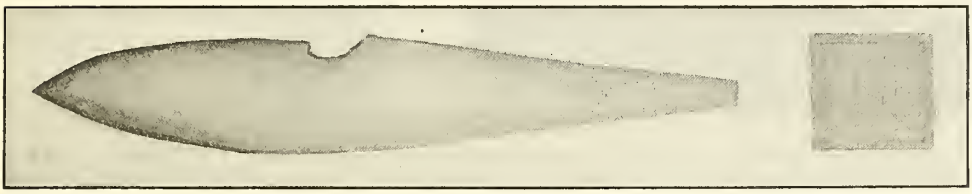
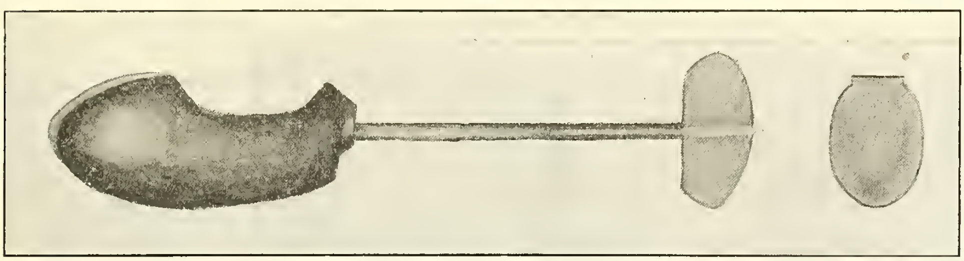
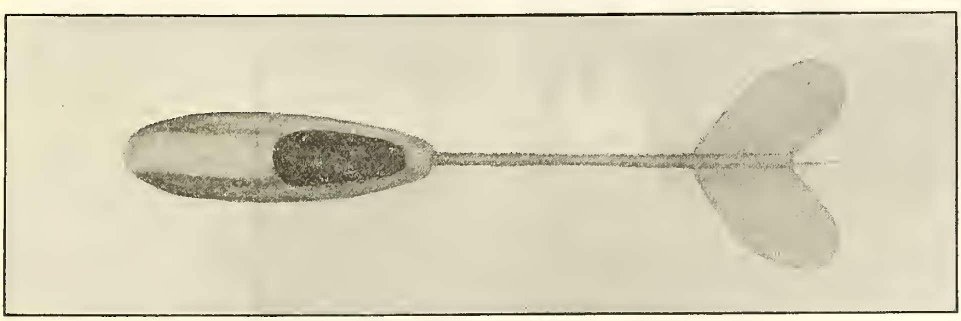
RESISTANCE OF BODIES—Continued.

R=resistance per square foot of maximum cross section at 70 miles per hour and angle of incidence of 0°.
A scale correction factor of 90 per cent has been applied to wind tunnel results.

	R.	Length: Max. depth.
 **Curtiss A–B scout (no radiator). Ref.: Curtiss Rept. No. 1400.**	2.1	4.8
 **Deperdussin (no radiator). Ref.: Eiffel.**	3.7	5.2
 **Deperdussin (no radiator). Ref.: Eiffel.**	5.0	5.3

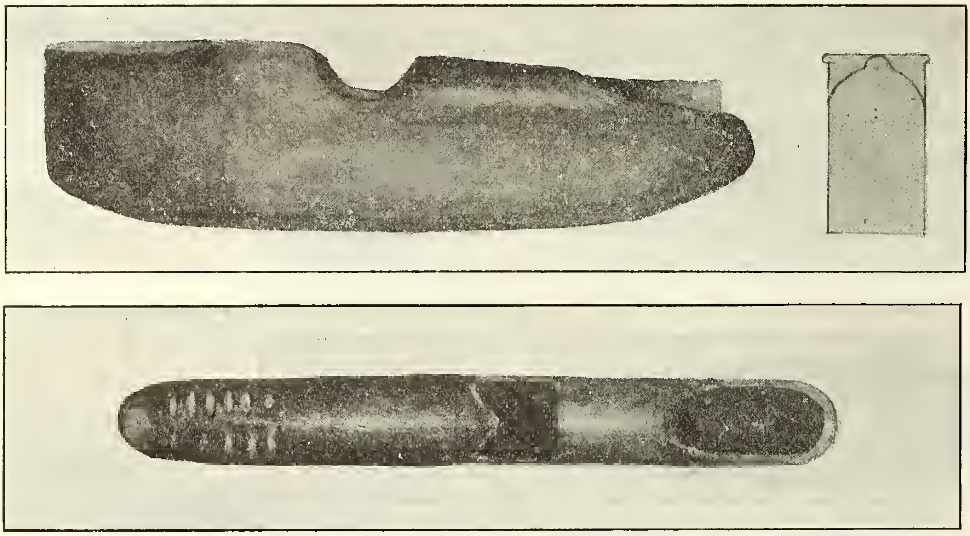
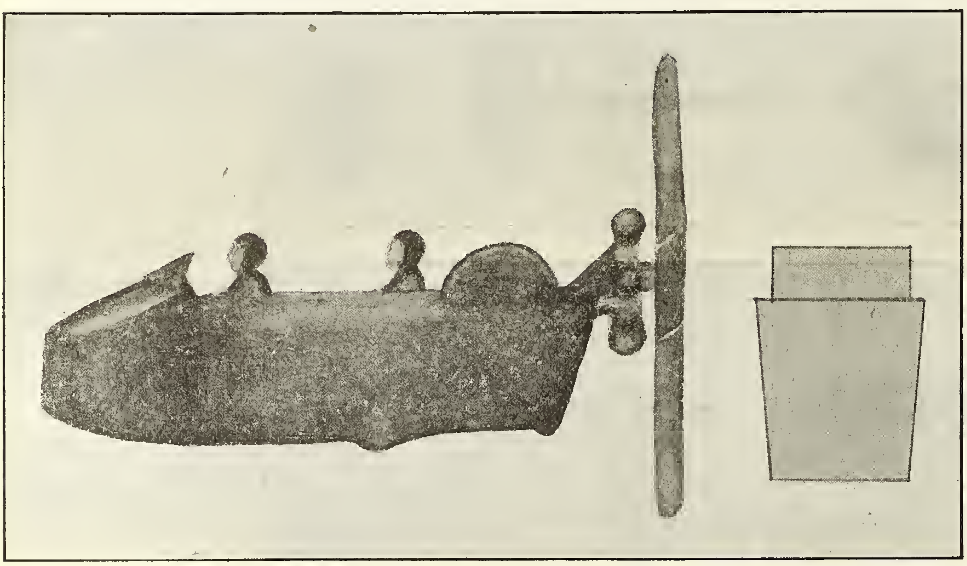
RESISTANCE OF BODIES—Continued.

R=resistance per square foot of maximum cross section at 70 miles per hour and angle of incidence of 0°.
A scale correction factor of 90 per cent has been applied to wind tunnel results.

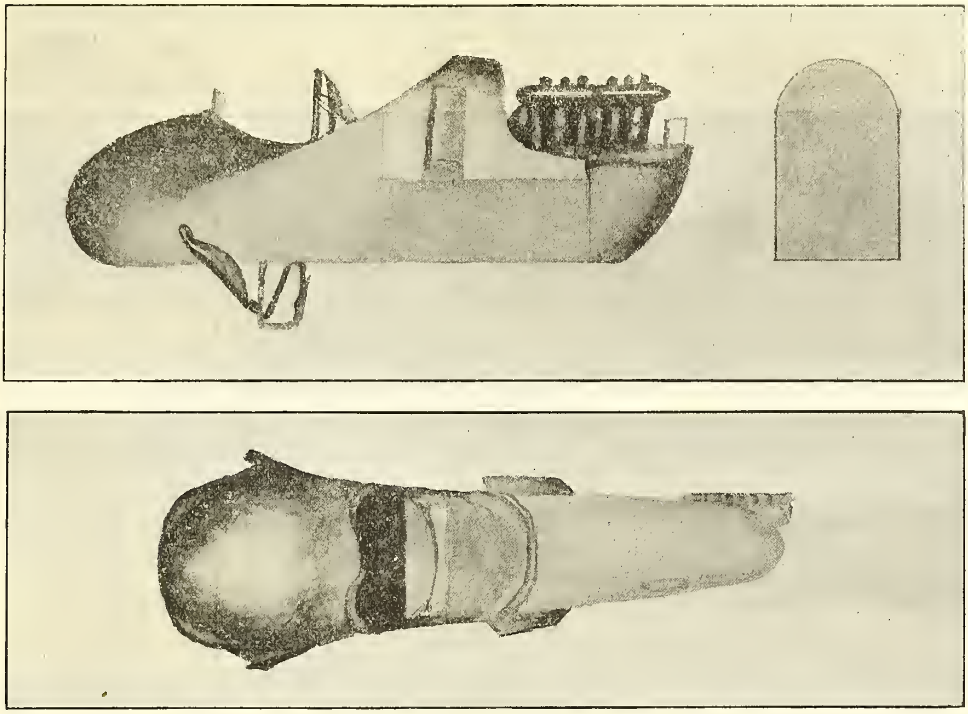
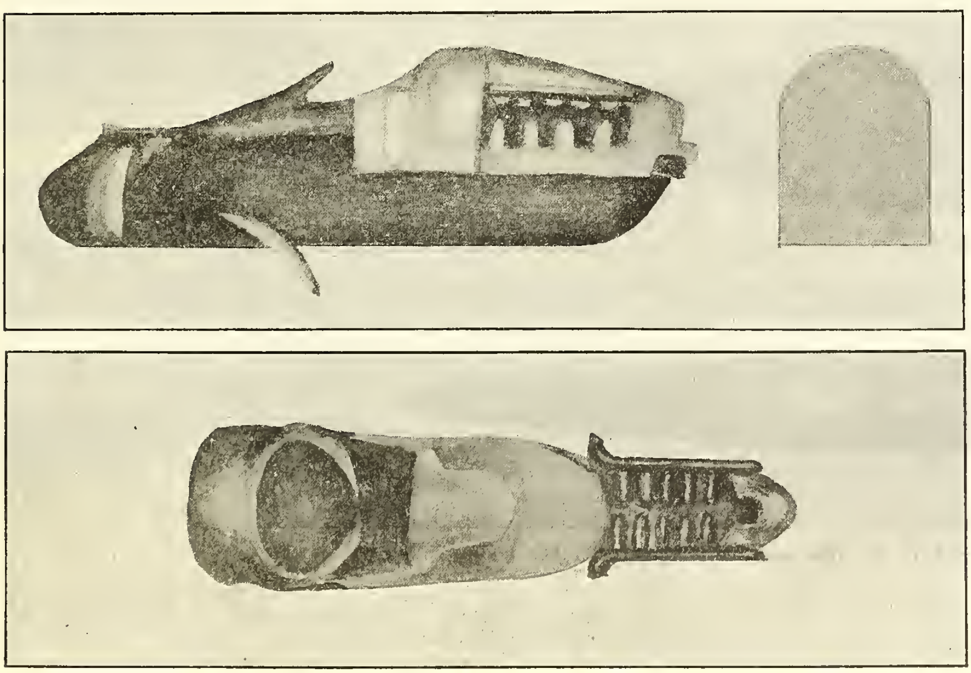
	R.	Length: Max. depth.
  **Curtiss V–1 (no radiator). Ref.: Curtiss Rept. No. 1473.**	2. 6	5. 7
  **No. 3 (no radiator). Ref.: A. C. A.**	3. 6 (with tail) 2. 2 (without tail)	3. 5

RESISTANCE OF BODIES—Continued.

R=resistance per square foot of maximum cross section at 70 miles per hour and angle of incidence of 0°.
A scale correction factor of 90 per cent has been applied to wind tunnel results.

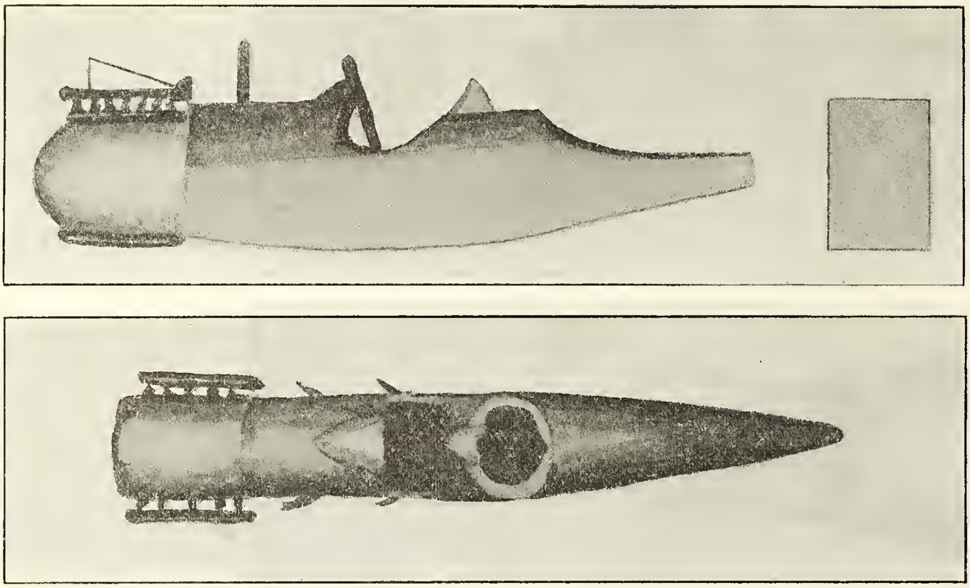
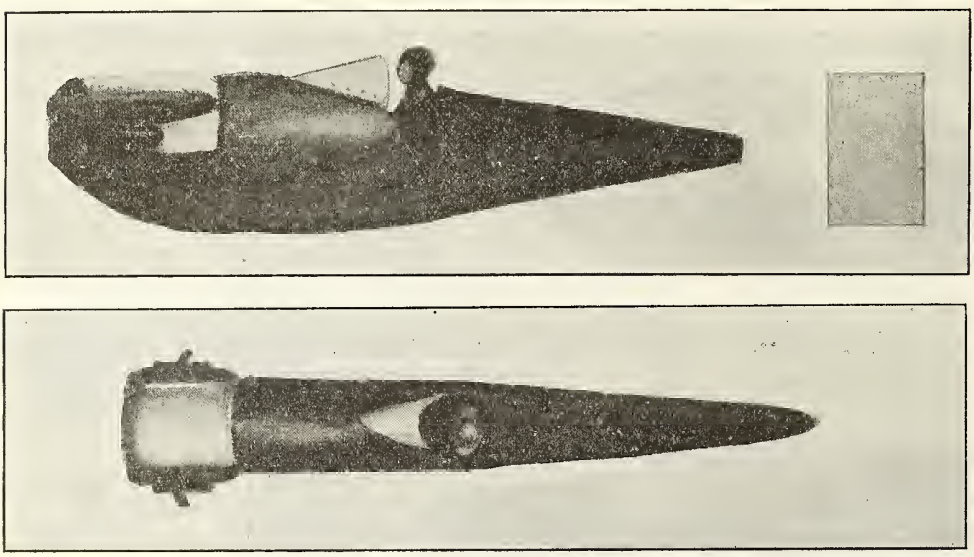
	R.	Length: Max. depth.
 N-1 seaplane (no radiator). Ref.: W. N. Y., No. 67.	4. 18	3. 6
 Farman No. 3 (no radiator). Ref.: Eiffel.	7. 2	4. 3

R=resistance per square foot of maximum cross section at 70 miles per hour and angle of incidence of 0°.
A scale correction factor of 90 per cent has been applied to wind tunnel results.

	R.	Length: Max. depth.
 **FE-2C. A. C. A. (No scale correction factor has been applied.)**	6. 4	4. 6
 **FE-2B. Ref.: A. C. A. (No scale correction factor has been applied.)**	6. 8	4. 6

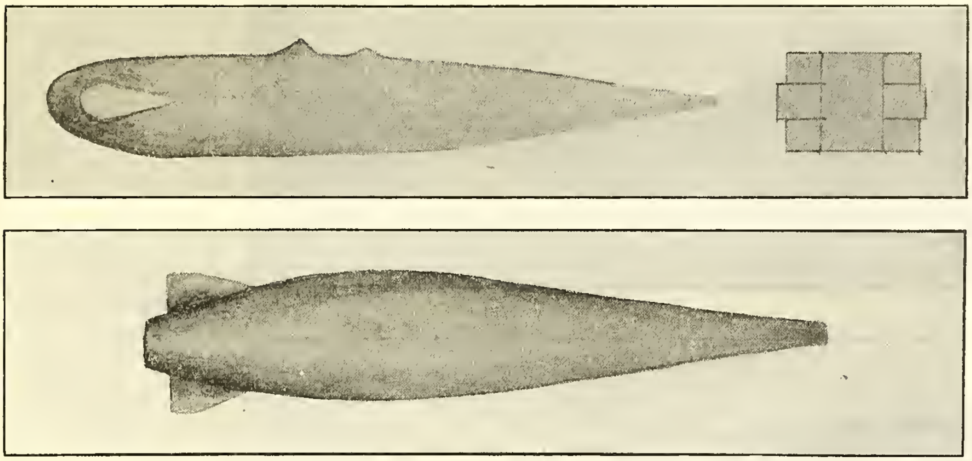
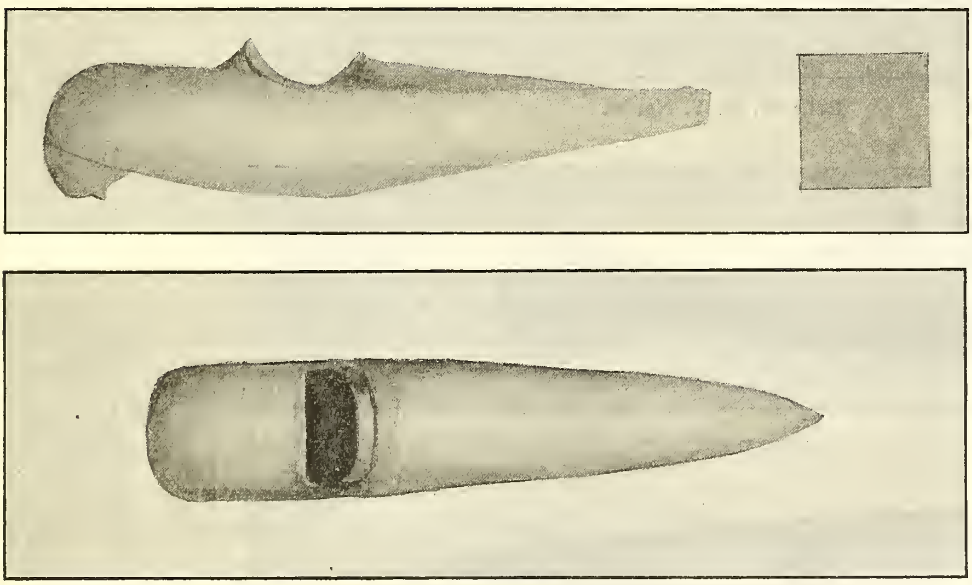
RESISTANCE OF BODIES—Continued.

R=resistance per square foot of maximum cross section at 70 miles per hour and angle of incidence of 0°.
A scale correction factor of 90 per cent has been applied to wind tunnel results.

	R.	Length: ax. depth.
 RE–8 (no radiator). Ref.: A. C. A. (No scale correction factor has been applied.)	5. 6	5. 7
 SE–5. Ref.: A. C. A. (No scale correction factor has been applied.)	6. 3	5. 6

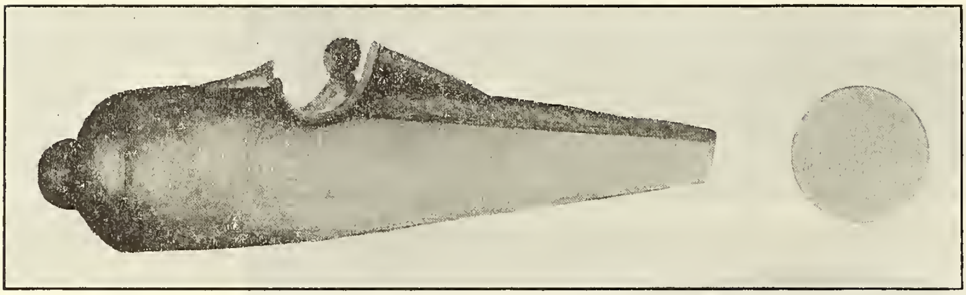
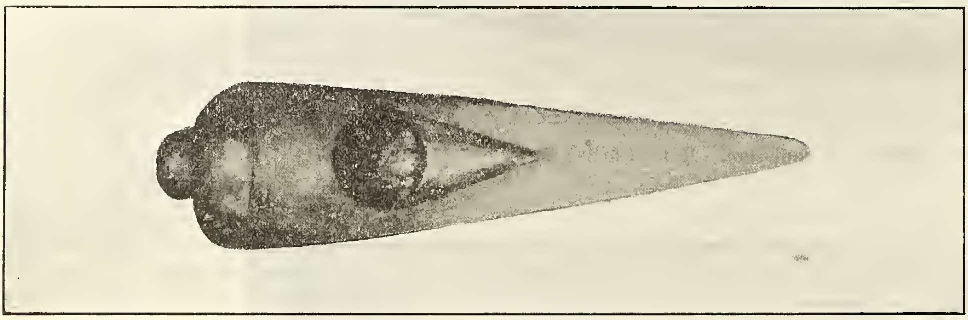
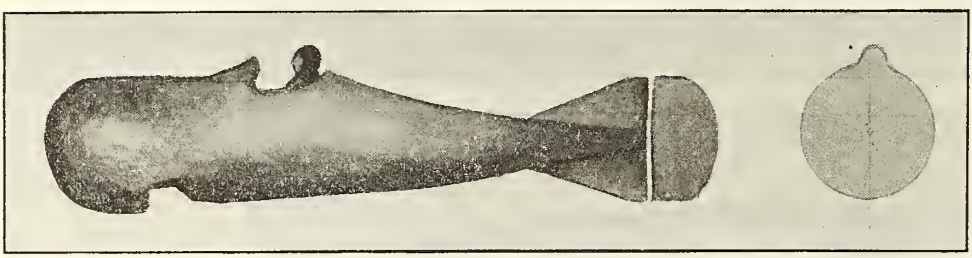
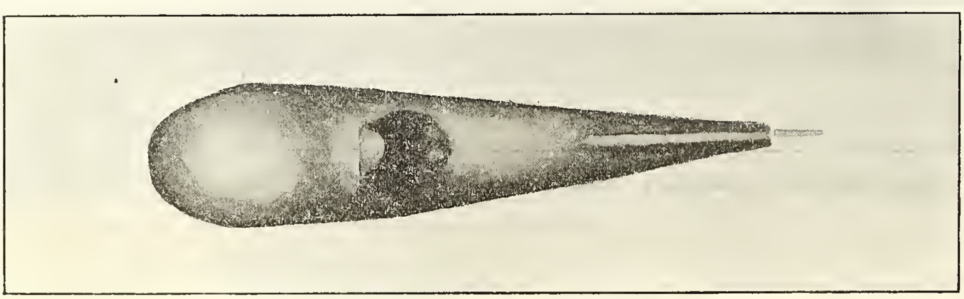
RESISTANCE OF BODIES—Continued.

R=resistance per square foot of maximum cross section at 70 miles per hour and angle of incidence of 0°.
A scale correction factor of 90 per cent has been applied to wind tunnel results.

	R.	Length: Max. depth.
 Curtiss torpedo plane (no radiator). Ref.: Curtiss Rept. No. 1530.	2.7	7.2
 Sopwith biplane type DS (no radiator). Ref.: A. C. A.	3.1	5.9

RESISTANCE OF BODIES—Continued.

R=resistance per square foot of maximum cross section at 70 miles per hour and angle of incidence of 0°.
A scale correction factor of 90 per cent has been applied to wind tunnel results.

	R.	Length: Max. depth.
  Ref.: W. N. Y. No. 20.	1. 9	4. 2
  SE-4A (no radiator). Ref.: A. C. A.	2. 4	4. 7

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AIRCRAFT DIVISION.
Engineering Section.

AIRCRAFT DESIGN DATA. NOTE NO. 11.

SAND LOAD TESTS FOR AIRPLANE WINGS.

STATIC SAND LOADING TEST.

All new types of airplanes shall, at the discretion of the bureau, be subjected to a static sand loading test.

The object of this test is to determine by experiment the factors of safety in an airplane. The actual loads under flying conditions are simulated, and by comparing the actual normal flying load with the sand load required to rupture the machine the factor of safety is obtained.

The sand load is applied to any part of the structure by means of sand bags. The bags are of heavy canvas construction and are filled with dry sand. They should be of varying weights, mostly 2, 5, and 10 pounds, with some 25 and 50 pounds. Each bag should be made up in sections, that is, quilted so as to fold or lie flat when used. Other necessary apparatus is linear wooden scales (graduated in tenths of an inch), jacks, and a level measuring in degrees.

DISTRIBUTION OF LOADING BETWEEN WINGS.

(*a*) In biplanes the distribution of loading between the wings is computed by the following formula:

$$W=\frac{11}{9}A_u\,x+A_l\,x.$$

W = total lift.

A_u = area upper wing.

A_l = area lower wing.

x = load per square foot on lower wing obtained by solving the above.

NOTE.—In all references to the upper or lower planes the machine shall be considered to be in normal horizontal flight.

(*b*) In triplanes the distribution of loading on the wings is computed by the following formula:

$$W=\frac{5}{4}A_u\,x+\frac{3}{4}A_m\,x+A_l x.$$

A_m = Area of middle wing.

Other notations same as for biplanes.

DISTRIBUTION OF LOADING ALONG THE SPAN.

When the variation of chord is small, the load is taken as uniform per foot run. Where the chord has an appreciable variation, as in the case of a tapered wing, the load per foot run will vary directly as the chord.

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DISTRIBUTION OF LOADING ALONG THE CHORD.

(See figure 1.)

The wing chord is divided into two parts beginning at the leading edge, the first part being six-tenths and the second part four-tenths.

Load on first part = F. S. × 0.8w.
Load on second part = F. S. × 0.4w.
Unit load (later defined) = w.
Factor of safety required = F. S.

The load is evenly distributed over each part and the center of gravity occurs at about four-tenths of the chord from the leading edge.

The airplane is placed upside down in such a position that the chord of the upper wing has an inclination of 15 degrees with the horizontal, the leading edge being on the higher level. This position gives a drag component along the chord of about one-fourth the loading, and approximately distributes the load equally between the front and rear spars. The wing structure including all wires, struts, etc. is to be set up as in normal flight. The fuselage or a dummy is to be placed on a suitable stand and trestles for preventing accident at the final break are placed at intervals under the wing and also to serve as a foundation for the jacks used to support the wing during the process of loading. These jacks are placed along the front and rear slightly to one side of each point of attachment of the linear scales described later. Before any loads are applied the wings are jacked up so as to be relieved of their own dead weight. Under this condition the wings are given their proper alignment and all wires and cables adjusted such that the initial tension is a minimum.

The sand bags are placed on the ground surrounding the wings in a well-ordered fashion. The wings are marked off with strings locating the load distribution relative to the chord.

Without removing the jacks the initial or zero deflection for no load is recorded. The following method of measuring deflection is recommended: linear graduated scales are secured to the under surface of the wings in a vertical position at strut points, points of attachment of overhang wires, midway between these points, and at the extreme tips of wings. Fixed light wires, tightly drawn in a horizontal direction are led past the edges of the scales. The deflection may then be determined by reading the movement of the scales relative to the wires.

The first load is now placed on the wings. The application of the load is made in increments in terms of a unit load. The unit load is equal to the gross weight of the airplane minus the weight of the wing structure. The gross weight is the weight of the complete machine fully loaded for flight. The test load is equal to the unit load multiplied by the factor of safety required. Since for zero reading the wings are not supporting even their own weight, the weight of sand placed on the wings for the first loading is equal to the unit load (w) minus the weight of the wing structure. Subsequent loads are each equal to the unit load. An orderly method of procedure is to be adopted and the sand bags are applied as nearly simultaneously and distributed as evenly as possible. The jacks are then released slowly and evenly and deflection readings taken. Before applying each new load, that is after taking each deflection reading, the jacks are replaced and set up just enough to bear lightly on the wing. The wings are loaded and deflections recorded as above for each additional unit load up to and including 80 per cent of the test load. At this point, the sand bags are removed and after a period of one hour the permanent deflection or set is recorded. The wings are again loaded to 80 per cent of the test load and the deflection taken. Additional loads of one-half the unit load are now added and deflections recorded until failure occurs.

All data should be carefully tabulated and deflection curves drawn for the different loadings for comparison with subsequent machines of the type.

As an alternative method of testing there may be allowed at the discretion of the bureau the sand loading of one set of right or left wings complete, assembled and attached to a jig instead of to the airplane body. The jig is specially prepared to receive the wing structure in an inverted position inclined at 15 degrees to the horizontal. Nose wires are led from the wings at the proper angle, and the entire set-up is made to conform to the installation on the complete machine. The sand loading test is then carried out in accordance with the method given for the complete airplane.

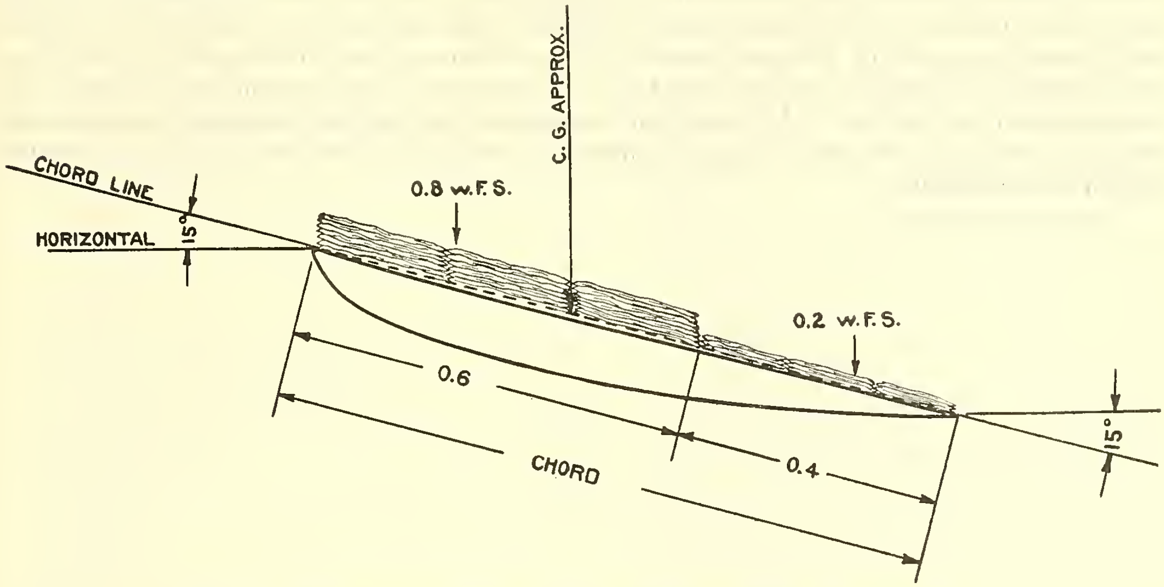

Fig. 1.--Diagram showing method and distribution of loading along the chord.

The machine tested to destruction is deemed to have passed the tests satisfactorily if found capable of maintaining the following loads:

Gross weight of airplane:	Unit loads before failure.
Under 5 tons	6
5 to 10 tons	5
Over 10 tons	4

When an airplane has been tested to destruction with satisfactory results, it is regarded as a standard for the type, and no other will, in general, be subjected to the same test. Subsequent machines may, at the discretion of the bureau, be tested in a similar manner up to two unit loads for airplanes under 10 tons gross weight, or one and a half units for heavier machines.

The airplane is deemed to have passed the tests satisfactorily if no deflection is more than 5 per cent greater than noted, with equal loading, in the machine tested to destruction. If however, the standard airplane showed an ultimate strength of x per cent greater than the factor of safety specified, the excess deflection permitted in subsequent machines will be $x+5$ per cent.

MULTI-ENGINE AIRPLANES.

For multi-engine airplanes where the engines are mounted between the wings and not on the center line of the airplane a special method of testing is adopted. In the case of a flying boat, for example, it is not practical to invert the whole machine. Either the whole or one half of the wing structure is mounted in a jig that will simulate exactly the mounting of the wings on the complete machine. Under no condition are any stressed wires or bracing to be omitted.

The procedure of testing is the same as previously outlined, but it is necessary to take care of the load imposed on the structure by the engines. At the center of gravity of attachment of the engine a lift of vertical force is applied. The magnitude of this lift force is equal to N times the weight of this part, where N is the number of unit loads applied to the wings at any instant of testing. This vertical load is obtained by running a rope or cable above the structure and over a block. At the end of the rope a dynamometer is attached or a platform on which sand bags are placed to give the desired loading. The friction in the block or pulley should be a minimum.

(Ref. N. Spec. Fr. Com. Br. A A.)

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AIRCRAFT DESIGN DATA. NOTE NO. 12.

WOOD IN AIRCRAFT CONSTRUCTION.

[Prepared by the Forest Products Laboratory, Forest Service, U. S. Department of Agriculture.]

CONTENTS.

98257—19—No. 12——1

MECHANICAL AND PHYSICAL PROPERTIES.

Wood differs from other structural materials in a great many ways, and the maximum efficiency in its use demands a thorough knowledge of the properties of wood and of the factors which influence these properties. In the following general discussion an attempt is made to explain the principal causes for the wide variations found in the strength of wood and to show how these variations may be largely eliminated in any group of material by proper specification and inspection.

VARIABILITY OF THE STRENGTH OF WOOD.

WOOD NON-HOMOGENEOUS.

Wood is exceedingly variable as compared with other structural materials. This variability is due to a number of factors, heretofore not well understood. For that reason any judgment of the strength of a piece was felt to be uncertain. The causes for variations in the properties of wood can now be given and their effects anticipated within reasonable limits.

VARIATION OF STRENGTH WITH LOCALITY OF GROWTH.

In some cases the locality of growth has an influence on the strength of the timber. For example, tests show a marked difference in strength between the Rocky Mountain and coast types of Douglas fir in favor of the coast type.

This influence of locality is usually overestimated. Different stands of the same species grown in the same section of the country may show as great differences as stands grown in widely separated regions, so that as a rule locality of growth can be neglected.

VARIATION OF STRENGTH WITH POSITION IN THE TREE.

In some instances specimens from different parts of the same tree have been found to show considerable difference in strength. In most cases, however, the wood of the highest specific gravity has the best mechanical properties regardless of its position in the tree. Where this is not the case, the toughest or most shock-resistant material is found near the butt. Above a height of 10 or 12 feet variations of mechanical strength correspond to the variations of specific gravity. Some variations with position in cross section or distance from the pith of the tree have been found which could not be entirely accounted for by differences in specific gravity.

VARIATION OF STRENGTH WITH RATE OF GROWTH.

Strength is not definitely proportional to rate of growth, either directly or inversely.

Timber of any species which has grown with exceptional slowness is usually below the average of the species in strength values.

Among many of the hardwood species, material of very rapid growth is usually above the average in strength properties. Notable exceptions to this are found, however, and rapid growth is no assurance of excellence of material unless accompanied by relatively high specific gravity. This is particularly true of ash.

In the coniferous species, material of very rapid growth is very likely to be quite brash and below the average strength.

VARIATION OF STRENGTH WITH AMOUNT OF SUMMER WOOD.

In many species the proportion of summer wood is indicative of the specific gravity, and different proportions of summer wood are usually accompanied by different specific gravities and strength values. However, proportion of summer wood is not a sufficiently accurate indicator of strength to permit its use as the sole criterion for the acceptance or rejection of airplane material. After some practice one should be able, through observation of the proportion of summer wood, to decide whether any particular piece is considerably below, considerably above, or near the required specific gravity. Caution must be observed in applying this to ash, and perhaps to other hardwoods, since rapid-growth ash is sometimes very low in specific gravity in spite of a large proportion of summer wood. In such cases careful examination will show that the summer wood is less dense than usual.

VARIATION OF STRENGTH WITH SPECIFIC GRAVITY.

A piece of clear, sound, straight-grained wood of any species is not necessarily a good stick of timber. To determine the quality of an individual stick by means of mechanical tests is extremely difficult, because the variations in strength of timber due to variations in moisture content, temperature, speed of test, etc., are so great. Furthermore, a test for one strength property does not always indicate what the other properties of the timber are. Without actual and complete tests, the best criterion of the strength properties of any piece of timber is its specific gravity or weight per unit volume, weight being taken when the wood is completely dry and volume when the wood is at some definite condition of seasoning or moisture content. Specific gravity based on oven-dry volume is greater than that based on the volume at any other moisture condition in proportion to the shrinkage which takes place as the moisture is driven out and the wood is reduced to the oven-dry condition.

Accurate determinations made on seven species of wood, including both hardwoods and conifers, showed a range of only about $4\frac{1}{2}$ per cent in the density of the wood substance, or material of which the cell walls is composed. Since the density of wood substance is so nearly constant, it may be said that the specific gravity of a given piece of wood is a measure of the amount of wood substance contained in a unit volume of it. Very careful analyses based on a vast amount of data have shown that wood of high specific gravity has greater strength than that of low specific gravity. Some fairly definite mathematical relations between specific gravity and the various strength properties have been worked out. Some of the strength properties (strength in compression parallel to grain and modulus of elasticity) vary directly as the first power of the specific gravity; others, however, vary with higher powers of the specific gravity, i. e., the strength property changes more rapidly than the specific gravity, a 10 per cent increase of specific gravity resulting in an increase in the strength properties of 15 per cent to even 30 per cent.

The rate of change in strength with changes of specific gravity is usually greater in individual specimens of a single species than in the averages for a number of species. This is illustrated by a comparison of figures 1 and 2. Figure 1 indicates that the modulus of rupture varies as the 5/4 power of the specific gravity when various species are considered, while figure 2 indicates that the relation of the crushing strength of individual specimens of white ash varies as the 3/2 power of the specific gravity. The modulus of rupture of spruce and of numerous other species has been found to vary as the 3/2 power of the specific gravity. Shock-resisting ability and other important properties vary as even higher powers of specific gravity. If an important airplane part is from wood 10 per cent below the specific gravity given in the specifications, it will not be just 10 per cent but at least 14.5 per cent inferior and perhaps more, depending on which particular property is of greatest importance in the part in question. If the specific gravity is 20 per cent low, the inferiority will not be less than 28.4 per cent. The

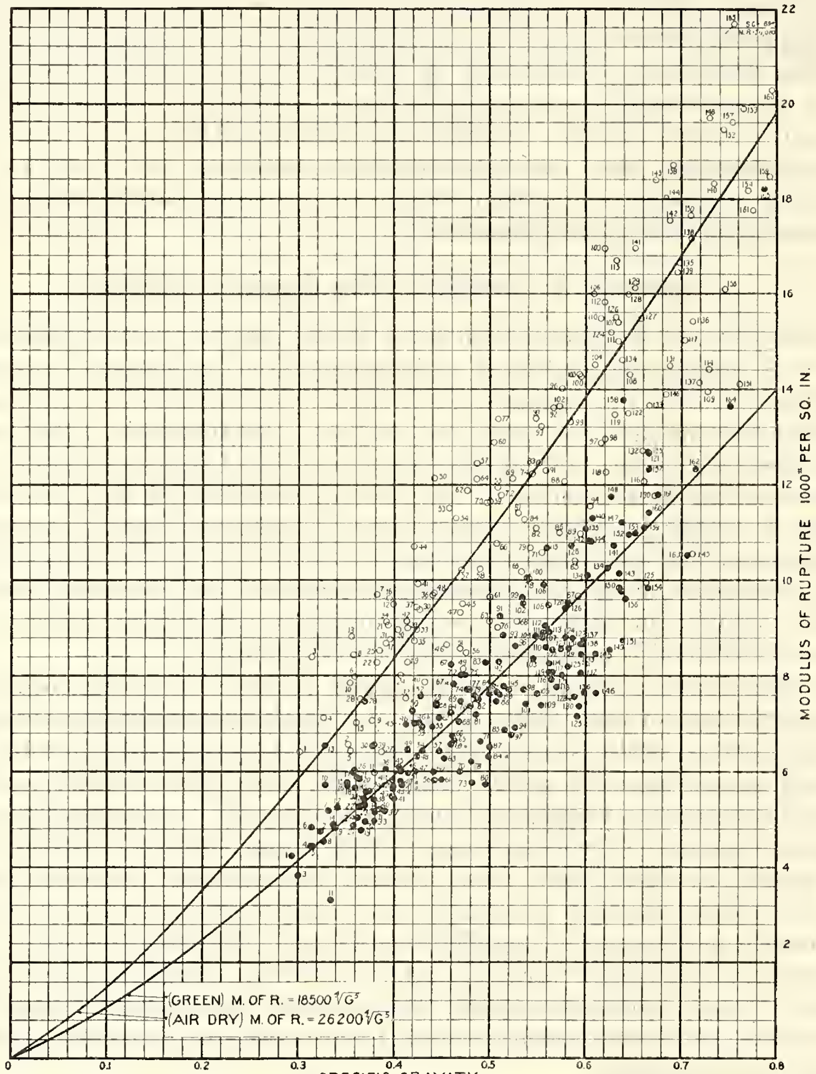


Fig. 1.—Relation between the modulus of rupture and specific gravity of various American woods.

lighter pieces of wood are usually exceedingly brash, especially when dry. The importance of admitting no material for airplane construction of lower specific gravity than given in the specifications is evident.

List of species and reference numbers for figure 1.

HARDWOODS.

Species.	Locality.	Reference No.	Species.	Locality.	Reference No.
Alder, red	Washington	30	Hickory—Continued.		
Ash:			Pignut	Pennsylvania	160
Biltmore	Tennessee	91	Do	West Virginia	161
Black	Michigan	60	Shagbark	Mississippi	140
Do	Wisconsin	70	Do	Ohio	152
Blue	Kentucky	99	Do	Pennsylvania	143
Green	Louisiana	93	Do	West Virginia	153
Do	Missouri	100	Water	Mississippi	141
Pumpkin	do	79	Holly, American	Tennessee	87
White	Arkansas	106	Hornbeam	do	149
Do	New York	128	Laurel, mountain	do	145
Do	West Virginia	83	Locust:		
Aspen	Wisconsin	23	Black	do	158
Largetooth	do	20	Honey	Indiana	162
Basswood	Pennsylvania	12	Madrona	California	101
Do	Wisconsin	5	Do	Oregon	128a
Beech	Indiana	110	Magnolia	Louisiana	66
Do	Pennsylvania	98	Maple:		
Birch:			Oregon	Washington	58
Paper	Wisconsin	73	Red	Pennsylvania	69
Sweet	Pennsylvania	129	Do	Wisconsin	92
Yellow	do	107	Silver	do	56
Do	Wisconsin	103	Sugar	Indiana	104
Buckeye, yellow	Tennessee	9	Do	Pennsylvania	108
Buckthorn, cascara	Oregon	84a	Do	Wisconsin	124
Butternut	Tennessee	27	Oak:		
Do	Wisconsin	21	Bur	do	125
Chinquapin, western	Oregon	48b	California black	California	80
Cherry:			Canyon live	do	163
Black	Pennsylvania	72	Chestnut	Tennessee	121
Wild red	Tennessee	24	Cow	Louisiana	133
Chestnut	Maryland	46	Laurel	do	116
Do	Tennessee	40	Post	Arkansas	130
Cottonwood, black	Washington	6	Do	Louisiana	137
Cucumber tree	Tennessee	59	Red	Arkansas	119
Dogwood:			Do	Indiana	118
Flowering	do	151	Do	Louisiana	117
Western	Oregon	125a	Do	Tennessee	97
Elder, pale	do	69a	Highland Spanish	Louisiana	94
Elm:			Lowland Spanish	do	142
Cork	Wisconsin, Marathon County.	126	Swamp white	Indiana	150
			Tanbark	California	115
Do	Wisconsin, Rusk County.		Water	Louisiana	111
			White	Arkansas	132
Slippery	Indiana	102	Do	Indiana	138
Do	Wisconsin	74	Do	Louisiana, Richland Parish.	136
White	Pennsylvania	55			
Do	Wisconsin	53	Do	Louisiana, Winn Parish.	131
Greenheart		165	Willow	Louisiana	109
Gum:			Yellow	Arkansas	122
Black	Tennessee	68	Do	Wisconsin	105
Blue (Eucalyptus)	California	147	Osage orange	Indiana	164
Cotton	Louisiana	76	Poplar, yellow (tulip tree).	Tennessee	35
Red	Missouri	54			
Hackberry	Indiana	90	Rhododendron, great	do	85
Do	Wisconsin	78	Sassafras	do	51
Haw, pear	do	146	Serviceberry	do	156
Hickory:			Silverbell tree	do	49
Big shellbark	Mississippi	135	Sourwood	do	89
Do	Ohio	154	Sumac, staghorn	Wisconsin	61
Butternut	do	139	Sycamore	Indiana	63
Mockernut	Mississippi	144	Do	Tennessee	65
Do	Pennsylvania	159	Umbrella, Fraser	do	45
Do	West Virginia	155	Willow:		
Nutmeg	Mississippi	112	Black	Wisconsin	11
Pignut	do	148	Western black	Oregon	43a
Do	Ohio	157	Witch hazel	Tennessee	114

List of species and reference numbers for figure 1—Continued.

CONIFERS.

Species.	Locality.	Reference No.
Cedar:		
Incense	California	26
Western red	Montana	2
Do	Washington	10
White	Wisconsin	1
Cypress, bald	Louisiana	62
Douglas fir	California	45a
Do	Oregon	67a
Do	Washington, Chehalis County.	46a
Do	Washington, Lewis County.	75
Do	Washington and Oregon.	67
Do	Wyoming	84
Fir:		
Alpine	Colorado	4
Amabilis	Oregon	39
Do	Washington	18
Balsam	Wisconsin	14
Grand	Montana	36
Noble	Oregon	16
White	California	17
Hemlock:		
Black	Montana	47
Eastern	Tennessee	52
Do	Wisconsin	15
Western	Washington	50
Larch, western	Montana	84
Do	Washington	64
Pine:		
Cuban	Florida	127
Jack	Wisconsin	43
Jeffrey	California	33
Loblolly	Florida	88
Lodgepole	Colorado	31
Do	Montana, Gallatin County.	35a

Species.	Locality.	Reference No.
Pine—Continued.		
Lodgepole	Montana, Granite County.	41a
Do	Montana, Jefferson County.	40a
Do	Wyoming	34
Longleaf	Florida	123
Do	Louisiana, Lake Charles.	113
Do	Louisiana, Tangipahoa Parish.	96
Do	Mississippi	95
Norway	Wisconsin	57
Pitch	Tennessee	71
Pond	Florida	86
Shortleaf	Arkansas	77
Sugar	California	22
Table Mountain	Tennessee	82
Western white	Montana	42
Western yellow	Arizona	19
Do	California	37
Do	Colorado	41
Do	Montana	32
White	Wisconsin	25
Redwood	California, Albion	28
Do	California, Korbel	13
Spruce:		
Engelmann	Colorado, Grand County.	8
Do	Colorado, San Miguel County.	3
Red	New Hampshire	44
Do	Tennessee	29
White	New Hampshire	7
Do	Wisconsin	38
Tamarack	do	81
Yew, western	Washington	134

The minimum strength values which may be expected of a particular lot of lumber can be raised a good deal by eliminating a relatively small portion of the lighter material. This lightweight material can, as a rule, be detected by visual inspection. In order to train the visual inspection and to pass judgment on questionable individual pieces, frequent specific gravity determinations are necessary.

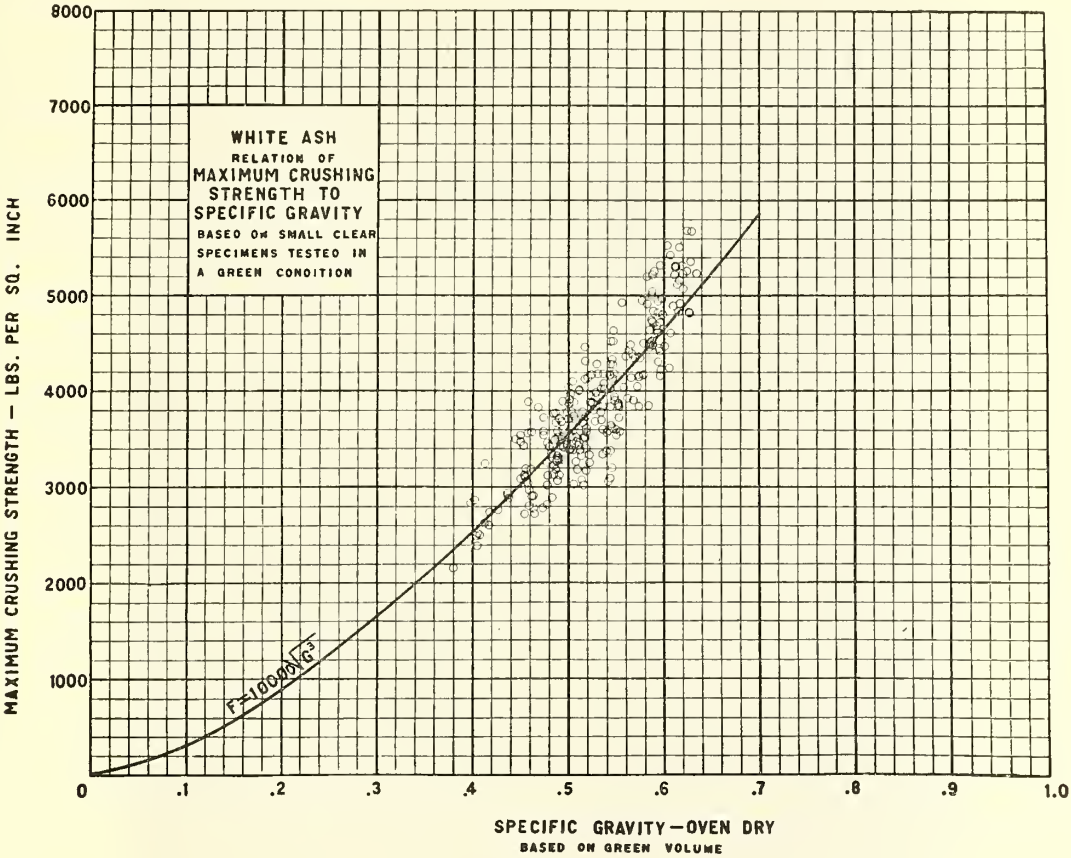


Fig. 2.

A specific gravity determination is relatively simple to make, and it is probably a better criterion of all the qualities of the piece than any single mechanical test which is likely to be applied; also the specific gravity determinations need no adjustment such as would be necessary on account of the varied conditions of a mechanical test.

VARIATION OF STRENGTH WITH MOISTURE CONTENT.

When a piece of green or wet wood is dried, no change in mechanical properties takes place until the fiber-saturation point is reached.* The changes beyond this point for small test specimens free from defects and very carefully dried are illustrated in figures 3 and 4. These

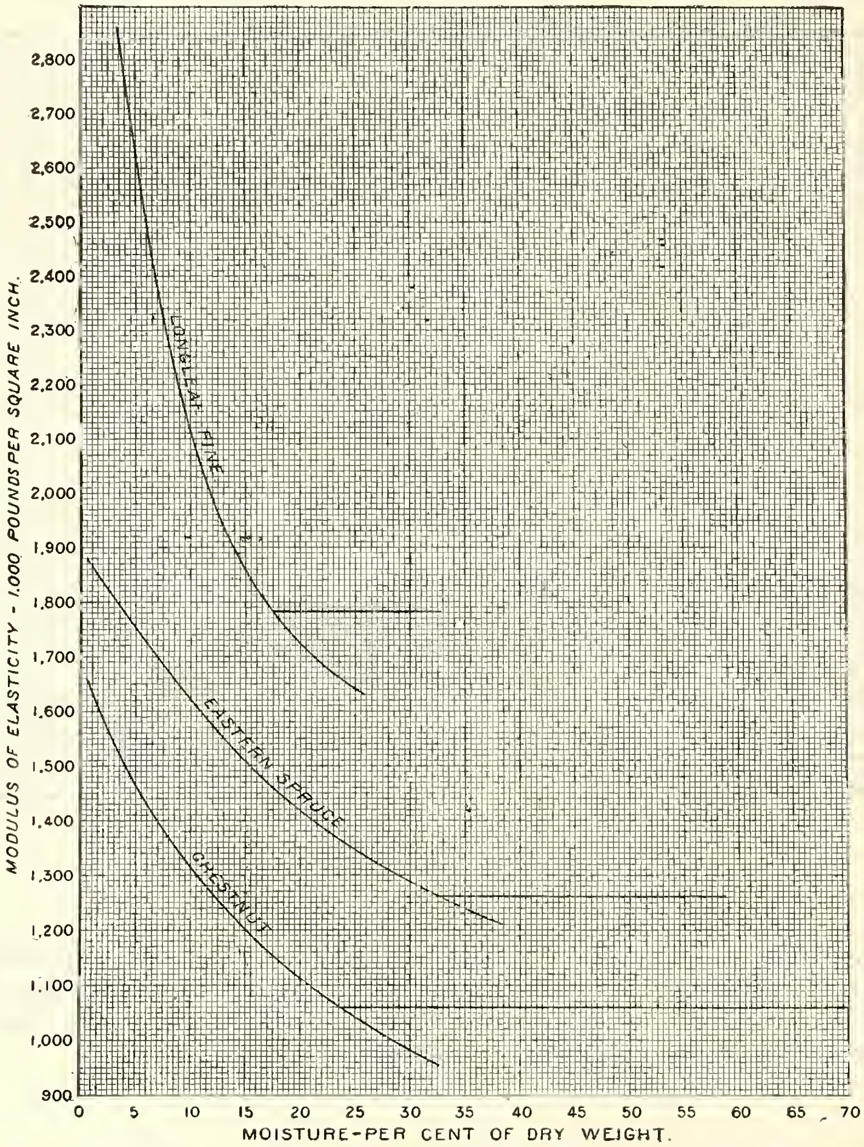


Fig. 3.—Relation between the stiffness (modulus of elasticity) in binding and moisture content, for three species.

figures show that the moisture content at the fiber-saturation point differs for different species. It will be noted that the influence of moisture is smaller in tests of shearing strength and compression perpendicular to the grain than in bending and compression parallel to the grain.

* The eucalypts and some of the oaks are exceptions to this rule.

Furthermore, there is no definite break at or near the fiber-saturation point in the moisture-strength curves for shear and compression perpendicular to the grain. In the case of shear this failure to show large increases in strength is probably due to checks which form as the material dries.

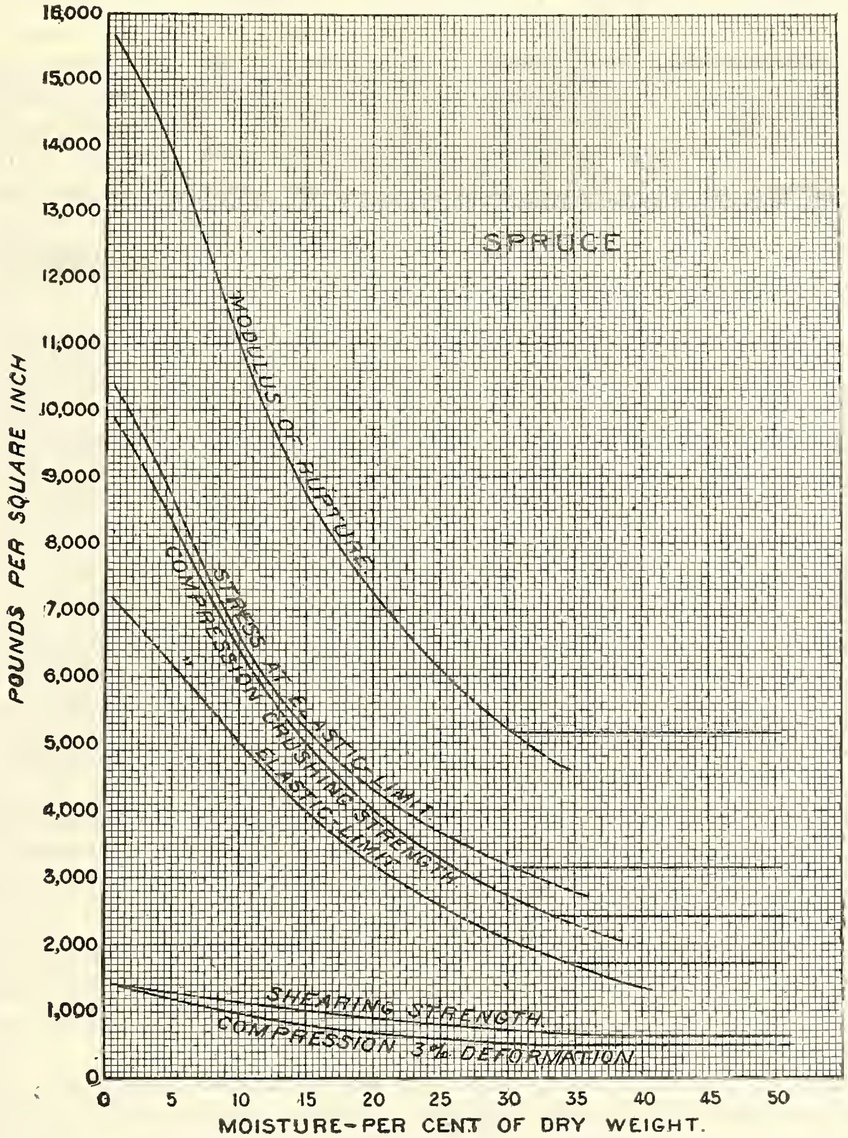


Fig. 4.—Comparison of the relation between strength and moisture content for red spruce in various kinds of tests. (The lowest curve is for compression at right angles to grain.)

The moisture content at the fiber-saturation point varies not only with the species but with different specimens of the same species. The percentage change of strength which results from a given change of moisture also varies with the species and with individual specimens of the species.

The form of the curves shown in figures 3 and 4 applies only to small clear pieces very carefully dried and having a practically uniform moisture content throughout. If the moisture be unequally distributed in the specimen, as is the case of large timbers rapidly dried or of "case-hardened" pieces, the outer shell may be drier than the fiber-saturation point while the inside still contains free water. The resulting moisture-strength curve will be higher than the curve from carefully dried pieces and will be so rounded off from the driest to the wettest condition as to obscure entirely the fiber-saturation point (see fig. 5).

The increase in strength which takes place in drying wood depends upon the specimen and upon the care with which the drying process is carried out. Furthermore, while the strength of the fibers is no doubt greatly increased by any reasonable drying process, the increase of the strength of a piece of timber taken as a whole may be very much less. Knots are more or less loosened, checking takes place, and shakes are further developed. In large bridge and building timbers these effects are so great that it is not considered safe to figure on such timbers having greater strength when dry than when green. When the pieces are small and practically free

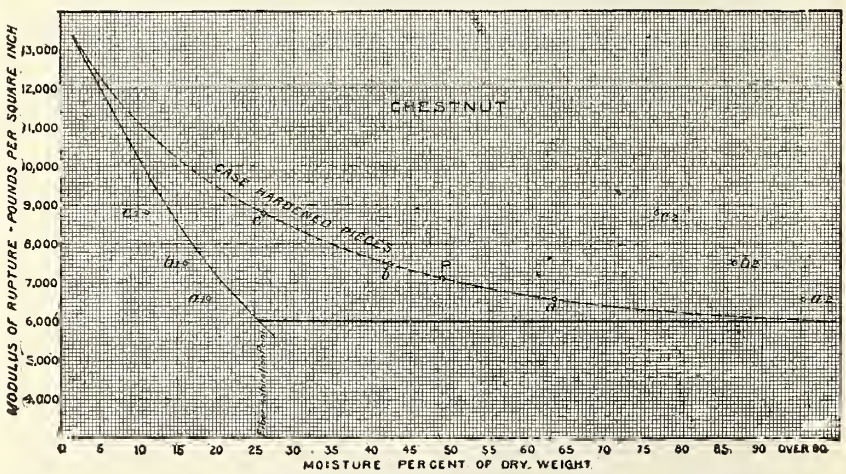


Fig. 5.—Effect of case-hardening upon the form of the moisture-strength curve in bending tests. The upper curve is from case-hardened specimens, the lower curve from uniformly dired specimens.

from defects, as in airplane construction, proper drying with careful control of temperature and humidity increases the strength of material very greatly. In whatever way wood is dried, upon its being resoaked and brought back to the original green or wet condition it is found to be weaker than it was originally. So when it is said that wood has been injured in the drying process it must be taken to mean that it is weaker than it should have been after drying and while still in a dried condition.

When a stick of timber dries out below the fiber-saturation point (that is, when it has lost all its free moisture and the moisture begins to leave the cell walls), the timber begins to shrink and change in its mechanical properties. Also numerous stresses are set up within the timber. Under severe or improper drying conditions the stresses may be great enough to practically ruin the material for purposes where strength is important. Improper drying conditions, however, do not of necessity mean fast drying conditions. When properly dried, the timber gains in its fiber stress at elastic limit, its modulus of rupture, maximum crushing strength, etc. It bends farther at the elastic limit when dry than when green, but does not bend so far at the maximum load. After having been bent to the maximum load dry timber breaks more suddenly than green timber of the same species—that is, dry timber is more brash than green, although it withstands greater stresses and is stiffer.

DEFECTS AFFECTING STRENGTH.

DIAGONAL AND SPIRAL GRAIN.

Diagonal grain is produced when the saw cut is not made parallel to the direction of the fibers. It can usually be avoided by careful sawing unless it is caused by crooks in the log. Spiral grain, on the other hand, results from a spiral arrangement of the wood fibers in the tree.

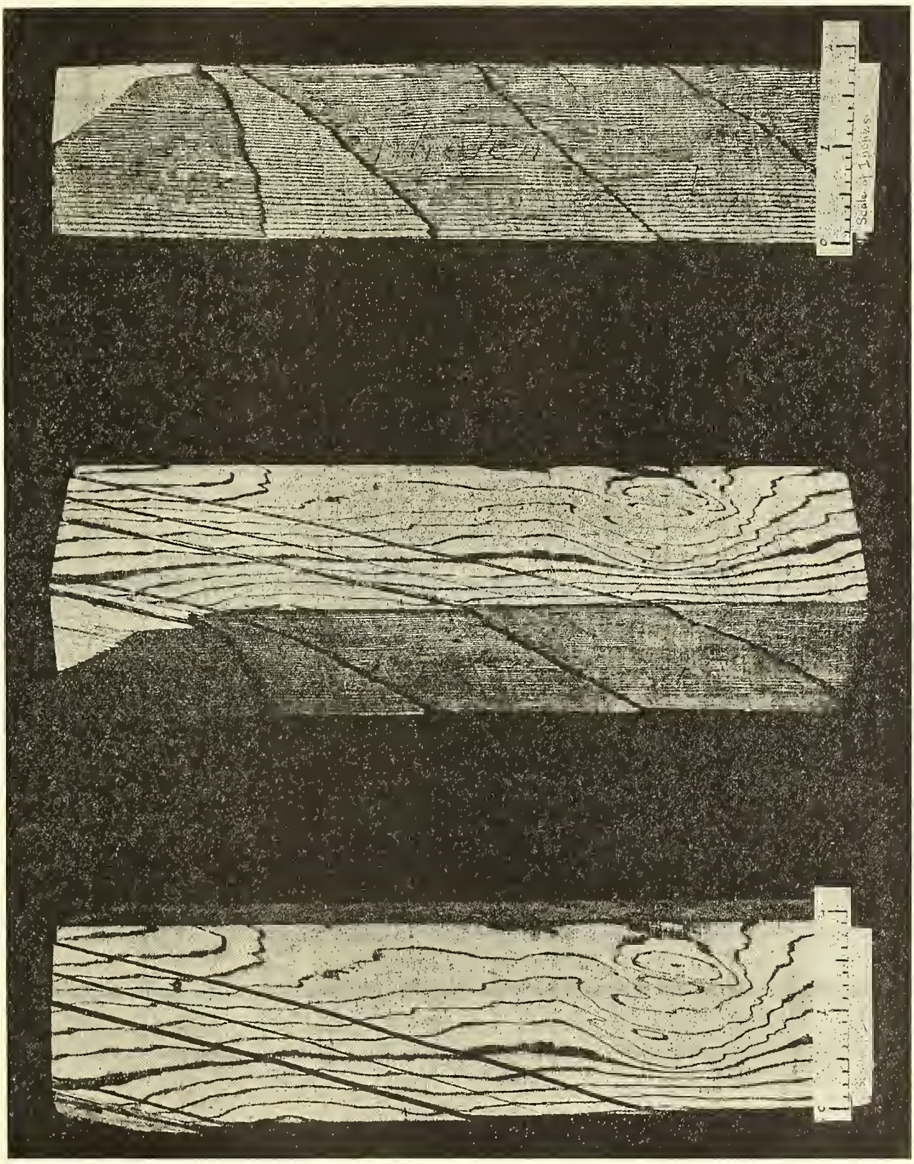


Fig. 6.—Spiral grain in Sitka spruce.

If a log is spiral grained, it is impossible to secure straight-grained material, except in small pieces, from the spiral-grained part. The effect of spiral grain is illustrated in figure 6, which shows three views of a piece of Sitka spruce. The center part of a log may be straight grained and the outer part spiral grained or vice versa.

Figures 7 to 14, inclusive, show the weakening effect of spiral or diagonal grain upon various strength properties of Sitka spruce and Douglas fir. The data are based upon about 1,400 static bending tests, made upon clear specimens, third point loading, 45-inch span. Similar impact bending tests have shown similar weakening with increasing slope of grain.

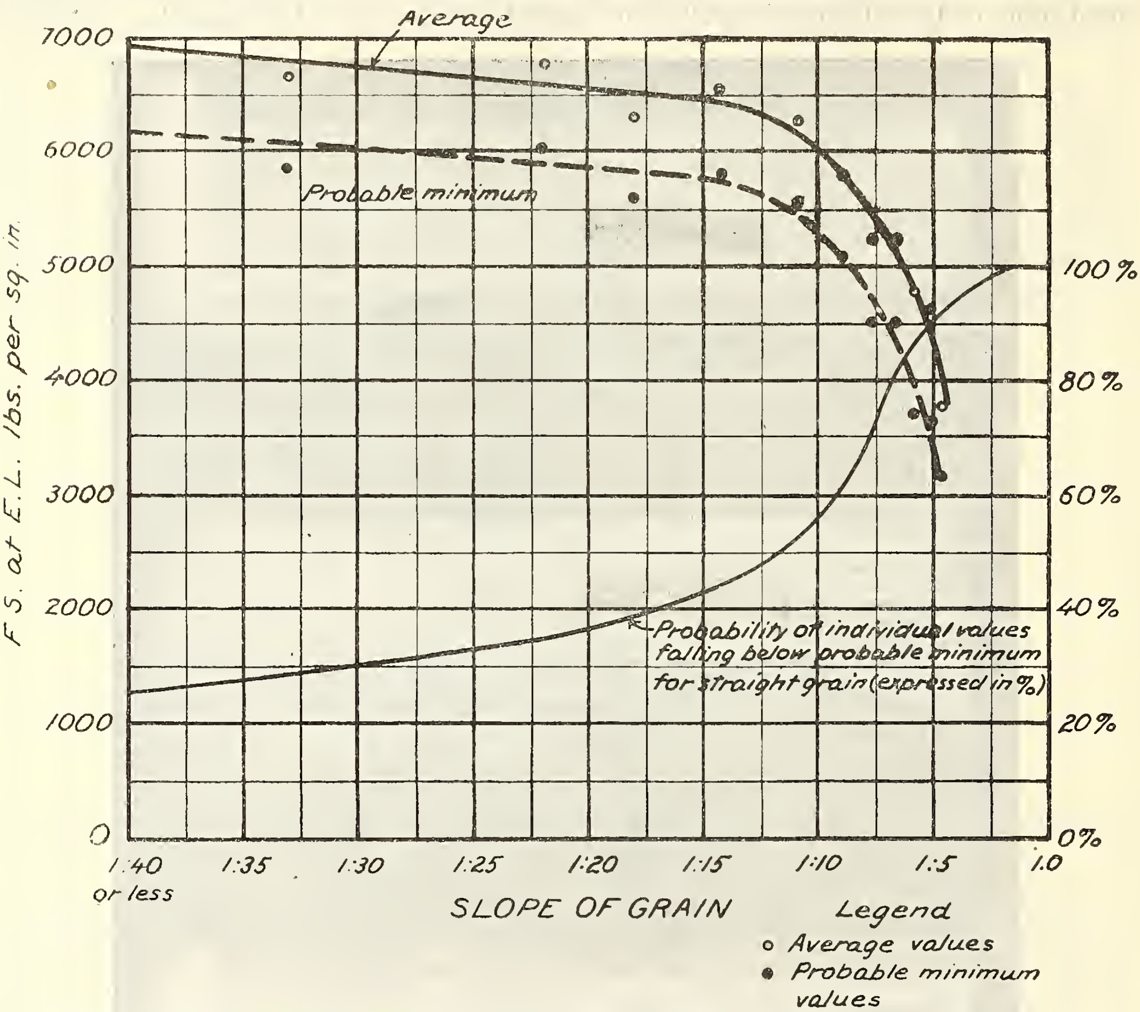


Fig. 7.—The effect of spiral and diagonal grain on the fiber stress at the elastic limit; Sitka spruce.

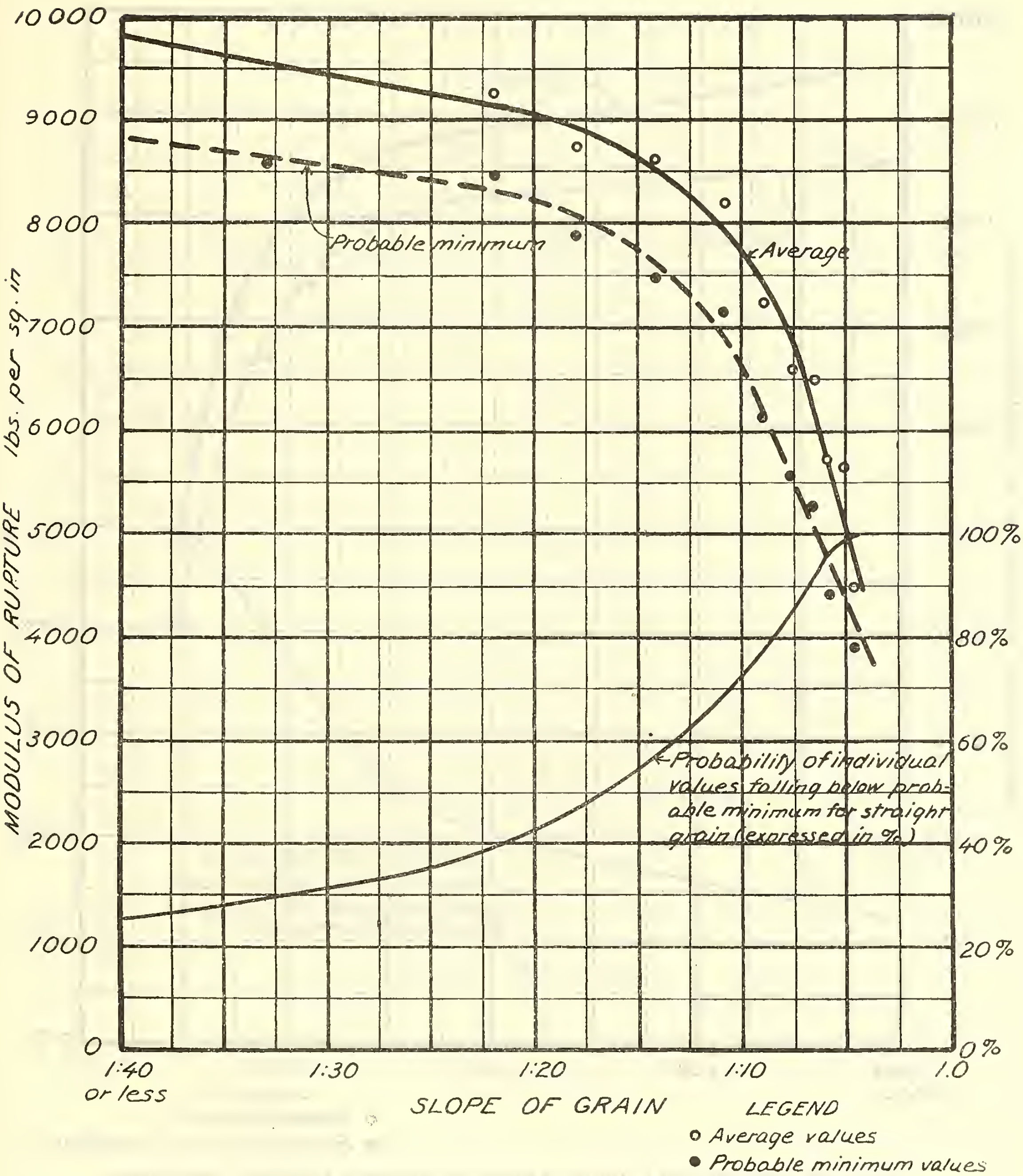


Fig. 8.—The effect of spiral and diagonal grain on the modulus of rupture; Sitka spruce.

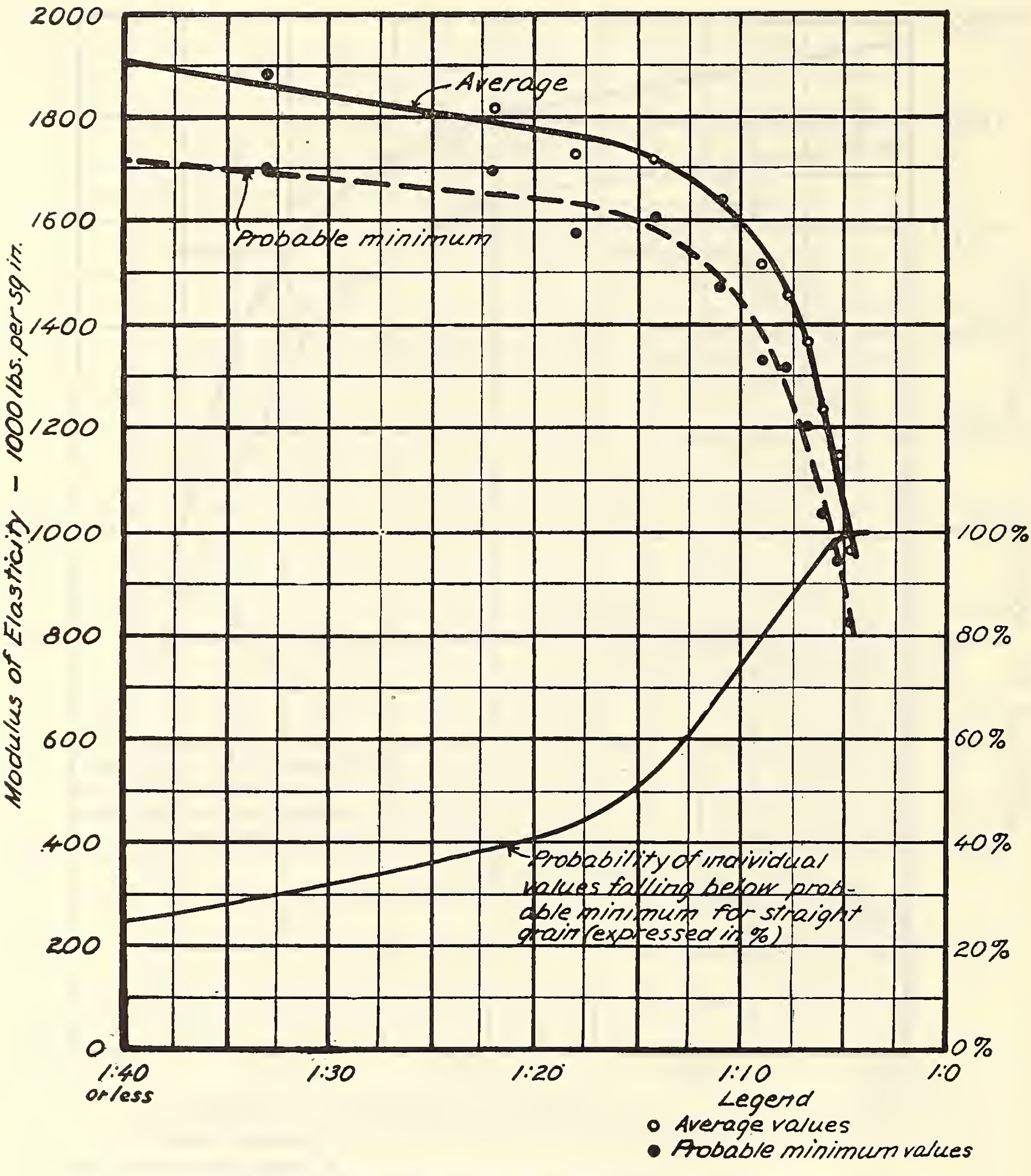


Fig. 9.—The effect of spiral and diagonal grain on the modulus of elasticity; Sitka spruce.

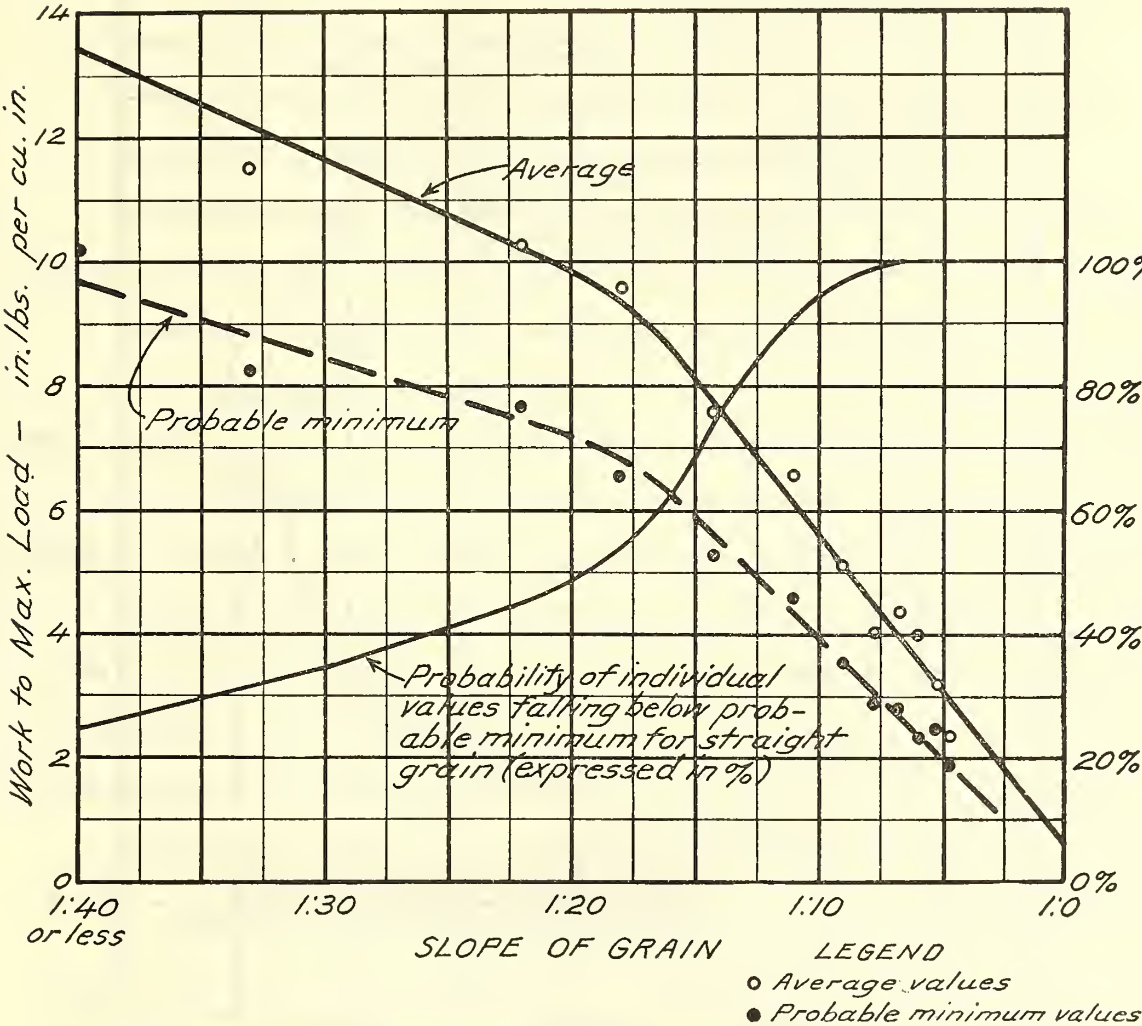


Fig. 10.—The effect of spiral and diagonal grain on the work to maximum load; Sitka spruce.

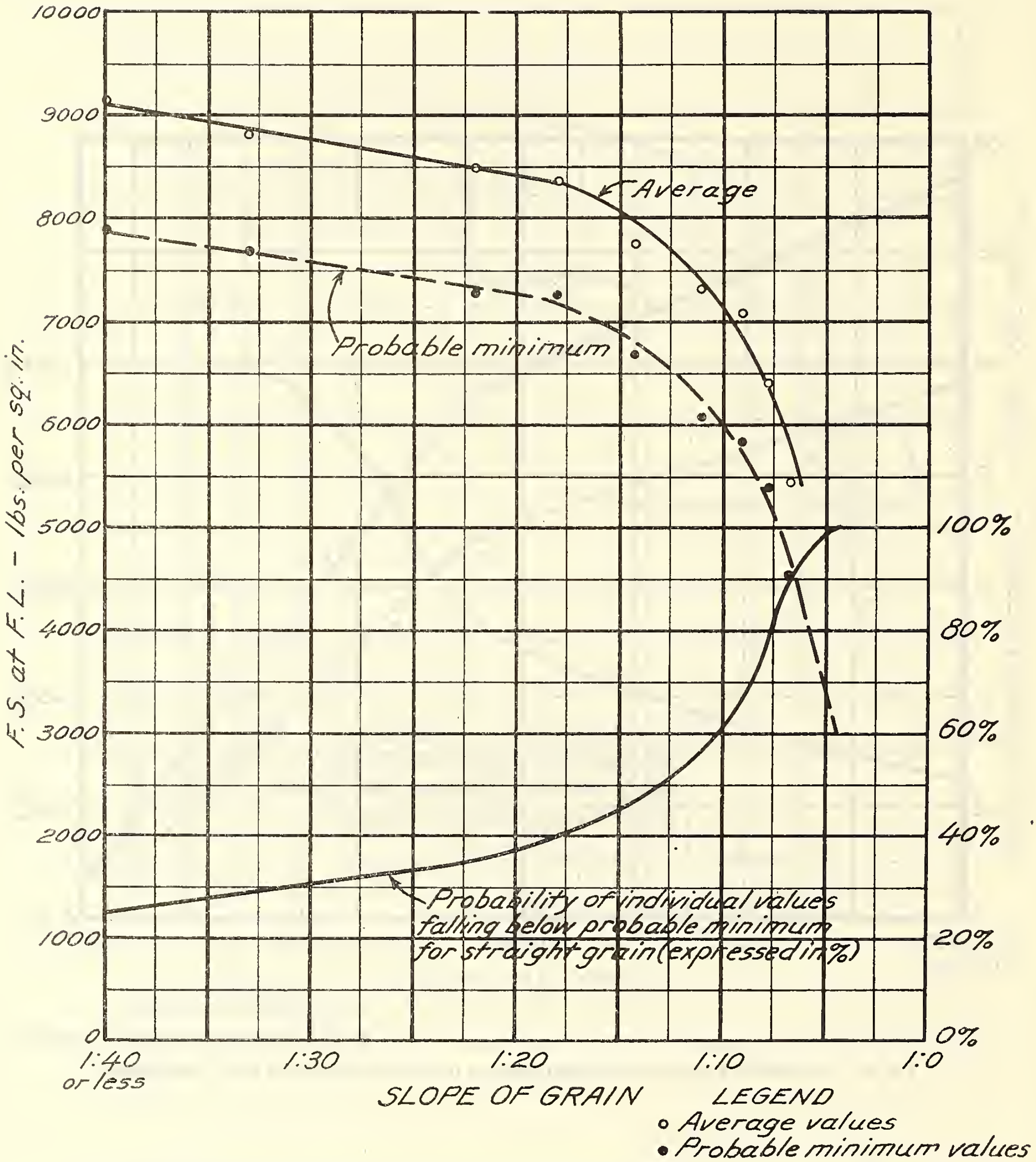


Fig. 11.—The effect of spiral and diagonal grain on the fiber stress at the elastic limit; Douglas fir.

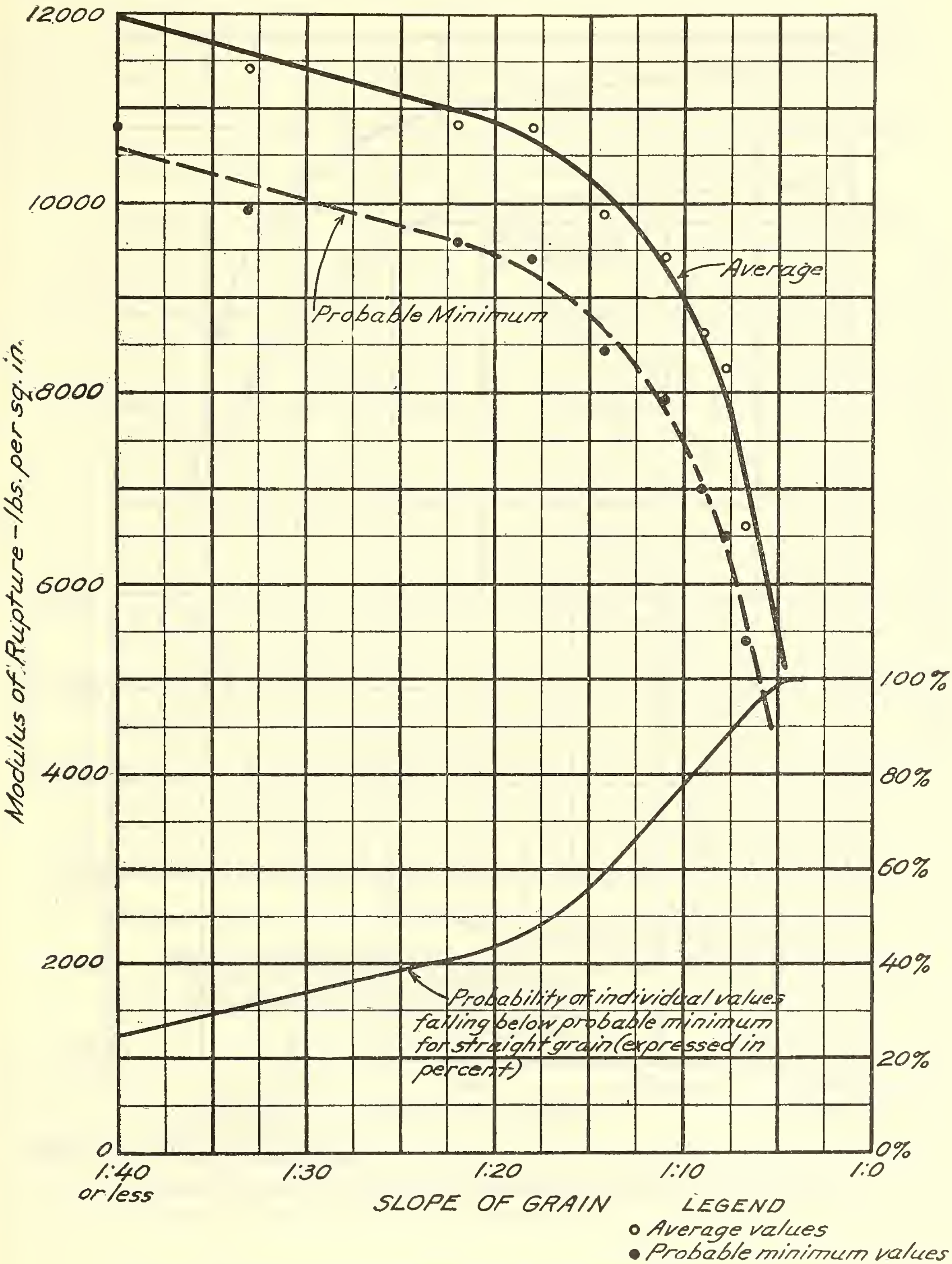


Fig. 12.—The effect of spiral and diagonal grain on the modulus of rupture; Douglas fir.

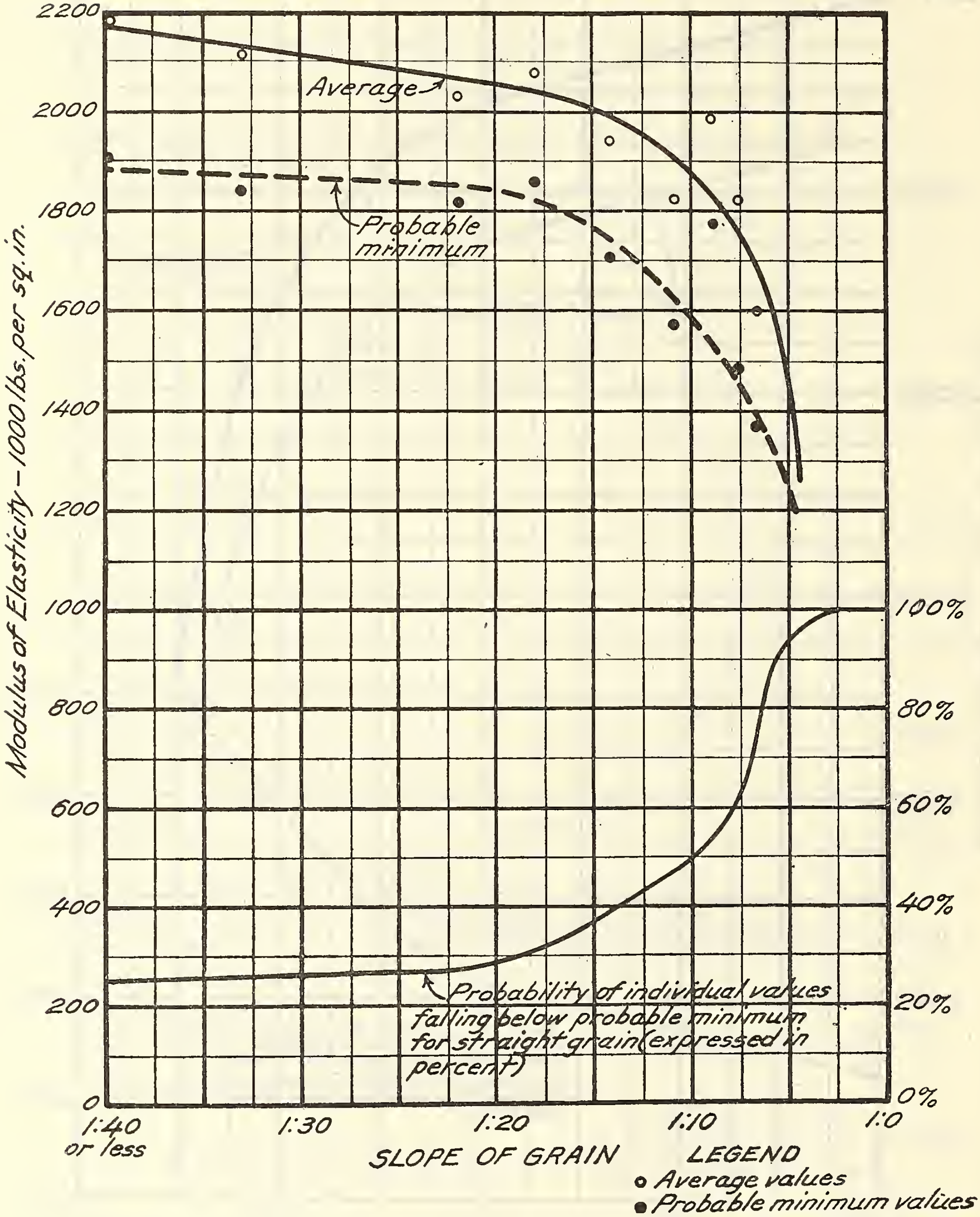


Fig. 13.—The effect of spiral and diagonal grain on the modulus of elasticity: Douglas fir.

The tests were made upon seasoned material, but since the moisture content of the individual specimens varied somewhat, it was necessary to reduce such properties as are materially affected by changes in moisture content to a uniform basis before comparisons could be made. Therefore, the values for fiber stress at the elastic limit, modulus of rupture, and modulus of elasticity have been reduced to 11 per cent by means of an empirical exponential formula. The work to the maximum load values were not reduced to a uniform moisture basis, since the correction would have been very small, and no greater accuracy would have been insured.

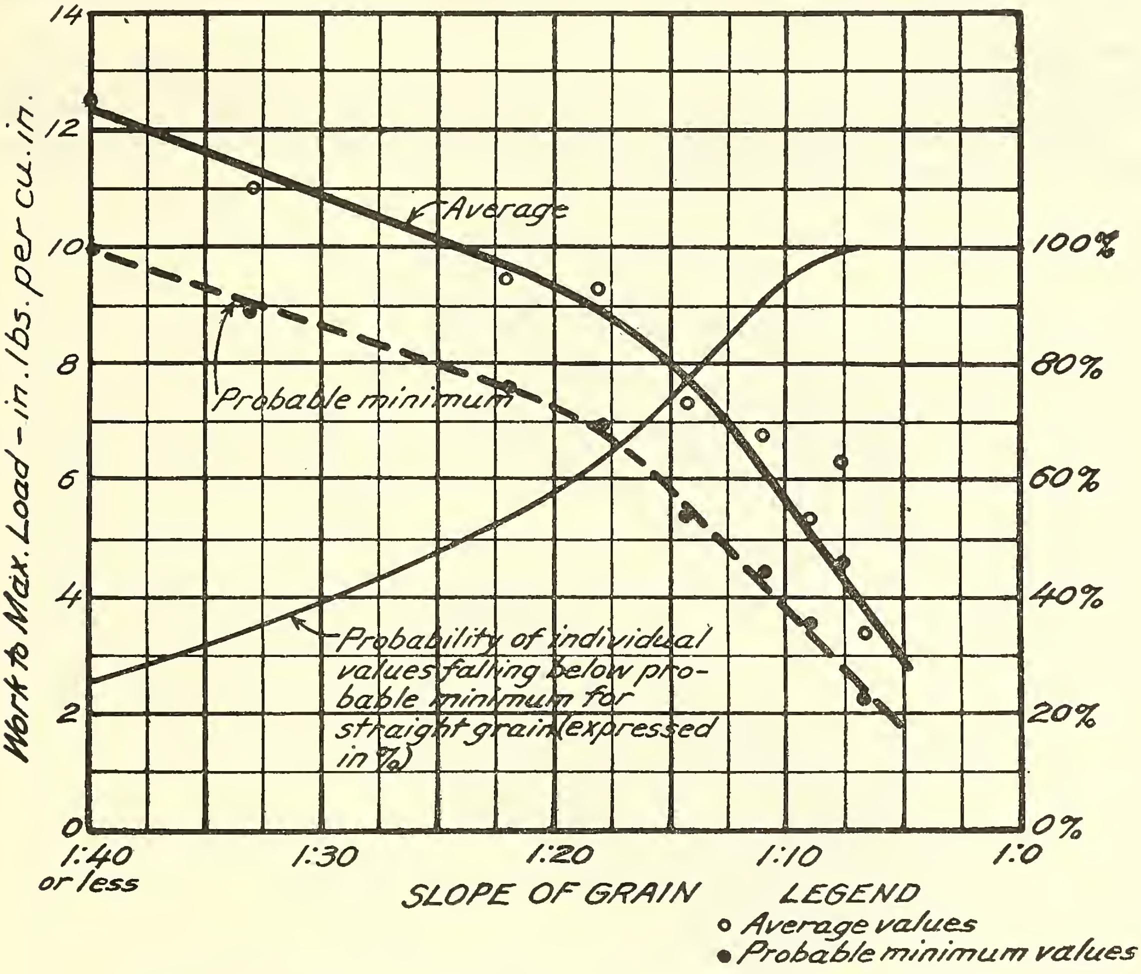


Fig. 14.—The effect of spiral and diagonal grain on the work to maximum load; Douglas fir.

In addition to the curve for average values based on test data, a curve for probable minimum values (broken line) was calculated and plotted. A third curve was also drawn showing the probability of individual values falling below the probable minimum value for straight-grained material. This probability is expressed in per cent and, as is to be expected, increases greatly as the slope of the grain becomes steeper.

The rate of falling off in strength increases abruptly at a slope between 1 in 20 and 1 in 15, and therefore this slope may be considered to be the critical one. It is to be noted, however, that even at slopes at 1 in 20 there is a decided weakening.

As a result of these tests it is recommended that for purposes of design the following values for moduli of rupture for spruce at 15 per cent moisture and different slopes of spiral or diagonal grain be strictly adhered to:

From straight to 1 in 25	7,900 pounds per square inch.
From 1 in 25 to 1 in 20	7,000 pounds per square inch.
From 1 in 20 to 1 in 15	5,500 pounds per square inch.

The effect of spiral grain upon the maximum crushing strength is much smaller than upon the modulus of rupture. The following stresses for different slopes of grain may be used with safety for compression members:

From straight to 1 in 25	4,300 pounds per square inch.
From 1 in 25 to 1 in 20	4,200 pounds per square inch.
From 1 in 20 to 1 in 15	3,800 pounds per square inch.

When the annual rings run diagonally across the end of a piece the true slope of diagonal grain can be obtained as shown by figure 15*a*.

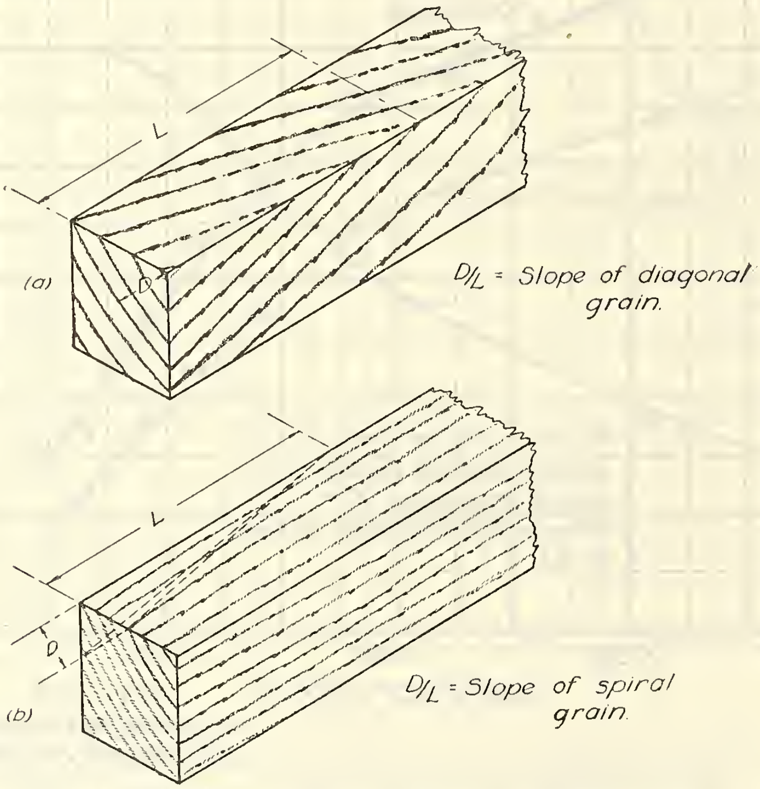


Fig. 15.—The measurement of the slope of diagonal and spiral grain.

The direction of spiral grain is indicated on a tangential (flat sawn) face by the direction of the resin ducts. These ducts, however, are often difficult to see. Drops of ink placed on tangential faces and allowed to spread are sometimes used to test for spiral grain. The ink will tend to follow the angle of the grain. The direction of spiral grain is, however, not given correctly by resin ducts or by spreading of ink unless these tests be applied to a truly tangential face. In figure 15, for instance, resin ducts or spreading of ink would be practically parallel to the edges whether the material was spiral grained or not. In such cases spiral grain can be detected only by splitting on a radial line (Fig. 15*b*) or by raising small splinters and observing if they have a tendency to tear deeper and deeper.

KNOTS.

The effect of knots depends upon their location with respect to the stresses to which the piece will be subjected, as well as upon their size and character. None but sound knots, firmly attached, should be permitted. Obviously, knots of any considerable size can not be allowed in any airplane parts because the parts themselves are comparatively small in cross section. Since the weakening effect of knots results from their disturbance of normal arrangement of fibers, their seriousness can best be decided from a consideration of the grain.

PITCH POCKETS.

Tests recently completed on 112 solid Douglas fir wing beams, made especially to study the effect of pitch pockets upon the strength of beams indicate that this effect may have been overrated in previous specifications. The tests were made over a 72-inch span under third-point loading. The following conclusions from these tests are presented in the form of specifications, and are intended to be applied to spruce and fir wing beams:

(*a*) In portions of the length where a slope of grain of 1 in 25 is the maximum allowed, pitch pockets 1½ inches in length and not to exceed one-eighth of an inch in width or depth may be allowed in any portion of the section except the outer quarters of the flange. No pitch pockets to be allowed in outer quarters of flange.

(*b*) Where a slope of spiral grain of 1 in 20 is allowed pitch pockets 2 inches in length and not to exceed one-fourth inch in width or depth may occur any place in the section except in the outer quarters of the flange. No pitch pockets to be allowed in outer quarters of flange.

(*c*), Where a slope of grain of 1 to 15 is allowed pitch pockets 1½ inches in length and one-fourth inch in width or depth may occur in the outer quarters of the flange, and pitch pockets 3 inches in length and one-fourth inch in width or depth may occur in any other portion of the section.

(*d*) Pitch pockets occurring in the web may not be closer together than 20 inches. If they are in the same annual ring, they may not be closer together than 40 inches. In other portions of the section these distances may be 10 inches and 20 inches, respectively.

Combining this specification with a knot and spiral-grain specification, the following table has been prepared; it is the intention that this table be used in drafting parts specifications for spruce and fir wing beams:

TABLE 1.—*Size and quantity of defects allowable with different slopes of grain.*

Allowable slope in grain not exceeding—	Knots.		Pitch pockets.	
	Maximum diameter permitted.	Minimum distance between any two.	Maximum length permitted.	Maximum width or depth permitted.
	Inches.	*Inches.*	*Inches.*	*Inches.*
1 inch in 25	$\frac{1}{8}$	10	$1\frac{1}{2}$	$\frac{1}{8}$
1 inch in 20	$\frac{3}{16}$	12	2	$\frac{1}{4}$
1 inch in 15	$\frac{1}{2}$	20	3	$\frac{1}{4}$

Supplementing the table are the following clauses:

1. All knots must be sound.

2. No defects must fall or cause irregular grain greater in slope than that allowable for cross grain in the outer quarter of the upper or lower flange; except that where a slope of 1 in 15 is allowed, pitch pockets 1½ inches long anu one-fourth inch wide or deep may be permitted.

3. Pitch pockets occurring in the web may not be closer together than 20 inches. If they are in the same annual ring, they may not be closer together than 40 inches. In other portions of the section these distances may be 10 inches and 20 inches, respectively.

4. The equivalent of the diameters specified may be allowed in a number of smaller knots, provided that they are not close together

COMPRESSION FAILURES AND "CROSS BREAKS."

All material containing compression failures and "cross breaks" should be eliminated from airplane parts where strength is of importance. The cause of certain "cross breaks" near the center of large logs such as are quite frequently found in mahogany is not known. Compression failures, which are, in fact, of the same nature as "cross breaks," are known frequently to be due to injury by storm in the standing trees, to carelessness in felling trees across logs, to unloading from a car across a single skid, or to injury during manufacture.

While some compression failures are so pronounced as to be unmistakable, others are difficult to detect. They appear as wrinkles across the face of the piece. Compression failures not readily apparent to the eye may seriously reduce the bending strength of wood and its shock-resisting ability, complete failure occurring suddenly along the plane of injury.

Figure 16 shows four samples of African mahogany containing compression failures which occurred during growth. These samples were later tested in static bending, and in all cases the compression failures developed during test followed those originally occurring in the samples. This is illustrated in figure 17.

BRASHNESS.

The term "brash," frequently used interchangeably with the term "brittle," when used to describe wood or failures in wood, indicates a lack of toughness. Brash wood, when tested in bending, breaks with a short, sharp fracture instead of developing a splintering failure and absorbs a comparatively small amount of work between the elastic limit and final failure. In impact tests brash wood fails completely under a comparatively small hammer drop.

DECAY.

The first effect of decay is to reduce the shock-resisting ability of the wood. This may take place to a serious extent before the decay has sufficiently developed to affect the strength under static load or to become evident on visual inspection. Unfortunately there is no method of detecting slight decay in wood except with a compound microscope. All stains and discolorations should be regarded with suspicion and carefully examined. It must be remembered that decay often spreads beyond the discoloration it causes and that pieces adjacent to discolored areas may already be infected. On the other hand, not all stains and discolorations are caused by decay of the wood. The blue sapstain of some hardwoods and of many coniferous woods, including spruce, and the brown stain of sugar pine are not caused by decay-producing organisms and do not weaken the wood.

INTERNAL OR INITIAL STRESSES IN WOOD.

WOOD FIBERS UNDER STRESS IN THE TREE.

Wood products are quite similar to metal castings as regards internal stresses. It is probable that wood fibers are continually under stress of some kind. The fact that freshly cut logs of some species split through the center (this frequently happens as the result of heavy shocks or jars and without the use of a wedge) is evidence of some tensile stresses in the outer portion of the tree and compression in the inner portion. These stresses are independent of the stresses due to the weight of the tree and pressure against it.

Fig. 16.—Compression failure occurring during growth. African mahogany

Fig. 17.—Influence of compression failure occurring during growth on failures in static bending. African mahogany.

INTERNAL STRESSES PRODUCED DURING DRYING.

The natural stresses may be partially or wholly relieved by sawing the tree into lumber, but other stresses are likely to be introduced by subsequent seasoning. Checking, honeycombing, warping, twisting, etc., are manifestations of the internal stresses which are produced in the drying of wood or whenever any change of moisture content takes place. Presumably such stresses are due to unequal distribution of moisture and consequent unequal shrinkage combined with more or less inherent lack of homogeneity.

Air drying for a number of years, which is practiced in some woodworking industries, has for its object the equalization of moisture and the relief of stresses induced in the early part of the drying. Careful and correct kiln drying followed by a period of seasoning under proper and controlled atmospheric conditions should produce results at least equal and probably superior to those obtained by long periods of air drying.

Relieving these internal stresses is important because they amount to an actual weakening of the material. If the fibers of a piece of wood are under stress when the piece is free, they are just that much less capable of resisting stresses of the same kind produced by exterior forces or loads applied to the piece.

INITIAL STRESSES PRODUCED IN ASSEMBLING.

When a member of any structure is stressed in assembling the structure and before any load is placed on it, it is said to be under initial stress. If the initial stress is of the same character as the stress for which the member is designed, it constitutes a weakening, for when the structure is loaded the safe working stress of the member will be reached just that much sooner. If this initial stress is opposite in character to that for which the member is designed, it amounts to a strengthening of the member, for when the structure is loaded the initial stress must be overcome before the member takes any of the stress for which it is designed.

Many of the curved parts of an airplane frame could be simply sprung to place on assembly. Were this done, they would be subjected to initial stress and usually of the same sign to which the member would later be subjected. In order to avoid initial stress, such parts are steam bent before assembly. It is desirable, of course, that this bending be so done as not to injure the material and to leave little tendency to spring back from the curves to which it is bent. In order that the material may be made sufficiently plastic to accomplish this result, it is essential that the steaming and bending be carried out while the wood is at a relatively high moisture content. If it is attempted on kiln-dry or thoroughly air-dry material, there is the tendency to spring back after the clamps are removed. Bending of such stock can not be compared to a considerable part of the bending done in other woodworking industries, where the strength of the wood is very greatly damaged by the bending process but without destroying its usefulness for the purpose for which it is intended. Some of the unexpected failures of bent parts in airplanes have doubtless been due to the initial stresses set up in the member during the bending.

WORKING STRESSES FOR WOOD IN AIRCRAFT CONSTRUCTION.

Table 2 gives strength values at 15 per cent moisture (which is probably close to the maximum moisture content of wood in a humid atmosphere) for use in airplane design, as well as the minimum specific gravity and average density which should be allowed. It is suggested that the working stresses for design be obtained by applying factors to the values for static load conditions as given in this table.

TABLE 2.—*Properties of various woods, strength values at 15 per cent moisture, for use in airplane design.*

Common and botanical names.	Specific gravity based on volume and weight when oven-dry.		Weight at 15 per cent moisture.	Shrinkage from green to oven-dry condition.		Static bending.				Compression parallel to grain; maximum crushing strength.	Compression perpendicular to grain; fiber stress at elastic limit.	Shearing strength parallel to grain.	Hardness, side; load required to imbed 0.444-inch ball to one-half its diameter.
	Average.	Minimum permitted.		Radial.	Tangential.	Fiber stress at elastic limit.	Modulus of rupture.	Modulus of elasticity.	Work to maximum load.				
HARDWOODS.													
			Lbs. per cu. ft.	*Per cent.*	*Per cent.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*1,000 lbs. per sq. in.*	*Inch lbs. per cu. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs.*
Ash, commercial white (Fraxinus americana, Fraxinus lanceolata, Fraxinus quadrangulata)	0.62	0.56	40	4.5	7.1	7,700	12,700	1,500	14.2	6,000	1,300	1,750	1,150
Ash, black (Fraxinus nigra)	.53	.48	35	5.0	7.8	5,800	10,500	1,400	14.1	4,900	800	1,350	740
Basswood (Tilia americana)	.40	.36	25	6.6	9.3	4,700	7,200	1,300	6.4	3,800	400	880	340
Beech (Fagus atropunicea)	.66	.60	41	4.8	10.6	7,400	12,600	1,500	13.3	5,900	1,100	1,700	1,060
Birch (Betula lutea, lenta)	.67	.61	43	7.0	8.5	8,400	13,500	1,800	17.6	6,600	1,000	1,620	1,070
Cherry, black (Prunus serotina)	.53	.48	35	3.7	7.1	7,300	10,600	1,400	12.0	5,800	700	1,500	830
Cottonwood (Populus deltoides)	.43	.39	28	3.9	9.2	4,500	7,000	1,200	7.3	3,800	400	800	380
Elm, rock (Ulmus racemosa)	.66	.60	44	4.8	8.1	6,700	12,500	1,400	19.3	5,800	1,200	1,650	1,200
Gum, red (liquidambar styraciflua)	.53	.48	34	5.2	9.9	6,700	10,400	1,400	11.0	4,900	700	1,500	650
Hickory (true hickories) (Hicoria glabra, laciniosa, alba, ovata)	.81	.73	50	7.3	11.4	8,900	16,300	1,900	28.0	7,300	1,800	1,800	....
Mahogany (true) (Swietenia mahagoni)	.54	.50	36	3.5	4.2	7,000	10,000	1,300	9.1	5,500	1,000	1,420	860
Mahogany, African (Khaya senegalensis)	.50	.46	34	4.8	5.5	7,100	10,400	1,400	10.3	5,100	900	1,270	730
Maple, hard (Acer saccharum)	.66	.60	42	4.8	9.2	8,100	12,900	1,600	12.9	6,500	1,200	1,990	1,200
Oak, commercial white (Quercus alba, macrocarpa, minor, michauxii)	.72	.65	46	5.3	9.2	6,700	12,000	1,400	12.7	5,900	1,300	1,760	1,270
Poplar, yellow (Liriodendron tulipifera)	.42	.38	28	4.1	6.9	4,800	7,500	1,300	6.2	4,100	400	900	370
Walnut, black (Juglans nigra)	.56	.52	38	5.2	7.1	7,900	11,900	1,500	13.1	6,100	1,000	1,300	950
CONIFERS.													
Cedar, incense (Libocedrus decurrens)	.36	.32	26	3.3	3.7	4,900	7,100	1,000	6.0	4,300	600	850	430
Cedar, Port Orford (Chamaecyparis lawsoniana)	.47	.42	31	5.2	8.1	6,200	10,300	1,700	9.7	5,300	700	1,160	580
Cedar, western red (Thuja plicata)	.34	.31	23	2.5	5.1	4,200	6,400	1,000	5.5	4,000	400	790	300
Cedar, white (northern) (Thuja occidentalis)	.32	.29	22	2.1	4.9	4,200	5,800	750	5.1	3,400	350	800	300
Douglas fir (Pseudotsuga taxifolia)	.52	.47	34	5.0	7.9	6,800	9,700	1,780	7.2	6,000	750	1,020	580
Pine, sugar (Pinus lambertiana)	.39	.36	27	2.9	5.6	5,300	7,400	1,100	5.0	4,300	540	950	410
Pine, western white (Pinus monticola)	.45	.40	29	4.1	7.4	5,100	7,800	1,400	6.9	4,800	480	670	360
Pine, white (Pinus strobus)	.39	.36	27	2.2	5.9	5,100	7,400	1,200	6.1	4,500	530	850	380
Pine, Norway (Pinus resinosa)	.51	.46	33	4.6	7.2	7,900	10,900	1,700	6.1	6,100	720	1,150	540
Spruce, red, white, Sitka (Picea rubens, canadensis, sitchensis)	.41	.36	27	3.9	7.5	5,100	7,900	1,300	7.4	4,300	500	920	430
Cypress, bald (Taxodium distichum)	.47	.42	31	3.8	6.0	6,100	8,800	1,300	6.8	5,400	670	940	460

Since it is impractical to season test specimens to precisely 15 per cent moisture, it was necessary to compute the strength values given in table 2 at this moisture from test data obtained at slightly different moisture contents. The formulæ used in these computations are presented here as a matter of record.

M less than 8, $$D_{15}=\frac{4(AD-B)}{19-M}+B$$

M 8 to 10, $$D_{15}=\frac{5(AD-B)}{20-M}+B$$

M 10 to 11, $$D_{15}=\frac{6(AD-B)}{21-M}+B$$

M 11 to 12, $$D_{15}=\frac{7(AD-B)}{22-M}+B$$

D_{15} = Strength at 15 per cent, AD = air dry strength value, B = green strength value, M = per cent of moisture.

The factors to be applied, and consequently the exact stress to be used in design, of course, will depend largely on the conditions to which it is assumed the machine will be subjected in flight. If they are the most severe which the machine is ever expected to sustain while in flight, the working stresses can be relatively high. If, on the other hand, the assumed conditions are only moderately severe, the stresses must be made lower in order to take care of exceptional conditions which may occur. It must also be remembered that working stresses can not be safely based on average strength figures, but must be lowered to a value which will be safe for the weakest piece likely to be accepted.

NATURE OF LOADING.

The time of duration of a stress on a timber is a very great factor in the size of the stress which will cause failure. A continuously applied load greater in amount than the fiber stress at elastic limit as obtained by the ordinary static bending test will ultimately cause failure.

The fiber stress at elastic limit in static bending for the dry material is usually somewhat more than nine-sixteenths of the modulus of rupture, and in compression parallel to the grain the elastic limit is usually more than two-thirds of the maximum crushing strength. Timber loaded slightly below the elastic limit will gradually give to loads and ultimately assume greater deflections than those computed by using the ordinary modulus of elasticity figures. In impact tests where a weight is dropped on the stick and the stress lasts for only a small fraction of a second, the stick is found to bend practically twice as far to the elastic limit as in static tests where the elastic limit is reached in about two minutes. The elastic stress developed in the stick under the blow is greater than the maximum stress obtained in the static test.

TENSILE STRENGTH.

In general data on the tensile strength of wood are little needed, and consequently there is very little data available. The following table presents a few figures on the tensile strength of several species tested green.

TABLE 3.—*Strength of various woods in tension parallel to grain.*

[From tests of small clear specimens of green timber.]

Species.	Number tests averaged.	Number trees represented.	Moisture content.	Specific gravity.	Tension parallel to grain average.	Probable variation of individual from average.
			Per cent.		*Lbs. per sq. in.*	*Lbs. per sq. in.*
Mahogany, African	20	5	49.7	c 0.457	15,110	2,075
Mahogany, Central American	27	7	50.1	c .492	16,400	2,400
Maple, sugar	6	4	47.1	.550	14,900	....
Oak, northern white a	50	18	49.9	c .645	14,012	2,900
Cedar, Port Orford	59	9	35.0	.399	11,730	1,210
Douglas fir (1)	63	10	24.1	c .530	16,200	1,735
Douglas fir (2)	48	10	23.0	c .477	13,300	2,050
Fir, white	10	10	50.0	.369	7,972	1,400
Hemlock, western	7	2	39 to 98	.390	7,716	1,570
Pine, Norway	4	2	31.0	.401	9,760	....
Pine, white	42	9	41 to 86	.351	9,580	1,405
Pine, alligator b	5	3	34.5	.500	9,880	....
Redwood	13	5	40 to 155	.400	9,600	1,170

a Not identified as to species.
b Araucaria from Chile, South America.
c Specific gravity based on oven-dry weight and volume. Other specific gravities based on oven-dry weight and volume as tested.

(1) Specimens from the 8 feet immediately above stump. (2) Specimens from the fifth 8-foot bolt above stump and higher. (1) and (2) rom same trees.

TORSIONAL STRENGTH.

Resistance to torsion is important in connection with control surface spars. The following fragmentary data are based on only 30 tests in all, 15 of each species:

TABLE 4.—*Torsional strength of commercial white ash and Sitka spruce.*

Properties.	White ash.	Sitka spruce.
Number of tests	15	15
Moisture, per cent of oven-dry weight	15.8	15.7
Specific gravity (based on oven-dry weight and oven-dry volume)	.62	.39
Shearing strength at elastic limit, pounds per square inch	1,753	1,090
Shearing strength at maximum load, pounds per square inch	2,371	1,651
Shearing modulus of elasticity, pounds per square inch	88,500	72,300
Work to elastic limit, inch-pounds per cubic inch	8.8	4.4
Work to first failure, inch-pounds per cubic inch (*1*)	24.0	19.7

(*1*) For the spruce and ash tested the first failure occurred at maximum load in all cases.

SHRINKAGE.

Ordinarily when a piece of green lumber is dried no change in dimensions takes place until the fiber saturation point is reached. The wood then begins to shrink in cross-sectional area until no further moisture can be extracted from the cell walls. It also shrinks longitudinally, but in most cases the amount of longitudinal shrinkage is so small as to be negligible.

The shrinkage in cross-sectional area in drying from the green to the oven-dried condition varies with different woods, ranging from as much as 22 per cent (based on the original area before drying begins) to as little as 6 per cent. When dry wood absorbs moisture it continues to swell until the fiber saturation point is reached. Figures 18, 19, and 20 illustrate the progress of shrinkage and swelling between zero moisture content and the fiber saturation point.

The shrinkage of wood, like its strength, is very closely related to its specific gravity. This illustrated by figure 21. On this curve, "Per cent shrinkage in volume" is the total shrinkage from fiber saturation to dryness. It will be noted that shrinkage, in general, increases with specific gravity. This relation in individual specimens of a single species (white ash) is shown in figure 22.

Radial shrinkage, or the shrinkage in width of quarter sawn boards, averages about three-fifths as great as tangential shrinkage, or the shrinkage in width of flat sawn boards.

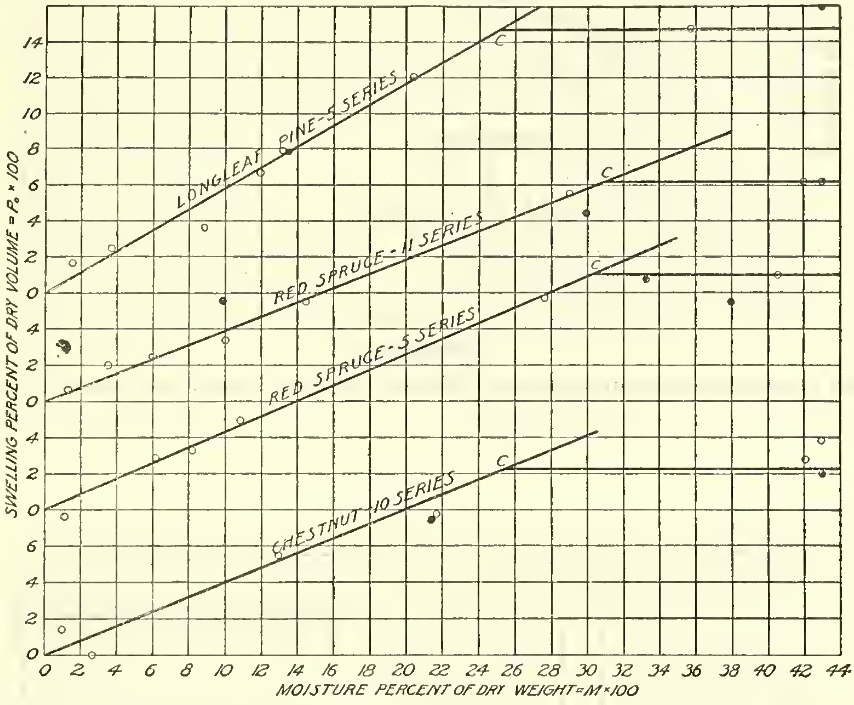


Fig. 18.—Relation between swelling and moisture. Each point is the average of from five to eleven specimens. Black dots indicate specimens that were kiln-dried and then allowed to reabsorb moisture. The fiber-saturation point is at c.

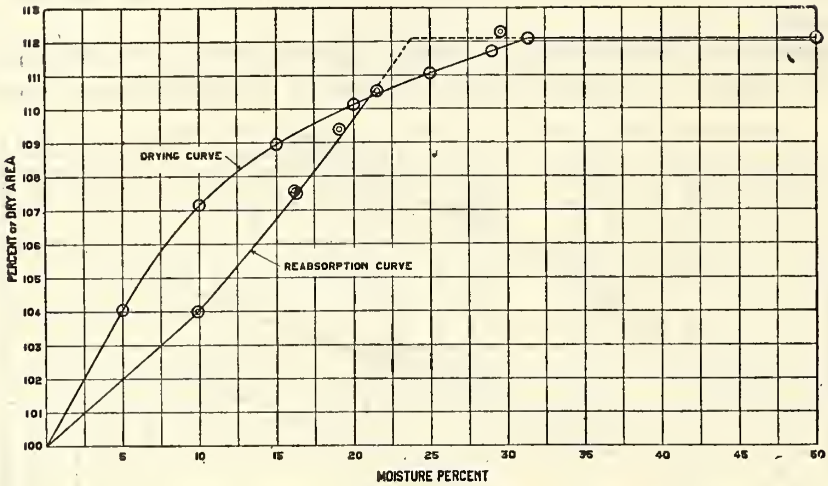


Fig. 19.—Relation between the moisture content and the cross section of small, clear pieces of western hemlock.

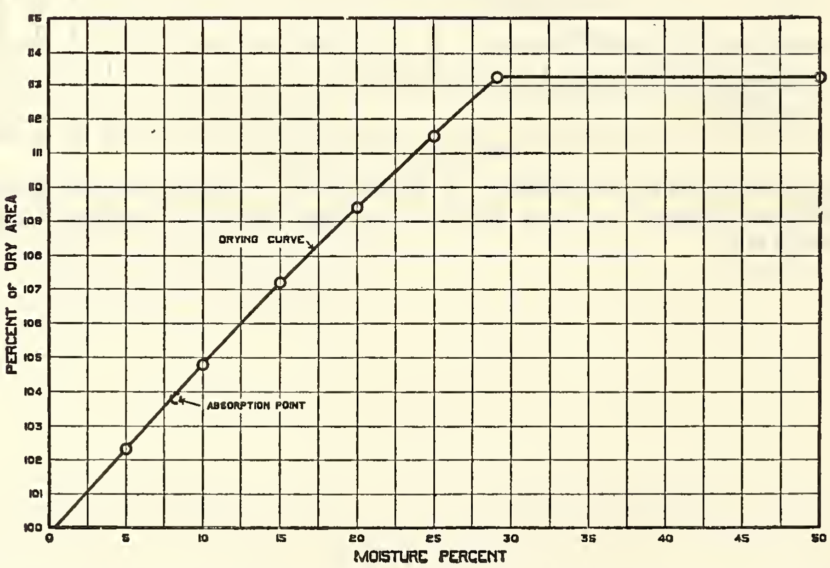


Fig. 20.—Relation between the moisture content and the cross section of small, clear specimens of western larch.

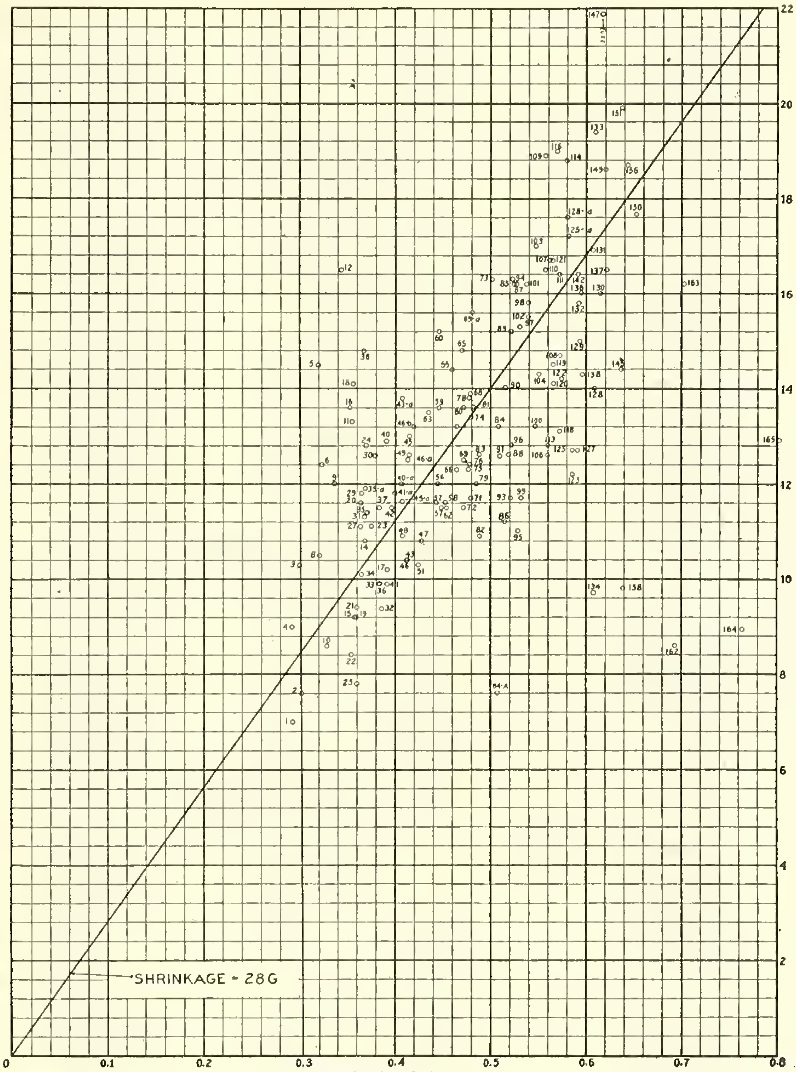


Fig. 21.—Relation between shrinkage in volume and specific gravity of various American woods.

List of species and reference numbers for figure 21.

HARDWOODS.

Species.	Locality.	Reference No.
Alder, red	Washington	30
Ash:		
Biltmore	Tennessee	91
Black	Michigan	60
Do	Wisconsin	70
Blue	Kentucky	99
Green	Louisiana	93
Do	Missouri	100
Pumpkin	do	79
White	Arkansas	106
Do	New York	128
Do	West Virginia	83
Aspen	Wisconsin	23
Largetooth	do	20
Basswood	Pennsylvania	12
Do	Wisconsin	5
Beech	Indiana	110
Do	Pennsylvania	98
Birch:		
Paper	Wisconsin	73
Sweet	Pennsylvania	129
Yellow	do	107
Do	Wisconsin	103
Buckeye, yellow	Tennessee	9
Buckthorn, cascara	Oregon	84a
Butternut	Tennessee	27
Do	Wisconsin	21
Chinquapin, western	Oregon	46b
Cherry:		
Black	Pennsylvania	72
Wild red	Tennessee	24
Chestnut	Maryland	46
Do	Tennessee	40
Cottonwood, black	Washington	6
Cucumber tree	Tennessee	59
Dogwood:		
Flowering	do	151
Western	Oregon	125a
Elder, pale	do	69a
Elm:		
Cork	Wisconsin, Marathon County.	126
Do	Wisconsin, Rusk County.	
Slippery	Indiana	102
Do	Wisconsin	74
White	Pennsylvania	55
Do	Wisconsin	53
Greenheart		165
Gum:		
Black	Tennessee	68
Blue (Eucalyptus)	California	147
Cotton	Louisiana	76
Red	Missouri	54
Hackberry	Indiana	90
Do	Wisconsin	78
Haw, pear	do	146
Hickory:		
Big shellbark	Mississippi	135
Do	Ohio	154
Butternut	do	139
Mockernut	Mississippi	144
Do	Pennsylvania	159
Do	West Virginia	155
Nutmeg	Mississippi	112
Pignut	do	148
Do	Ohio	157
Hickory—Continued.		
Pignut	Pennsylvania	160
Do	West Virginia	161
Shagbark	Mississippi	140
Do	Ohio	152
Do	Pennsylvania	143
Do	West Virginia	153
Water	Mississippi	141
Holly, American	Tennessee	87
Hornbeam	do	149
Laurel, mountain	do	145
Locust:		
Black	do	158
Honey	Indiana	162
Madrona	California	101
Do	Oregon	128a
Magnolia	Louisiana	66
Maple:		
Oregon	Washington	58
Red	Pennsylvania	69
Do	Wisconsin	92
Silver	do	56
Sugar	Indiana	104
Do	Pennsylvania	108
Do	Wisconsin	124
Oak:		
Bur	do	125
California black	California	80
Canyon live	do	163
Chestnut	Tennessee	121
Cow	Louisiana	133
Laurel	do	116
Post	Arkansas	130
Do	Louisiana	137
Red	Arkansas	119
Do	Indiana	118
Do	Louisiana	117
Do	Tennessee	97
Highland Spanish	Louisiana	94
Lowland Spanish	do	142
Swamp white	Indiana	150
Tanbark	California	115
Water	Louisiana	111
White	Arkansas	132
Do	Indiana	138
Do	Louisiana, Richland Parish.	136
Do	Louisiana, Winn Parish.	131
Willow	Louisiana	109
Yellow	Arkansas	122
Do	Wisconsin	105
Osage orange	Indiana	164
Poplar, yellow (tulip tree).	Tennessee	35
Rhododendron, great	do	85
Sassafras	do	51
Serviceberry	do	156
Silverbell tree	do	49
Sourwood	do	89
Sumac, staghorn	Wisconsin	61
Sycamore	Indiana	63
Do	Tennessee	65
Umbrella, Fraser	do	45
Willow:		
Black	Wisconsin	11
Western black	Oregon	43a
Witch hazel	Tennessee	114

List of species and reference numbers for figure 21—Continued.

CONIFERS.

Species.	Locality.	Reference No.
Cedar:		
Incense	California	26
Western red	Montana	2
Do	Washington	10
White	Wisconsin	1
Cypress, bald	Louisiana	62
Douglas fir	California	45a
Do	Oregon	67a
Do	Washington, Chehalis County.	46a
Do	Washington, Lewis County.	75
Do	Washington and Oregon.	67
Do	Wyoming	48
Fir:		
Alpine	Colorado	4
Amabilis	Oregon	39
Do	Washington	18
Balsam	Wisconsin	14
Grand	Montana	36
Noble	Oregon	16
White	California	17
Hemlock:		
Black	Montana	47
Eastern	Tennessee	52
Do	Wisconsin	15
Western	Washington	50
Larch, western	Montana	84
Do	Washington	64
Pine:		
Cuban	Florida	127
Jack	Wisconsin	43
Jeffrey	California	33
Loblolly	Florida	88
Lodgepole	Colorado	31
Do	Montana, Gallatin County.	35a

Species.	Locality.	Reference No.
Pine—Continued.		
Lodgepole	Montana, Granite County.	41a
Do	Montana, Jefferson County.	40a
Do	Wyoming	34
Longleaf	Florida	123
Do	Louisiana, Lake Charles.	113
Do	Louisiana, Tangipahoa Parish.	96
Do	Mississippi	95
Norway	Wisconsin	57
Pitch	Tennessee	71
Pond	Florida	86
Shortleaf	Arkansas	77
Sugar	California	22
Table Mountain	Tennessee	82
Western white	Montana	42
Western yellow	Arizona	19
Do	California	37
Do	Colorado	41
Do	Montana	32
White	Wisconsin	25
Redwood	California, Albion	28
Do	California, Korbel	13
Spruce:		
Engelmann	Colorado, Grand County.	8
Do	Colorado, San Miguel County.	3
Red	New Hampshire	44
Do	Tennessee	29
White	New Hampshire	7
Do	Wisconsin	38
Tamarack	do	81
Yew, western	Washington	134

SUITABILITY OF VARIOUS AMERICAN WOODS FOR AIRCRAFT CONSTRUCTION.

The difficulty of securing ample supplies of the woods heretofore considered as the standards for aircraft construction has made it necessary to consider the substitution of other species. It must must be realized that aircraft can, if necessary, be made from practically any species of wood which will furnish material in the required sizes, and progress in laminating and splicing has done much to increase the utilization of smaller sized material. It must also be borne in mind that the differences in suitability are slight for a number of species and that high-grade stock of a species considered to be inferior may actually be better than lower grade stock of

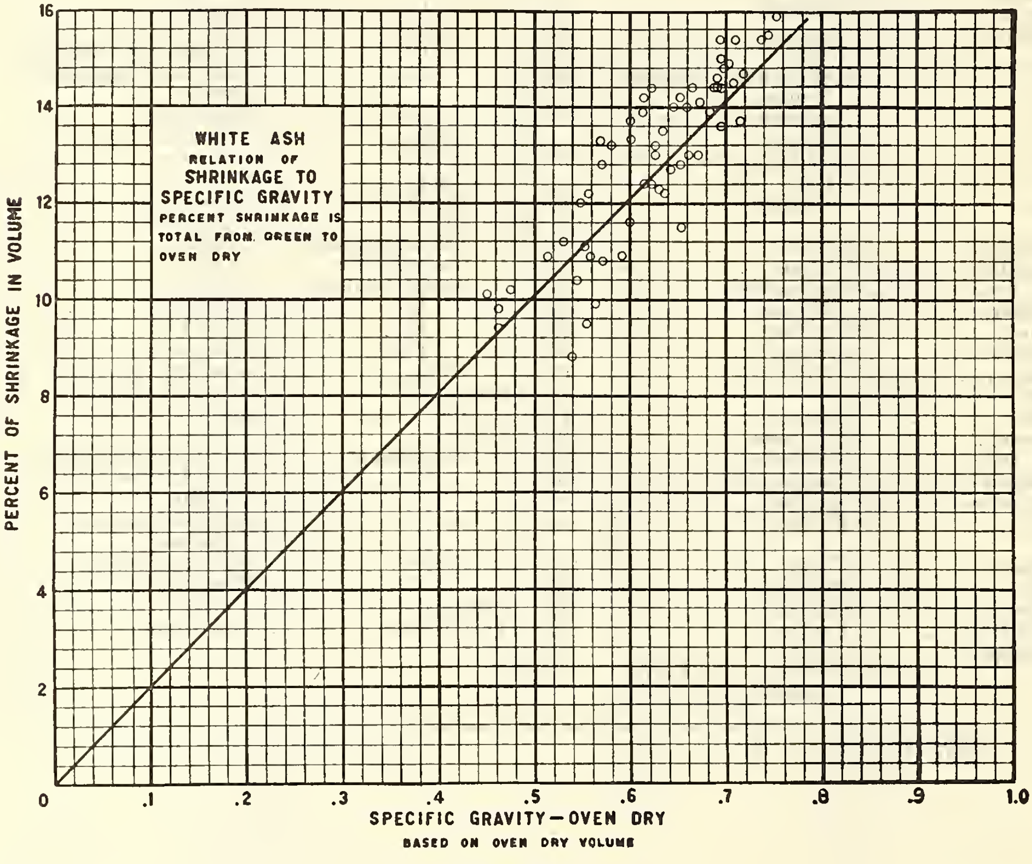


Figure 22.

the species considered superior. In other words, it may be preferable to change species and keep the grade up rather than to lower the grade and use the same species.

In order to give a general idea of the relative properties of the more common American species of timber, with respect to their use in aircraft, a short statement concerning each has been prepared. In those cases in which the species might possibly be considered as a substitute for spruce its properties are compared with those of spruce.

CONIFEROUS SPECIES.

Incense cedar.—This species is somewhat lighter than spruce, but lacks considerably in stiffness and does not possess the toughness of spruce. It might be substituted for spruce for parts which are not highly stressed.

Port Orford cedar.—Port Orford cedar is somewhat heavier than Sitka spruce and equals or exceeds it in all its strength properties. Recent data upon this species indicate that it is not as strong as originally supposed, but still show it to be equal to spruce, although of slightly greater weight.

Western red cedar.—Western red cedar is lighter than spruce and below it in all its strength properties. It is more difficult to dry, but could probably be used with success in many parts where spruce is now used, but could not be used in parts which are highly stressed.

White cedar.—White cedar is very low in all its strength properties. It is a comparatively small tree and could hardly be considered as a possibility for use for the larger members.

Bald cypress.—Bald cypress is slightly heavier than spruce. Its average figures show it somewhat superior to spruce when used in the same sizes. The great variability in the wood of this species has, however, prevented its recommendation for aircraft construction. Cypress is very wet in its green condition and is considered much more difficult to dry and glue than many other species.

Yellow cypress.—Data on this species are not very complete. The indications are that it is too low in stiffness to be a satisfactory substitute for spruce.

Douglas fir from the Pacific coast.—Douglas fir from the Pacific coast is considerably heavier than spruce and all its strength properties are equal to or exceed those of spruce. It is quite probable that the bulk of good wing-beam stock will come from second-cut logs and that the weight and corresponding strength values will run slightly lower than the average of the species. Douglas fir is considerably harder to dry than spruce and more inclined to shakes and to check during manufacture and to develop these defects in service. It is inclined to break in long splinters and to shatter when hit. The use of Douglas fir in the manufacture of wing beams requires considerably more care than is necessary with spruce, but it should give excellent results (from the strength standpoint) when substituted for spruce in the same sizes.

Douglas fir, Rocky Mountain type.—The Rocky Mountain type of Douglas fir is much smaller than the coast type, is quite knotty and somewhat brash, and probably would not be satisfactory as a substitute for spruce.

Alpine fir.—The Alpine fir so far tested was very low in weight and in all its strength properties. This material was from small knotty trees and should not be used except to resist low stresses. It is quite possible that the wood in more extensive stands of comparatively large Alpine fir will be heavier and stronger than that already tested.

Amabilis fir.—The amabilis fir so far tested was slightly heavier than spruce and in most of its strength properties it was practically the equal of spruce. Sufficient data are not at hand to determine how this material will kiln dry nor to determine its working properties. If it can be kiln dried and worked satisfactorily, indications are that it will be a fairly satisfactory substitute for spruce in spruce sizes in wing beams, struts, and other highly stressed parts.

Balsam fir.—Balsam fir is somewhat lighter than spruce and considerably lower in all its strength properties. It does not give promise of being satisfactory in airplane construction.

Grand fir, noble fir, and white fir.—The grand fir so far as tested was slightly heavier than spruce, while the noble and white fir were slightly lighter. In strength properties these species compare very favorably with spruce except in the case of the shock-resisting ability of white fir, which is a little low. This, however, may be accidental. The statement made concerning amabilis fir will apply to these species also.

Black hemlock.—Black hemlock is quite a little heavier than spruce and lacking in stiffness.

Eastern hemlock.—On a basis of strength properties alone eastern hemlock appears to be a substitute for spruce, but the lumber is shaky and liable to heart rot, has numerous knots, and develops shakes and checks in service. It need not, therefore, be considered.

Western hemlock.—Western hemlock is heavier than spruce, but not quite so heavy as Douglas fir. It is low in shock-resisting ability, but on a basis of strength alone it might serve as a substitute for spruce in spruce sizes. No data are available concerning proper kiln-drying methods and the possibility of manufacturing conditions which would cause this species to be rejected.

Western larch.—Butts of the western larch tree are very heavy. The material is shaky and is hard to dry. It would not seem feasible to use this species for aircraft in view of the supply of more suitable species.

Cuban pine.—Cuban pine is entirely too heavy to be considered.

Jack pine.—The jack pine so far tested was 9 per cent heavier than spruce and was lacking in stiffness.

Jeffrey pine.—Jeffrey pine is especially lacking in stiffness.

Loblolly pine.—Loblolly pine is quite heavy. It is very variable in its properties and need not now be considered.

Lodgepole pine.—Lodgepole pine is somewhat low in its shock-resisting ability and slightly low in stiffness. If extensive stands of large trees can be located, there is a possibility that it might be found practicable to use some of this species.

Longleaf pine.—This material is considered too heavy for use in airplanes without redesign.

Norway pine.—Indications are that Norway pine can be used as a substitute for spruce in spruce sizes. More data are needed as to kiln drying and the difficulties which may be met in manufacture.

Pitch and pond pine.—Pitch and pond pine are both heavy, and it is not likely that they would ever be needed in aircraft work.

Shortleaf pine.—The lighter material from the shortleaf pine could be used for aircraft construction, but probably would not be as satisfactory as Douglas fir, since weight for weight it shows a lower modulus of rupture and stiffness.

Sugar pine.—Sugar pine is quite low in shock-resisting ability and stiffness and is quite variable. It probably would not, therefore, be a suitable substitute for spruce.

Table mountain pine.—Table mountain pine has about the properties of shortleaf pine. It probably would not produce clear material satisfactory for aircraft stock.

Western white pine.—Western white pine is slightly heavier than spruce and shows up well in all its strength properties except hardness. It is more difficult to dry than the eastern white pine, but probably could be substituted for spruce in spruce sizes.

Western yellow pine.—Strength data show the western yellow pine to be lacking in shock-resisting ability and stiffness. It is also quite variable. It is not considered a good substitute for spruce.

Eastern white pine.—Tests to date show eastern white pine somewhat below spruce in hardness and rather low in shock-resisting ability. It, however, runs quite uniform in its strength properties, is very easily kiln dried without damage, works well, stays in place well, and is recommended for aircraft construction as a substitute for spruce in spruce sizes.

Redwood.—The data available on redwood are not comparable to those on other species and are too erratic to form a very definite judgment of the species. The indications are that the material is quite variable in its properties and likely to be very brash.

Engelmann spruce.—Engelmann spruce is quite light and low in all its strength properties.

Tamarack.—Tamarack is too heavy to be substituted for spruce. It probably would not furnish clear material.

Yew.—This wood is very heavy. The tree is small and crooked.

HARDWOODS.

Red alder.—Data on this species are very meager, but it is probably not available in sizes sufficiently large to make it of importance.

Biltmore ash.—Biltmore ash should be considered along with white ash and may be used for longerons and other work where strength, stiffness, and ability to steam bend are of importance.

Black ash.—Black ash is very low in stiffness. It is an exceedingly tough species. It is one of the best native species for steam bending. It can not be used, however, where strength and stiffness are of great importance, as in places where white ash is used.

Blue, green, and white ash.—These species are known commercially as white ash and are very desirable for use in longerons and other places where steam bending, great strength, and stiffness are required.

Oregon ash.—Oregon ash appears to be about equal to the eastern white ash, although the data on this species are somewhat meager.

Pumpkin ash.—Pumpkin ash as a species is somewhat lighter than the white ashes. It is considerably less stiff than the white ash. Commercially the term is made to include the weak, soft material from all the other species of ash.

Commercial white ash.—Commercial white ash includes the Biltmore, blue, green, and white ash already mentioned.

Aspen.—Aspen is quite soft and lacking in stiffness.

Basswood.—Basswood is light in weight and low in practically all its strength properties. It is one of the best species to receive nails without splitting and is used extensively for webs, veneer cores, and similar work.

Beech.—Beech is quite heavy and has about the strength properties of sweet and yellow birch and hard or sugar maple. It might be used to some extent in propellers but not extensively in other aircraft parts.

Paper birch.—Paper birch is rather low in its stiffness and high in weight.

Sweet and yellow birch.—Sweet and yellow birch are quite heavy, hard, and stiff. They have a uniform texture and take a fine finish. On account of their hardness and resistance to wear they can be used to face other woods to protect them against abrasion.

Yellow buckeye.—Yellow buckeye is low in its weight and all its strength properties.

Cascara buckthorn.—Cascara buckthorn is a small tree and need not be considered.

Butternut.—Butternut is lacking in stiffness and probably need not be considered.

Western chinquapin.—Western chinquapin is a small tree and need not be considered.

Black cherry.—Black cherry is a very desirable propeller wood.

Wild cherry.—Wild cherry is a small tree and lacking in stiffness.

Chestnut.—Chestnut is somewhat heavier than spruce and is quite deficient in stiffness.

Cottonwood.—The cottonwood so far tested was slightly heavier than spruce. It is soft, low in its strength as a beam or post, and lacks stiffness. It is very tough, however, does not split in nailing, and bends well. Cottonwood can not well be substituted for spruce in wing beams and long struts but can be used in minor parts.

Black cottonwood.—Black cottonwood is low in weight and all its strength properties.

Cucumber tree.—The wood of the cucumber tree is somewhat heavier than spruce and shows up well in all its strength properties. It is one of the few hardwoods which gives promise of being a good substitute for spruce in wing beams and struts.

Flowering and western dogwood.—The dogwood trees are too small to be considered.

Elder, pale.—Elder is too small a tree to be considered.

Elm, cork (rock elm).—Cork elm is slightly heavier than ash. It is low in stiffness and very resistant to shocks. It steam bends well and if properly dried can be used for longerons as a substitute for ash. Considerably more care is necessary in the drying of elm in order to have it remain in shape as it twists and warps badly when not held firmly.

Slippery elm.—Slippery elm is somewhat lighter than cork elm, but when of equal density may be used as cork elm.

White elm.—Very dense pieces of white elm have the requisite density and strength to be used along with cork elm. Most of the white elm, however, is quite light. It is lacking in stiffness, but steam bends well. It could probably be used to excellent advantage in the bent work at the ends of the wings, rudders, elevators, etc. Considerable care would be necessary in order to hold this material in place while drying, as it warps badly.

Black gum.—Black gum is considerably heavier than spruce and not nearly so stiff. It probably will be but little used in aircraft.

Blue gum (eucalyptus).—Eucalyptus grown in this country is quite heavy. It has large internal stresses, swells and shrinks excessively, twists badly in drying, and is very difficult to dry. Under present conditions it probably should not be used in aircraft.

Cotton gum (Tupelo).—This species is considerably heavier than spruce, but not nearly so stiff. At present it probably should not be considered for aircraft.

Red gum.—Red gum is considerably heavier than spruce and superior to it in strength properties. On account of its locked grain and its tendency to twist, warp, and check it probably should not be used in place of spruce. There is some prospect, however, that carefully quarter-sawed material of this species can be used in propellers.

Hackberry.—The denser pieces of hackberry might be substituted for ash in longerons.

Pear haw.—Pear haw is a very small tree and of no importance in this connection.

True hickories, including shellbark, mockernut, pignut, and shagbark.—These species are heavier than ash and are very tough and strong. They could be substituted for ash in longerons, but would probably not give quite as good service for the same weight.

Pecan hickories, including butternut, nutmeg, pecan, water.—These hickories are considerably inferior to the true hickories, especially in their ability to resist shock, and probably would not make satisfactory substitutes for ash.

American holly.—This species is lacking in stiffness and probably is of no importance in airplane construction.

Hornbeam, California laurel, mountain laurel, black locust, honey locust, madrona.—The laurels, locusts, and madrona are all heavy woods and probably have little use in aircraft construction.

Magnolia.—Magnolia has approximately the same properties as cucumber wood, to which it is closely related, and could probably be used as a substitute for spruce in wing beams and longerons.

Oregon maple.—Oregon maple has about the same properties as silver maple. It is a little more stiff and not quite so resistant to shock. There is probably little use for either of these species in aircraft.

Red maple.—Red maple is somewhat heavier, stiffer, and stronger than silver maple. Red maple might possibly be used in propeller work, but would give much softer propellers than sugar maple.

Sugar maple.—Sugar maple is quite heavy, hard, and stiff. It could be used with birch in propeller manufacture. It has very uniform texture and takes a fine finish. On account of its hardness and resistance to wear it is very often used to face other woods to protect them against abrasion.

Silver maple.—Silver maple is the lightest and softest of all the maples. It is much too soft to be considered as a substitute for sugar maple and lacks the stiffness to make it a satisfactory substitute for spruce.

The oaks.—The oaks need not be considered as substitutes for spruce, but they play an important part in the manufacture of propellers. The oaks are all quite heavy and hard. The oaks, even when a single botanical species is considered, are extremely variable in their strength properties. The differences in the average strength properties of the various eastern oaks are not great, and greater differences might readily be found among different logs of any one species. The white oaks, as a rule, shrink and swell more slowly with changes in the weather than do the red oaks. The radial shrinkage of the oaks is about one-half the tangential shrinkage. This accounts for the much greater value of quarter-sawed oak over plain-sawed oak for propeller construction. The southern-grown oaks are much more difficult to dry than are the northern oaks. Experiments are being made in the drying of both northern and southern red and white oaks. The northern white oaks when quarter-sawed and carefully dried make very satisfactory propellers. It is possible that quarter-sawed northern red oak will also make fairly satisfactory propellers but with this disadvantage: It is more subject to defects in the living tree, decays more readily, and changes more rapidly with changes in weather conditions. To be satisfactory in this work the southern oaks will require exceeding care in drying, as they are very difficult to dry without checking, honeycombing, and casehardening.

Osage orange, persimmon.—Osage orange and persimmon have other very important uses and are probably of no importance in aircraft construction.

Yellow poplar.—Yellow poplar is but little heavier than spruce, and while rather low in shock-resisting ability has good working qualities, retains its shape well, is comparatively free from checks, shakes, and such defects. It would probably be a fairly satisfactory substitute for spruce in wing beams and struts. It offers no manufacturing difficulties.

Rhododendron, sassafras, service berry, silverbell, sourwood, sumac.—These species probably have no place in aircraft construction.

Sugarberry.—This species is closely related to the hackberry and the denser pieces might be substituted for ash in longeron construction.

Sycamore.—The trees are very shaky and probably would not furnish material suitable for aircraft.

Fraser umbrella.—This species is closely related to the cucumber and magnolia previously discussed and has similar properties. The clear stock obtained might be used as a substitute for spruce.

Willow, black and western black, witch hazel.—Willow and hazel probably are of no use in aircraft construction.

Walnut, black.—Black walnut has many very important uses and need not be considered as a substitute for spruce. This species probably makes the best propellers of any of the native species. It is somewhat difficult to dry, but stays in place unusually well and is hard enough to resist wear.

SYNOPSIS OF COMMENTS AS TO SUBSTITUTES FOR SPRUCE.

The following species range in weight from that of spruce to 25 per cent heavier than spruce. The data available indicate strongly that these species can be substituted for spruce in highly stressed parts using the spruce design: Port Orford cedar, coast type Douglas fir, eastern and western white pine, yellow poplar, cucumber tree and magnolia. The following species give promise of furnishing substitutes for spruce, but more experiments are needed in order to overcome known difficulties before these species can be recommended: Bald cypress, amabilis fir, grand fir, noble fir, white fir, lodgepole pine, Norway pine, and redwood. The following species are lighter than spruce, but could be used in parts where the stresses are relatively low: Incense cedar, western red cedar, and Alpine fir.

As conditions change other species will doubtless come into consideration as substitutes for spruce.

STORAGE AND KILN DRYING OF LUMBER.

The proper piling of lumber and timber for air seasoning or as temporary storage previous to kiln drying is extremely important. Green or partially dry stock is subject to various forms of deterioration, such as staining, decay, severe checking, and (especially in hardwoods) insect attack. During warm, humid weather staining may take place in a few days and decay may weaken the wood in a few months.

Proper piling of such stock will tend to reduce the deterioration to a minimum. All lumber or timber which is to be stored any length of time should be piled on solid foundations with stickers between each two courses, and should have some protection from the sun and rain. Whenever possible, the stock should be piled in a shed with open sides. If this is not practicable, each pile should be covered so as to keep out rain and snow. Green hardwoods, especially oak, frequently check severely at the ends. This can be avoided to a large extent by coating the ends with linseed-oil paint.

Stock should be cut up into as small sizes as is practicable before kiln drying. Large pieces usually check severely because the outer portion dries and shrinks considerably faster than the inner core, which always dries slowly. Timbers which contain the pith and which are to be cut into smaller sizes later should at least be cut through the pith once, or, better, be quartered before being stored away. This will avoid the large checks which are commonly produced in the seasoning of timbers containing the pith by reason of the tangential shrinkage being greater than the radial shrinkage.

RULES FOR PILING LUMBER.

1. The foundations should be strong, solid, and durable, preferably concrete piers with inverted rails or I beams for skids. If this is impracticable, creosoted or naturally durable wooden timbers should be used.
2. Each foundation should be level.
3. The foundations should not be over 4 feet apart for lumber, but may be farther apart for larger timbers. For woods which warp easily or for stock less than 1 inch in thickness foundations should not be over 3 feet apart.
4. If the piles are in the open, they should have a slope from front to rear of 1 inch for every foot in length.
5. The foundations should be sufficiently high to allow the free circulation of air underneath the piles, and weeds or other obstructions to circulation should be removed.
6. Boards of equal length should be piled together with no free unsupported ends.
7. A space of about three-fourths of an inch should be left between boards of each layer and from 1 to 2 inches between timbers of each layer.
8. The stickers should be of uniform thickness, preferably seven-eighths of an inch for 1-inch lumber and $1\frac{1}{2}$ inches for thicker stock.
9. Stickers should be placed immediately over the foundation beams and kept in vertical alignment throughout the piles. Their length should be slightly in excess of the width of the pile.
10. The front and rear stickers should be flush with or protrude slightly beyond the ends of the boards.

KILN DRYING OF WOOD.

ADVANTAGES OF KILN DRYING.

The chief objects of kiln-drying airplane stock are (*a*) to eliminate most of the moisture in green or partly dried stock more quickly than can be done in air drying and (*b*) to reduce the moisture content of the wood below that attained in ordinary air drying, so that no more drying, with consequent checking, warping, and opening up of seams will occur after the wood is in place. Other advantages incident to kiln drying are that a smoother surface can be obtained on kiln-dried stock and that glues will hold better.

THE ELIMINATION OF MOISTURE FROM WOOD.

Green lumber may contain from about one-third to two and one-half times its oven-dry weight of water. Expressed in percentage, this is from $33\frac{1}{3}$ to 250 per cent moisture based on the oven-dry weight. The moisture content of green lumber varies with the species, the position in the tree, whether heartwood or sapwood, the locality in which the tree grew, and the drying which has taken place since the tree was cut. As a rule sapwood contains more moisture than heartwood, although in some species, especially in butt logs, the heartwood contains as much moisture as the sapwood. Thoroughly air-dried lumber may contain from about 10 to 20 per cent moisture for inch stock and more for thicker material.

Much of the moisture in green wood is contained in the cell cavities (like honey in a comb), and the rest is absorbed by the cell walls. When wood is drying the moisture first leaves the cell cavities and travels along the cell walls to the surface, where it is evaporated. When the cell cavities are empty but the cell walls are still saturated a critical point is reached, known as the fiber-saturation point. Wood does not shrink or increase in strength while seasoning until it has dried below the fiber-saturation point, which usually ranges between 25 and 30 per cent moisture, but may be less or more, and in spruce usually is between 30 and 35 per cent. This has an important bearing on the drying operation, since no casehardening, checking, or warping can occur so long as the moisture content is above the fiber-saturation point in all parts of the stick.

In practice the stock should be dried to a moisture content slightly less than it will ultimately have when in use. This may be as low as 6 per cent for interior work and not so low for wood to be exposed to weather.

Two steps are necessary in the drying of lumber—(*a*) the evaporation of moisture from the surface, and (*b*) the passage of moisture from the interior to the surface. Heat hastens both these processes. For quick drying as high a temperature should be maintained in the kiln as the wood will endure without injury. Some woods (especially coniferous woods) will endure higher temperatures than others. The general specifications for kiln-drying airplane stock which follow give the temperatures at which a kiln should be operated to prevent injury to lumber to be used for airplanes.

The lumber in a kiln is heated and evaporation is caused by means of hot air passing through the piles. To insure proper drying throughout the piles a thorough circulation of air is necessary. The lumber must be properly piled and the kiln constructed so as to make the necessary circulation possible.

Dry hot air will evaporate the moisture from the surface more rapidly than it can pass from the interior to the surface, thus producing uneven drying, with consequent damaging results. To prevent excessive evaporation and at the same time keep the lumber heated through, the air circulating through the piles must not be too dry; that is, it must have a certain humidity. The specifications give the proper humidities at which to operate the kiln for drying airplane stock.

THREE ESSENTIAL QUALITIES OF A DRY KILN.

The merits of any method of drying airplane woods depend upon the extent to which it affects the mechanical properties of the stock and upon the uniformity of the drying. In order that complete retention of properties and uniform drying may be guaranteed, it is essential that the circulation, temperature, and humidity of the air be adequately controlled. In this connection circulation does not mean the passage of air through flues, ducts, or chimneys, but through the piles of lumber, and the terms temperature and humidity control apply to the air within the piles of lumber in the kiln.

Control of air circulation involves rate or speed and uniformity. A uniform passage of air through all portions of the piles of lumber is the most essential quality in a kiln. If the circulation can be made both uniform and rapid, all portions of the pile will dry quickly and at the same rate. Furthermore, uniform and rapid circulation of air are necessary before the control of temperature and humidity within the piles of lumber is possible.

When unsaturated air at any given temperature enters a pile of lumber containing moisture, it exchanges heat for moisture, is cooled, and rapidly approaches saturation. With green wood and a sluggish circulation, the cooling is very rapid. The rate of cooling decreases as the lumber dries, and if the circulation is increased the loss of heat in passing through the pile is less. So if the air moves rapidly through certain parts of the piles and slowly through others, the different parts of the piles will be at different temperatures. The temperature of the air within the lumber can not be maintained at any given value unless the circulation of air is uniform at all points in the pile. Even though the air moves at uniform speed from one side of a pile of lumber to the other, if the speed is too slow the air loses its heat and approaches saturation rapidly. In general a wide variation in the temperature of the lumber in different parts of the kiln is proof of very uneven or slow circulation. Inadequate circulation and temperature control render the control of humidity and uniform drying impossible.

Humidity is of prime importance, because the rate of drying and the prevention of checking and casehardening are directly dependent thereon. It is generally true that the surface of the wood should not dry more rapidly than the moisture transfuses from the center to the surface. The rate of evaporation must be controlled, and this can be done by means of the relative humidity. Stopping the circulation to obtain a high humidity or increasing the circulation by opening ventilators to reduce the humidity is not good practice. Humidity should be raised, if necessary, to check evaporation without reducing the circulation.

DEFECTS DUE TO IMPROPER DRYING.

Casehardening and honeycombing.—When the surface of a piece of lumber is dried more rapidly than the moisture can pass to it from the interior, unequal moisture conditions exist in the lumber. The moisture in the outer layers falls below the fiber saturation point. The outer layers then tend to shrink but are held from shrinking by the more moist interior, which has not yet started to shrink; so the surface either checks or dries in a stretched condition, usually both. Later, as the interior dries it also tends to shrink normally, but in turn is held by the outside, which has become "set" or "casehardened." Consequently, the interior dries under tension, which draws the outer layers together, closing up all checks and producing compression. Casehardened lumber, when resawed, will invariably cup toward the inside if the interior if the lumber is dry (fig. 23). If the tension in the interior of the wood is severe enough, it may produce radial checks which do not extend to the surface. Wood with such checks is said to be honeycombed or hollow-horned (fig. 24). Casehardening and honeycombing can practically be prevented by regulating the humidity so that the evaporation from the surface does not take place too rapidly.

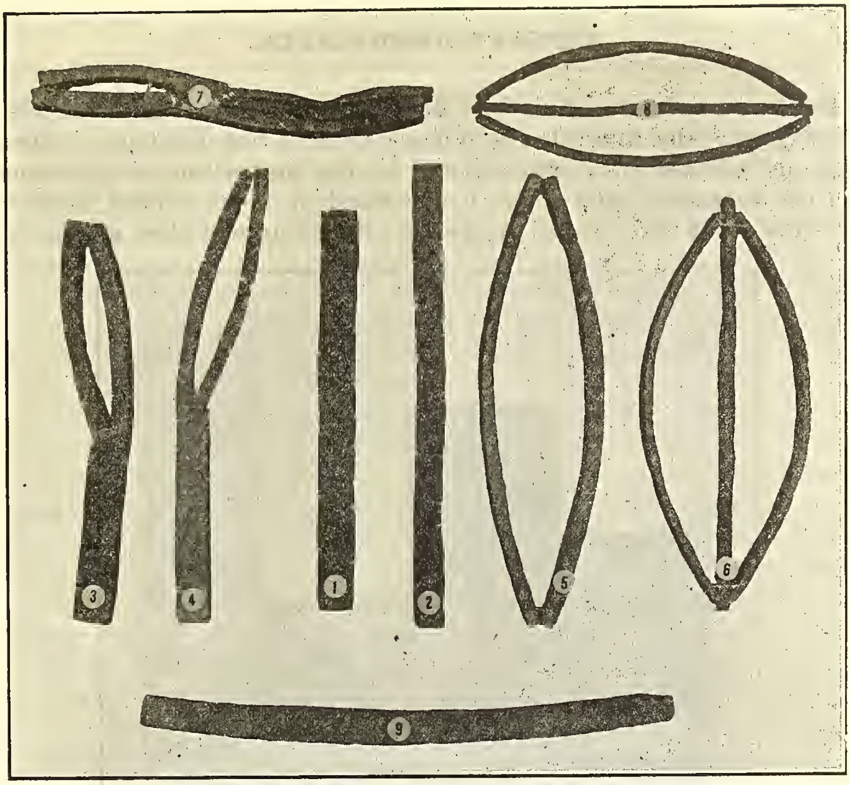


Fig. 23.—Sections of casehardened western larch boards. Nos. 1 and 2 are original sections; Nos. 3 to 8 are resawed sections showing cupping; No. 9 is one-side surfaced.

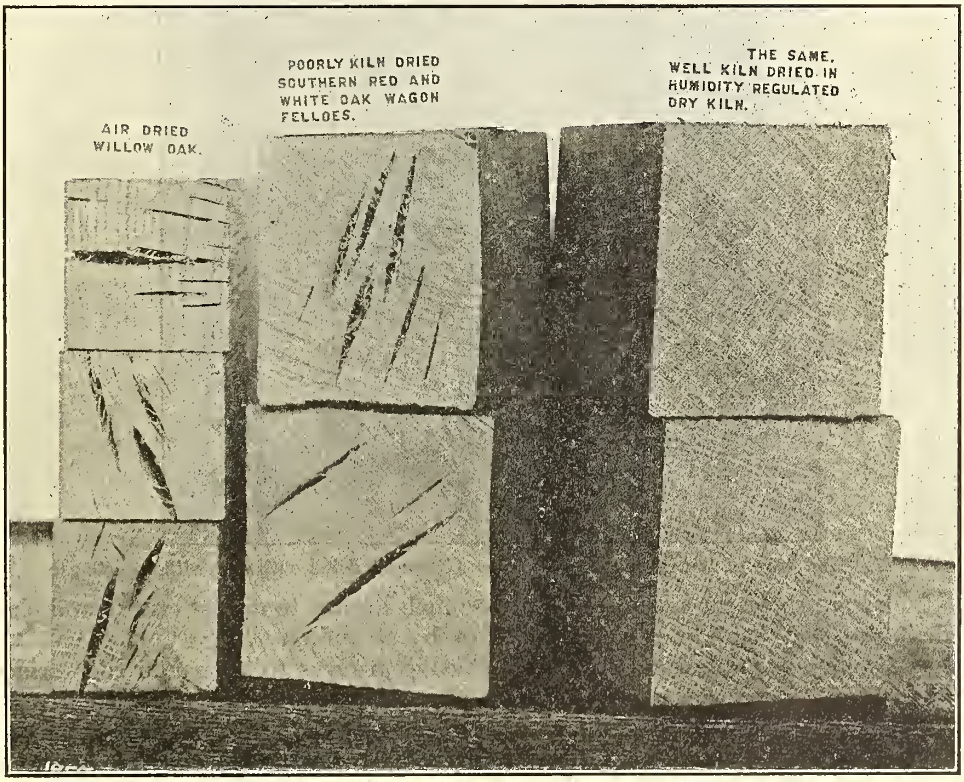


Fig. 24.—Oak stock honeycombed by air drying and improper kiln drying. Also similar stock properly dried.

If wood becomes casehardened in kiln drying, it may be brought back to normal condition by steaming, provided that checks and cracks have not developed. Steaming softens the outer fibers and relieves the stresses caused by the contraction of the outer shell. Care must be taken not to steam wood which has checked or honeycombed from casehardening enought to part the fibers and weaken the piece. Steaming will close up the cracks but will

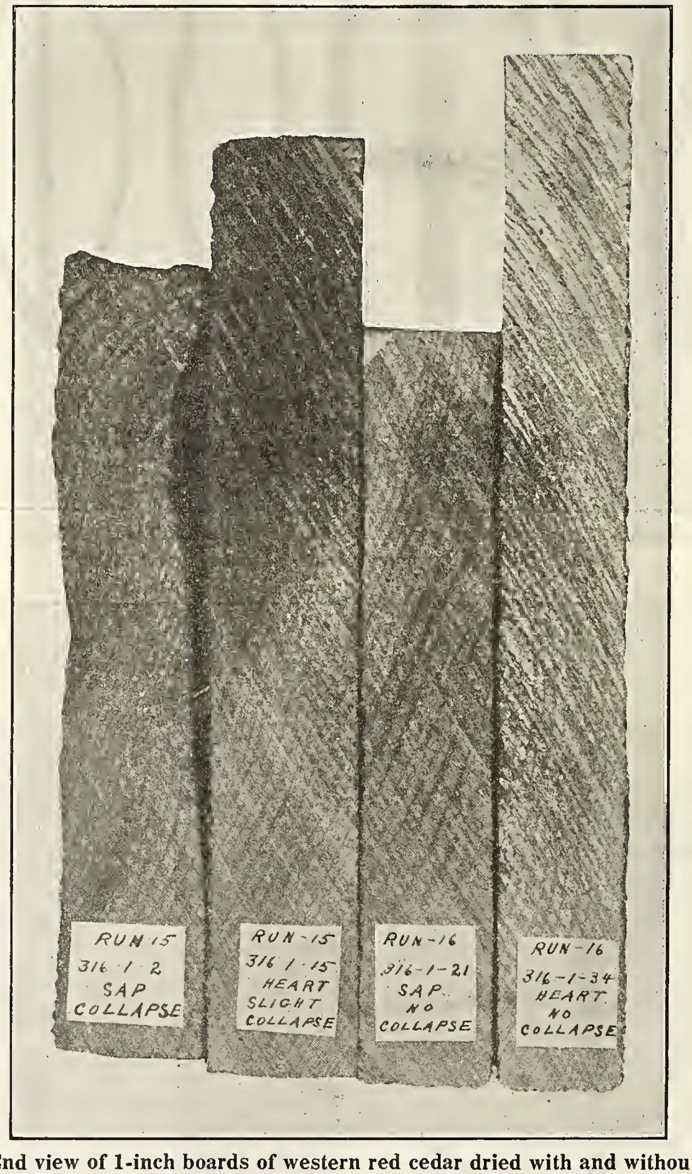


Fig. 25.—End view of 1-inch boards of western red cedar dried with and without collapse.

not restore the strength of the piece. It will be much harder to detect cracks and checks due to casehardening if they have been closed up again by steaming.

Collapse.—Collapse is abnormal shrinkage causing grooves to appear in the surface of the lumber or a genaral distortion of the surface (fig. 25). It is produced when wet lumber is dried at too high a temperature. The heat and moisture cause the cell walls to become soft and plastic. As the water leaves the cell cavities the moist cell walls are drawn together if no air is present. This causes the cells to flatten, and a general reduction in the cross section takes place. Collapse occurs especially in such woods as western red cedar, redwood, white oak, and others which readily become soft and plastic when hot and moist. It can be avoided by not allowing the temperature to rise too high while the wood is still moist (at or above the fiber saturation point).

Brashness.—High temperature treatments of all kinds, whether steam or hot air, are injurious to lumber, causing it to turn darker and become brash. The injuries thus sustained increase with the temperature and length of time the wood is exposed to such severe conditions. No definite rule can be laid down as to what conditions of temperature wood will endure without becoming brash. If the temperatures prescribed in the specifications (see p. 68) are not exceeded, no difficulty will be experienced in this respect.

METHODS OF TESTING CONDITIONS DURING DRYING.

In drying airplane stock it is advisable to test conditions in the kiln at frequent intervals so that the operator will be able to make any changes promptly that the tests indicate are necessary to maintain the proper rate of drying and to prevent injury to the lumber. A continuous record of proper conditions during kiln drying is a strong assurance of satisfactory stock. The following tests will aid the inspector in keeping check on drying conditions.

1. Preliminary test:
   - (*a*) Initial moisture conditions in the lumber.
   - (*b*) Preparation and placing of samples.
   - (*c*) Initial weights and placing of whole pieces.
   - (*d*) Determination of direction, uniformity, and rate of air circulation.
   - (*e*) Location and calibration of instruments.
2. Current tests:
   - (*a*) Determination of current temperatures.
   - (*b*) Determination of current humidities.
   - (*c*) Determination of circulation.
   - (*d*) Weighing of samples and determination of current moisture conditions
3. Final tests:
   - (*a*) Average kiln-dry moisture condition of samples.
   - (*b*) Distribution of moisture in the kiln-dry samples.
   - (*c*) Determination of casehardening in kiln-dry samples.
   - (*d*) Average kiln-dry moisture condition of whole pieces.
   - (*e*) Calculation of initial moisture condition of whole pieces.
   - (*f*) Distribution of moisture in kiln-dry whole pieces.
   - (*g*) Distribution of casehardening in kiln-dry whole pieces.
   - (*h*) Determining the effect of the process on the toughness and strength of the kiln-dry stock.

In making these tests the following instruments and material will be needed:

- 1 sensitive equal arm balance (capacity, 0.1 to 250 grams).
- 1 drying oven in which the air can be heated to and held at 212° F.
- 1 can of asphalt paint and a brush.
- 1 sensitive platform scale (capacity, 0.01 to 250 pounds).
- 1 electric flash light (lantern type recommended).
- 12 packages of punk sticks.
- 3 accurate standardized ordinary glass thermometers (60° to 230° F. by 2° intervals).
- 2 accurate standardized glass wet and dry bulb hygrometers with extra wicks (60° to 230° F. by 2° intervals).
- Access to a laboratory equipped with machines for making impact, static bending, hardness, compression parallel to the grain, and other tests.
- Waxed or oiled paper.

1. *Preliminary tests.*—(*a*) Initial moisture condition: Select at least three representative pieces for each 10,000 board feet of stock to be dried. Cut about 2 feet from one end of each. Then cut a 1-inch section, a 24-inch sample, and a second 1-inch section in succession. Immediately weigh the two 1-inch sections to an accuracy of one-tenth of 1 per cent. Mark the initial weights on the section and dry them to constant weight in the oven heated to 212° F. Reweigh them to the same accuracy and determine the per cent initial moisture content of the samples from the formulæ:

$$\text{Per cent initial moisture content} = \frac{\text{Initial weight—oven-dry weight}}{\text{oven-dry weight}} \times 100$$

(*b*) Preparation and placing of samples: Immediately after cutting the 24-inch samples described under (*a*) paint the ends of the samples with a heavy coat of asphalt paint. Then weigh them separately on the platform to an accuracy of one-tenth of 1 per cent. Mark the initial weights on the samples and place them in the piles so as to come under the most severe, least severe, and average drying conditions, and so as to be subjected to the same drying conditions as the adjacent pieces. Where the circulation of air is vertical, place samples near the tops, centers, and bottoms of the piles, and where the circulation is lateral place them near the sides where the air enters and leaves the piles and near the centers of the piles.

(*c*) Initial weights and placing of whole pieces: In addition to the 24-inch samples it is desirable to select several representative whole pieces of stock and weigh them to an accuracy of one-tenth of 1 per cent on the platform scale. Mark the weights on the pieces and place them at various points near the tops, edges, bottoms, and centers of the piles.

(*d*) Determination of the direction, uniformity, and rate of air circulation: In order to insure correct placing of samples, whole pieces, and instruments it is necessary that the direction of the circulating air be known. To determine this light a few punk sticks, take the flash light, enter the kiln, close the door, and determine the direction, uniformity, and rate of motion of the circulating air in the spaces around the piles and through the piles by observing the smoke from the burning punk.

(*e*) Location and calibration of instruments: Having determined the direction in which the air passes through the piles, place the bulb of the recording thermometer in contact with a standardized glass thermometer close to the pile at the center of the side where the air enters the pile. If the circulation is up through the piles, place the thermometer bulbs close under the bottom center; if it is down through the lumber, place the bulbs close to the top center, and if the air moves through the pile laterally, place the bulbs close to the center of the side where the air enters the pile. It is also desirable to know the variation of temperature in different parts of the piles and kiln. To determine this variation, place several of the standardized thermometers in the tops, bottoms, edges, and centers of the piles and at different points in the kiln. In order to calibrate a recording thermometer, place the bulb in contact with a standardized glass thermometer in the kiln and adjust the stylus until it agrees with the glass thermometer. The temperature must not be fluctuating, as is often the case where it is controlled by a thermostat. It is best to use a steady steam pressure in the heating pipes while calibrating instruments. Never attempt to calibrate a recording thermometer out of its place in the kiln.

To determine humidity, place the standardized glass wet and dry bulb hygrometer near the bulb of the recording thermometer, so as to indicate the humidity of the air entering the piles at the tops, bottoms, or edges, as the case may be.

2. *Current tests.*—(*a*) Determination of current temperatures: If any part of a pile is exposed to direct radiation from the heating pipes, place a thermometer near the side so exposed.

This will indicate whether or not any part is subject to higher temperature than that indicated by the recording instrument. If possible, allow no direct radiation on the lumber. The temperature of the air entering the piles must be known at all times, preferably by means of recording thermometers with extension bulbs which have been calibrated in place, as directed under 1 (*e*).

The temperatures in the tops, bottoms, edges, and centers of the piles and at different points in the kiln should be determined occasionally by using standardized thermometers located as directed under 1 (*e*).

(*b*) Determination of current humidities: Never attempt to determine the relative humidity of the air where the bulbs of the hygrometer are exposed to direct radiation. Where direct radiation may take place, it is necessary to shield the hygrometer from the heating pipes before readings are taken. The relative humidity of the air entering the piles must be indicated at all times by means of standardized glass wet and dry bulb hygrometers placed as directed under 1 (*e*). Before reading the hygrometer fan the bulbs briskly for about a minute. An air circulation of at least 15 feet per second past the wet bulb is necessary for an accurate humidity reading. The wick should be of thin silk or linen and it must be free from oil or dirt at all times. It should come into close contact with as much of the bulb as possible. Knowing the correct wet and dry bulb hygrometer readings, the relative humidity may be determined from the humidity diagram, figure 26.

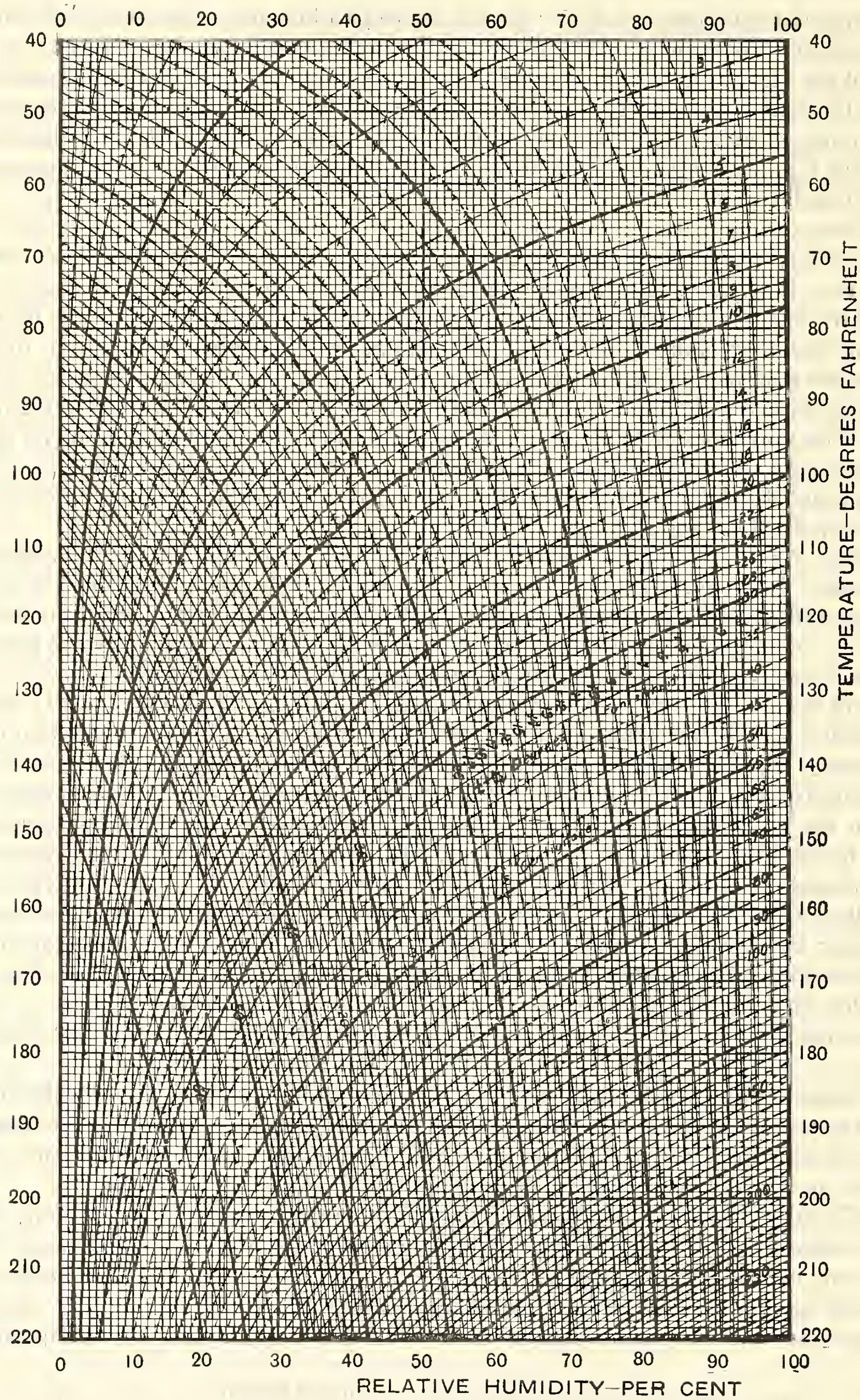
Relative humidity is shown on the horizontal scale and Fahrenheit temperature on the vertical scale. The curves running from the top left to the bottom right part of the chart are for various differences in the wet and dry bulb readings. The curves are numbered near the center of the chart above the heading "$(t—t^1)$ degrees Fahrenheit." To get the relative humidity, follow the curve which is numbered to correspond to the difference of the wet and dry bulb readings till it intersects the horizontal line numbered to correspond to the dry bulb reading. Directly below this intersection in a vertical line will be found the relative humidity on the bottom scale. Example: Dry bulb reading, 120; wet bulb reading, 113; difference, 7. Curve 7 intersects horizontal line 120 at vertical line 79. Relative humidity is 79 per cent.

When the humidity is desired in a Tiemann kiln, use the set of curves running from the top right to the bottom left part of the chart. Locate the lower of the two thermometer readings on the scale at the right of the chart. This is the reading of the thermometer in the baffle box. Follow along parallel to the nearest curve till the horizontal line is crossed whose number is the higher thermometer reading. Vertically below this point of intersection on the lower scale will be found the relative humidity. Example: Baffle thermometer reading, 112°; flue thermometer reading, 120°. Start at 112 on right-hand scale, follow parallel to curve 28 till horizontal line 120 is crossed. This point falls on vertical line 80. Relative humidity is 80 per cent.

(*c*) Determination of circulation: During each drying operation the circulation of the air should be tested several times, as under 1 (*d*). As the lumber becomes drier, it has less cooling effect on the air, and this may change the circulation in the kiln. If this occurs, corresponding changes in the location of instruments should be made.

(*d*) Weighing of samples and determination of current moisture condition: The 24-inch samples, placed as directed under 1 (*b*), should be weighed daily to an accuracy of one-tenth of 1 per cent on the platform scale. From test 1 (*a*) the initial moisture contents of these samples are known. Their initial weights were determined by test 1 (*b*). Knowing their initial moisture contents and weights, their oven-dry weights may be computed from the formula:

$$\text{Oven-dry weight of sample} = \frac{\text{initial weight}}{100 + \text{initial moisture content}} \times 100$$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130
140
150
160
170
180
190
200
210
220
RELATIVE HUMIDITY—PER CENT
TEMPERATURE—DEGREES FAHRENHEIT

Fig. 26.

Having the calculated oven-dry weights and daily weights of the samples, their current moisture contents may be computed from the formula:

$$\text{Current moisture content of sample} = \frac{\text{current weight—oven-dry weight}}{\text{oven-dry weight}} \times 100$$

Therefore, since the samples were cut from representative stock, the drying rate of the material is known currently.

3. *Final tests.*—(*a*) Average kiln-dry moisture condition of samples: When the current moisture contents of the samples indicate that the material is dried to the required point, three 1-inch sections are cut from the center of each sample. One section from each sample is used to determine the average kiln-dry moisture content of each sample by the method of test 1 (*a*). This test must be made immediately after sawing.

(*b*) Distribution of moisture in kiln-dry samples: A thin shell (about one-fourth inch) is split from the four outer surfaces of the second 1-inch section cut from each sample. The outsides and centers are tested for moisture content separately and immediately after sawing by the method of 1 (*a*). The results of this test show the distribution of moisture in cross section of the samples. The difference between the moisture contents of the outer shells and the centers shows whether or not the distribution is sufficiently uniform across the sections.

(*c*) Determination of casehardening in kiln-dry samples: The first indication of case-hardening is surface checking. The next sign of case-hardening is honeycombing or interior checking along the medullary rays. This defect can not always be detected by a superficial inspection. It is necessary to cut the stock to discover it. Occasionally it is evidenced by a bulging of the surface over the honeycombed part. Often neither of these defects is present. In this case the third 1-inch section from each sample is resawed two or three times from one end down to within about half an inch of the other end (see fig. 23). If the material is case-hardened and dry, it will pinch the saw; if it is not dry at the time of sawing, the cupping of the outer prongs will increase upon further drying. If the kiln-dried samples show case-hardening, the material should be steamed until the resawed sections do not pinch the saw in this test.

(*d*) Average kiln-dry moisture condition of the whole pieces: When the kiln is unloaded, the whole pieces from different parts of the piles and kiln are weighed and then cut as follows: Remove about 2 feet from one end and then cut off three 1-inch sections. The average kiln-dry moisture contents of the whole pieces are determined from one section as in test 3 (*a*). The other sections are used as stated in 3 (*f*) and 3 (*g*).

(*e*) Calculation of initial moisture condition of whole pieces: From the kiln-dry weights and kiln-dry moisture contents of the whole pieces, their oven-dry weights may be computed from the formula:

$$\text{Oven-dry weight of whole pieces} = \frac{\text{kiln-dry weight}}{100 + \text{kiln-dry moisture content}} \times 100$$

Knowing the initial weights and oven-dry weights of the whole pieces, their initial moisture contents are computed from the formula:

$$\text{Initial moisture content of whole pieces} = \frac{\text{initial weight—oven-dry weight}}{\text{oven-dry weight}} \times 100$$

Therefore the initial and kiln-dry moisture conditions of the samples, whole pieces, and the average stock are known.

(*f*) Distribution of moisture in kiln-dry whole pieces: This test is a duplicate of test 3 (*b*).

(*g*) Determination of case-hardening in kiln-dry whole pieces: This test is a duplicate of test 3 (*c*).

(*h*) To determine the effect of drying on the strength of the stock: It is practically impossible to determine the effect of the process of drying on the properties of the stock by inspection unless some visible defect has developed. This is not usual, and as the inspector can not always resort to mechanical tests he should be able to show from his operation records that conditions in the kiln have been kept within the specifications recommended as safe for kiln-drying airplane stock.

Detailed instructions for the kiln drying of various airplane woods have been prepared and issued in the form of a specification. This specification, which follows, is based upon a great many experimental kiln runs and strength tests upon matched specimens. Part of the matched specimens were tested while green, part were tested after air drying under shelter, and part were kiln dried to the same degree as the air-dried specimens and then tested. In this way the effect of kiln drying as compared to air drying was investigated and the conditions of kiln drying were determined for most rapid drying without decreasing the strength below that obtained in air drying to the same degree.

SPECIFICATION FOR KILN DRYING FOR AIRCRAFT STOCK.

GENERAL.

1. This specification covers general requirements for kiln drying wood for airplane stock.

2. The kiln-drying operations shall be so conducted that the wood will not lose any strength, toughness, or other physical property as compared to wood air dried to the same degree of dryness.

MATERIAL.

3. Only one species and approximately one thickness shall constitute a kiln charge. A difference of not to exceed one-half inch in the thickness of single pieces will be allowed.

PILING.

4. The boards shall be piled so that the horizontal width of the spaces between them will be at least 1 inch for each inch of board thickness, but in no case shall the horizontal width of such spaces exceed 3 inches. The boards must be held flat and straight while drying.

5. For stock up to four-quarters (1 inch) in thickness the crossers shall be at least 1 inch thick and not over $1\frac{1}{2}$ inches wide.

6. For stock from four to twelve quarters (1 to 3 inches) in thickness the crossers shall be at least $1\frac{1}{2}$ inches thick and not over $1\frac{1}{2}$ inches wide.

7. For stock over twelve quarters (3 inches) in thickness the thickness of the crossers shall be increased in the above proportion but must not exceed 2 inches in any case.

8. The crossers shall be placed directly over one another and not over 3 feet apart in the courses.

9. The lumber must be so disposed in the kiln as to permit of easy access on both sides of the pile and the taking of temperature and humidity readings whenever required by the inspector.

INSTRUMENTS.

10. At least one recording thermometer or recording hygrometer of approved make shall be used in each dry kiln compartment.

11. Recording thermometers and hygrometers shall be checked at least once every kiln run with a standard thermometer or a glass thermometer calibrated to an accuracy of 1° F. This comparison shall be made with the thermometers placed so as to record the maximum temperature of any portion of the pile.

12. *Thermometers.*—Thermometer bulbs must be shielded from direct radiation from steam pipes, wet lumber, cold walls or surfaces, and must receive a free circulation of air.

13. The inspector may, at his discretion, place other thermometers at any point in the pile.

14. *Hygrometer.*—Humidity readings shall be made at least three times daily or more often as the inspector may desire, according to standard methods approved by the inspector, at the same points where the bulbs for the recording thermometers and hygrometers are placed.

15. The following shall constitute a standard method: Use a glass or recording wet and dry bulb hygrometer with distilled water and with the wick changed at least once a week; produce a circulation of air past the wet bulb of at least 15 feet per second before reading.

16. Hygrometer bulbs must be shielded from direct radiation of steam pipes, wet lumber, and cold walls or surfaces, and must receive a free circulation of air.

STEAMING.

17. *At the beginning of the drying operations.*—Green wood is to be steamed at a temperature not to exceed 15° F. higher than the initial drying temperature specified in tables 5 and 6 for six hours for each inch of thickness. Humidity during steaming period must be 100 per cent, or not below 90 per cent, in every portion of the pile.

18. Previously air-dried wood is to be steamed at a temperature not to exceed 30° F. higher than the initial drying temperature specified in tables 5 and 6 for eight hours for each inch of thickness. Humidity during steaming period must be 100 per cent, or not below 90 per cent, in every portion of the pile.

19. *Near the end of the drying.*—If on official test the stock shows serious casehardening it shall be steamed at a temperature not to exceed 20° F. higher than the final drying temperature specified in tables 5 and 6 for not more than three hours. After steaming it shall be redried.

TEMPERATURE AND HUMIDITY.

20. Operating conditions are specified in tables 5 and 6, but lower temperatures and higher humidity conditions are permissible.

21. The progression from one specified stage to the next must proceed without abrupt changes.

22. *Green wood (above 25 per cent moisture) over 3 inches thick.*—Reduce the temperature values given in tables 5 and 6 by 5° F. for each inch increase in thickness.

23. *Air-seasoned wood (below 25 per cent moisture) over 3 inches thick.*—Reduce the temperature values given in tables 5 and 6 by 5° F. for each inch increase in thickness.

TABLE 5.

Stage of drying.	Drying conditions.	
	Maximum temperature.	Minimum relative humidity.
	°F.	*Per cent.*
At the beginning	120	80
After fiber saturation is passed (25 per cent)	125	70
At 20 per cent moisture	128	60
At 15 per cent moisture	138	44
At 12 per cent moisture	142	38
At 8 per cent moisture	145	33
Final	145	33

24. Table 5 applies to the following woods:

Ash, white, blue, and Biltmore.
Birch, yellow.
Cedar, incense.
Cedar, northern white.
Cedar, western red.
Cedar, Port Orford.
Cypress.
Pine, sugar.
Pine, white (Idaho or eastern).
Spruce, eastern (red or white).
Spruce, Sitka.
Fir, Douglas.

TABLE 6.

Stage of drying.	Drying conditions.	
	Maximum temperature.	Minimum relative humidity.
	°F.	Per cent.
At the beginning	105	85
After fiber saturation is passed (25 per cent)	110	73
At 20 per cent moisture	117	62
At 15 per cent moisture	129	46
At 12 per cent moisture	135	42
At 8 per cent moisture	135	40
Final	135	40

25. Table 6 applies to the following woods:

Cherry.
Mahogany.
Oak, white and red.
Walnut, black.
Maple.

TESTS DURING DRYING.

26. Samples shall be inserted in the pile in such manner that they will be subjected to the same drying conditions as that portion of the pile where inserted. They shall be so placed that they can be removed for periodical weighing in order to ascertain the average moisture content of the pile at any time.

27. Three samples shall be used for each 10,000 board feet or less of material in the pile. Each sample is to be 2 feet long and shall not be cut nearer than 2 feet to the end of one of the pieces to be dried.

28. The original moisture content of the samples shall be determined from sections 1 inch thick cut from both ends of the sample at the time it is sawed from the stick. This determination shall be made as provided in the specifications. (See Appendix, p. 147.)

29. Before placing them in the pile, the ends of the samples must be given a thorough coating of asphaltum varnish to prevent end drying.

30. The samples shall be weighed to an accuracy of one-tenth of 1 per cent immediately after cutting the moisture sections and before placing in the kiln. They shall be weighed at least daily when the time of drying is 10 days or less, and at least every other day when the time of drying is more than 10 days.

31. The samples shall be placed in the pile and distributed so that they will be exposed to the average, most rapid, and slowest drying, except that they shall not be placed on the top or bottom layers. The samples placed in the portion of the pile where drying is most rapid shall control the regulation of the temperature and humidity.

32. After obtaining the dry weight of the samples, the average moisture condition of the pile during drying shall be determined after each weighing.

33. The following example will illustrate the method employed:

Original weight of sample = 7.35 pounds.
Original moisture per cent (average of the two 1-inch sections) = 47.
Calculated dry weight of sample = 7.35 divided by 1.47 = 5.00 pounds.
Current weight = 6.23 pounds.
Moisture in samples = 6.23 − 5.00 = 1.23 pounds.
Current moisture per cent = (1.23 divided by 5.00) × 100 = 24.6.

34. Continuous and permanent records must be kept of the temperature and humidity observations and the percentage of moisture in the lumber in the kiln.

TESTS AFTER DRYING.

35. Standard moisture content and case-hardening tests shall be made before the lumber is removed from the kiln. Material for these tests shall be taken from four boards for each 5,000 board feet or less of material in the pile. Pieces selected must fairly represent the dried stock and shall be taken from different parts of the pile. At his discretion, the inspector may select other pieces for tests. Sections for these tests shall not be cut nearer than 2 feet to the ends of the pieces.

36. Three adjacent sections 1 inch thick shall be cut from the centers of each test piece of stock. Each section must be weighed within five minutes to prevent moisture evaporation.

37. The first section (A, fig. 27) shall be dried whole and the average moisture content obtained as provided in specifications.

38. The second section (B, fig. 27, moisture distribution) shall be cut into an outer shell ¼ inch wide and an inner core ½ inch wide. The moisture content of the outer shell and inner core shall be determined.

39. The third section (C, fig. 27) shall be sawed parallel to the wide faces of the original board into tongues or prongs, leaving about ½ inch of solid wood at one end of the section. For material less than 2 inches thick two saw cuts shall be made and for material more than 2 inches thick five saw cuts shall be made. In sections having six prongs the second prong from each side shall be broken out, leaving two outer and two central prongs. The center prong shall be removed from sections having only three prongs.

40. The third section shall then be allowed to dry for 24 hours in the drying room and any curving of the prongs noted.

41. If the prongs remain straight, perfect conditions of stress and moisture content are indicated.

42. If the outer prongs bend in, conditions of casehardening are indicated.

43. Only very slight casehardening is permissible.

FINAL MOISTURE CONDITIONS.

44. An average dryness of approximately 8 per cent, unless otherwise specified,* shall be required. A moisture content of from 5 to 11 per cent is permissible in individual sticks.

45. The variation in moisture content between the interior and exterior portions of the wood, as shown by the "moisture distribution section" provided for in paragraph 38, must not exceed 4 per cent.

SEASONING.

46. Before manufacture the wood shall be allowed to remain in a room, with all parts under uniform shop conditions, at least two weeks for 3-inch material and other sizes in proportion.

*See Note 3.

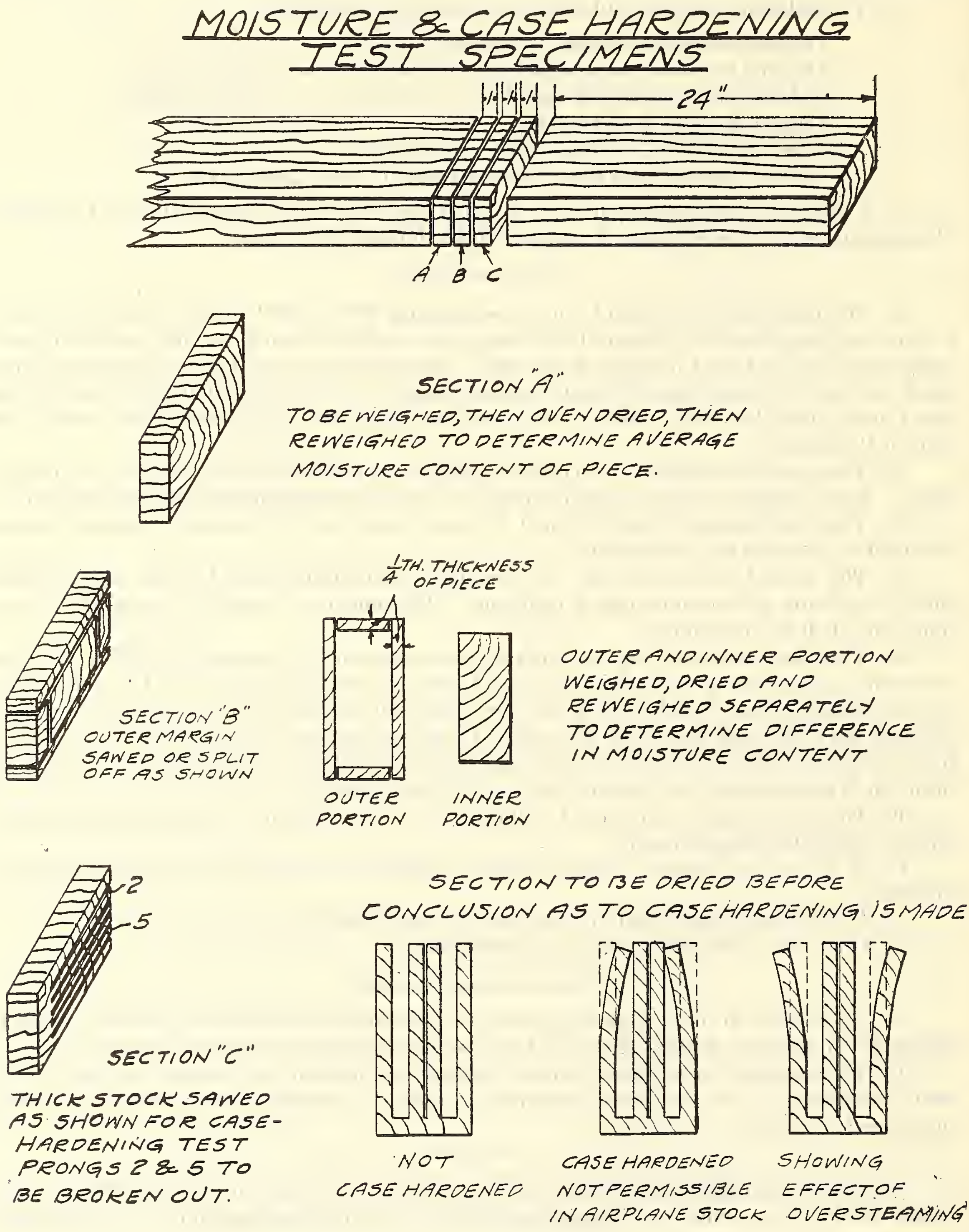

Fig. 27.—Moisture and casehardening test specimens.

STEAMING AND BENDING OF ASH FOR LONGERON CONSTRUCTION.

47. The ash shall be cut in the form of rough squares sufficiently large to allow for shrinkage and finish.

48. Where it is necessary to bend this material, it shall be steamed in the green condition (more than 18 per cent moisture), bent on forms, and then kiln dried, as provided in paragraph 23.

49. Steaming shall be conducted at a temperature not to exceed 212° F. for a period not longer than six hours and the bending shall be accomplished while the material is hot.

INSPECTION.

50. At all stages of the process the lumber shall be subjected to inspection by the inspection department.

51. The inspector shall mark all lumber with the official acceptance or rejection symbol.

52. The inspector shall have free access to every part of the kiln at all times and shall be afforded every reasonable opportunity to satisfy himself that this specification is being complied with.

NOTE 1. *Steaming.*—It has been found possible to dry spruce satisfactorily without steaming to relieve casehardening. A preliminary steaming is given at low temperature, and after the drying has been completed the material is held in the kiln for 24 hours, with a humidity of 75 per cent or 80 per cent, at room temperature.

NOTE 2. *Tests during drying.*—(Paragraph 31.) The most rapid drying sample should not be confused with the sample of lowest moisture content. If the original moisture content was practically the same for all samples, then at any stage of the run the low sample would be the most rapid drying. However, the original moisture content is not likely to be uniform for the whole charge, and with stock of varying moisture content the run should be controlled for the stock of high moisture content. Other things being equal, the sample with the highest moisture content will dry the most rapidly, so that in such a case the specification would still hold. It would therefore be desirable to place the high original sample where it will be the most rapid drying sample. Otherwise it would be necessary to take into account the high stock—possibly specify following the average of the samples on the entering air side of the pile provided the average is not more than 10 per cent below the high sample.

NOTE 3. *Final moisture content.*—For naval aircraft, it has been found desirable to have the moisture content on removal from the kiln about 12 per cent. The maximum individual variation allowed should not be over 3 per cent.

TREATMENT OF WOOD AFTER REMOVAL FROM THE KILN.

Lumber should be retained for at least two weeks after removal from the dry kiln in a shed or room where the conditions are approximately the same as in the shop where the material is to be worked up. The necessity for this will be understood upon consideration of the following facts: When lumber is drying in the kiln the outer surface is necessarily somewhat drier than the interior. In good methods of drying this difference is a minimum and in bad methods of drying it is excessive; but it exists to a certain extent in all methods of drying. When the lumber has been dried down to a point somewhat below the condition to which it will finally come when exposed to the normal shop working conditions, it will gradually reabsorb moisture on the outside. Thus, thoroughly kiln-dried lumber, if it has stood in an unheated room for some time, will be found to be drier on the inside than it is on the surface, though the difference is likely to be very small. Since differences in moisture content are indicative of internal stresses existing in the wood, it is evidently desirable to have the moisture distribution as uniform as possible before the lumber is made up into finished products; otherwise the adjustment of stresses, when the lumber has been cut up, will cause warping, checking, or other troubles.

Just how long lumber should remain in the shop air after being kiln-dried will depend, of course, upon a great many circumstances. Generally speaking, the longer it remains the better it will be, provided the moisture conditions of the room in which it is stored are suitable. The same kind of a test as has been explained for casehardening occurring in the dry kiln will apply as a test of the lumber after remaining in storage, to see whether the internal stresses have been neutralized.

Even if casehardening has been removed in the dry kiln by resteaming at the end of the drying period, there may still exist within the lumber slight differences in moisture content which will gradually adjust themselves under proper storage conditions, so that material which has been steamed before removal from the kiln is also benefited by being allowed to stand in the room before it is manufactured. Recent experiments have shown that the length of time required for kiln-dried stock to reach a state of equilibrium under shop conditions after removal from the kiln may be reduced very materially by allowing it to remain in the kiln for about 24 hours, after the drying has been completed, at a humidity of 75 per cent or 80 per cent and shop temperature.

Ideal conditions for the storage and manufacturing of lumber require regulation of the humidity, which should be kept slightly below that of the average conditions to which the lumber is to be subjected after it is put into service. The nearer these conditions are actually met in practice the better are the results to be expected, particularly where requirements are so exacting as in the construction of airplanes.

CHANGES OF MOISTURE IN WOOD WITH HUMIDITY OF AIR.

Wood is a hygroscopic material; that is, it has the property of absorbing moisture from the air or surrounding medium. It has already been explained that there are two different kinds of moisture found in wood, namely, free water, which occupies the openings in the cell structure of the wood, and hygroscopic water, which is actually taken into the cell walls and which upon being removed or added to wood causes shrinkage or swelling.

There is a definite moisture content to which wood will eventually come if it is held in an atmosphere which is at a constant humidity and temperature. The moisture content of wood will vary with the average atmospheric conditions, also with the size of the material. Thus, ordinary lumber which is stored in the open during the summer months for sufficient time will eventually attain a moisture content of from 8 to 15 per cent, and wood stored indoors in a heated building will in time fall to about 5 or 6 per cent because of the lower relative humidity. If the relative humidity is constant, an increase in temperature decreases the moisture-holding power of the wood. However, the moisture content is not appreciably affected by temperature within a range of 25° to 30° F.

Figure 28 shows the relation between the moisture content of wood and the humidity conditions of the atmosphere. The data for the curve were obtained by keeping the wood at a constant humidity and temperature until no further change in moisture occurred. This curve can be used as an aid in controlling the moisture conditions of wood, the approximate atmospheric condition being known, and in determining the proper humidities for storing lumber in order to secure a certain moisture content and give uniform material for use in fine wood jointing, propellers, etc. It is of importance to have wood to be used for propellers of uniform moisture content. The curve may be used also to prepare wood for use in a given locality, such as the border States, where the humidity is usually very low. Propellers for use under such conditions should be made up at a low moisture content, in order that there may be less tendency for moisture changes to take place when they are put in service. It must be remembered that this curve must not be used for dry-kiln work because of the fact that the dry-kiln temperatures used are higher than those at which the data were collected Furthermore, the curve represents the ultimate moisture content at a given temperature and humidity, and in the case of large pieces of wood this moisture content would not be reached for a long period of time. Kiln drying tends to reduce the hygroscopic properties of wood, hence curves for kiln-dried wood are lower than the one given. For example, wood that had been dried to 2 per cent moisture, or less, if subjected to humidities between 30 and 70 per cent, would probably show a corresponding moisture content about 1½ to 2½ per cent lower than in the curve in figure 28.

VENEER AND PLYWOOD.

VENEER.

Veneer may be loosely defined as thin wood. It usually varies in thickness from one-hundredth inch to one-eighth inch, though it is commercially possible to cut it thinner, and thicker sizes are to be obtained. However, in general, veneer used in aircraft falls within the limits stated.

There are three common methods of manufacturing veneer, as follows: (1) The rotary process, (2) the slicing process, (3) the sawing process.

By far the greater portion of all veneer manufactured is made by the rotary process. Veneer made by this process is all slash cut, and the length along the grain is limited by the length of the veneer lathe. Rotary veneer longer than 100 inches is more or less uncommon.

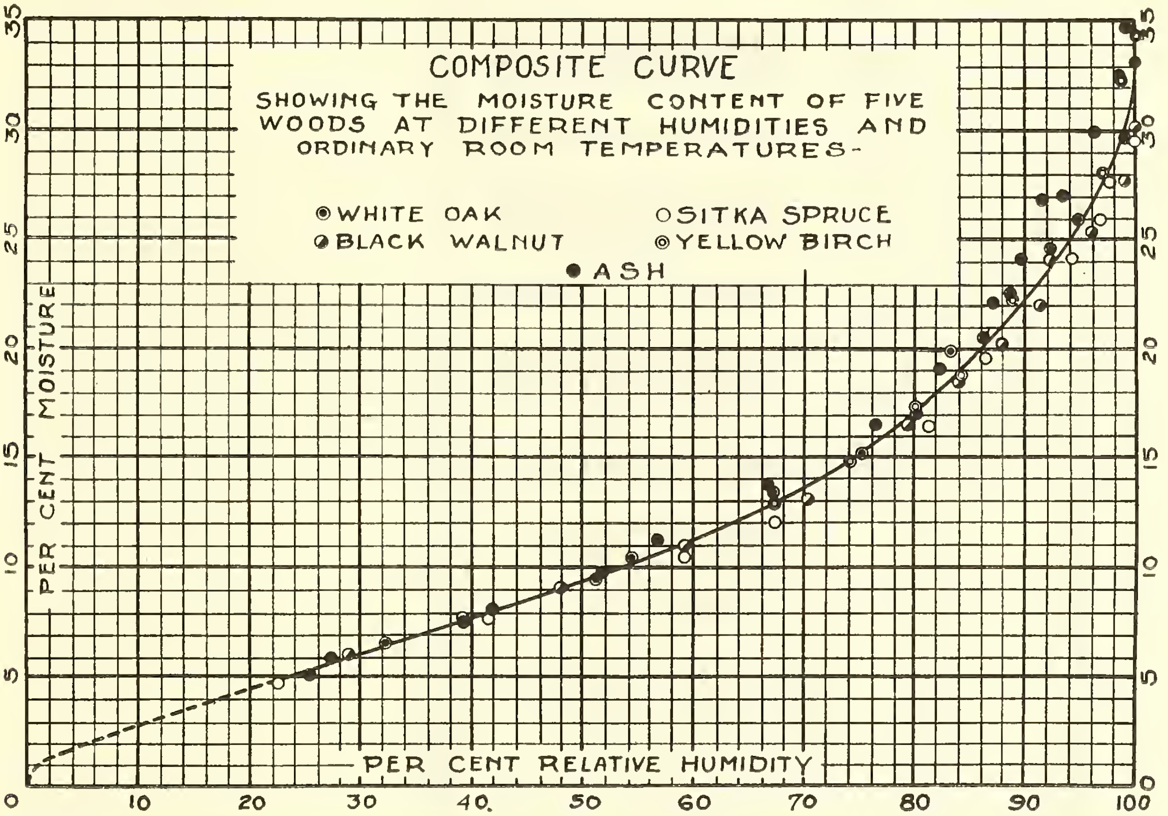

Fig. 28.—Composite curve of moisture content of fine woods at different humidities and ordinary room temperature.

Sliced veneer is usually manufactured only from the finer woods. On account of the fact that it is possible to produce quartered veneer on slicing machines, and the waste on account of saw kerf is absent, this method of manufacture is preferred where pattern is important and the value of the wood is great. The length parallel to the grain of sliced veneer is limited by the length of the knife.

Sawed veneer can be produced in almost any reasonable length and from any kind of stock. The material produced may be either quartered or slash. In general, sawed veneer will not be specified for aircraft uses, to the exclusion of rotary stock, except where it is necessary to have extra long lengths or quartered stock or for some other reason it is impossible to secure the stock by rotary cutting. It may happen, for instance, that the stock from which the veneer is to be cut can not be handled to advantage in a rotary lathe on account of its shape.

TABLE 7.—*Comparative strengths of 5 species of sawed, sliced, and rotary-cut plywood.*

Each panel composed of 3 plies of $\frac{1}{16}$-inch veneer; grain of successive plies at right angles; casein glue used.

	Method of manufacture.	Average panel thickness.	Specific gravity.†	Per cent moisture.	Column-bending modulus. Parallel.* Number of tests.	Column-bending modulus. Parallel.* Pounds per square inch.	Column-bending modulus. Perpendicular.* Number of tests.	Column-bending modulus. Perpendicular.* Pounds per square inch.	Tensile strength. Parallel.* Number of tests.	Tensile strength. Parallel.* Pounds per square inch.	Tensile strength. Perpendicular.* Number of tests.	Tensile strength. Perpendicular.* Pounds per square inch.	Splitting resistance. Number of tests.	Splitting resistance. Total work in splitting, inch-pounds.	Splitting resistance. Splitting modulus, inch-pounds per inch.
		Inches.													
Ash, commercial white	Sawed	0.206	0.55	11.4	10	8,220	10	2,160	10	6,810	10	4,310	20	910	4,400
Do	Sliced	.183	.56	10.9	10	9,670	10	1,940	10	7,040	10	4,770	20	690	3,790
Do	Rotary cut	.193	.52	12.0	10	7,180	10	1,810	10	4,290	10	3,180	20	650	3,350
Birch	Sawed	.194	.65	10.1	8	10,520	8	2,660	8	8,600	8	6,590	16	1,620	8,360
Do	Sliced	.169	.65	9.4	9	9,670	9	1,760	9	9,230	9	5,760	18	1,460	8,660
Do	Rotary cut	.182	.61	10.3	10	11,330	10	2,340	10	11,350	10	5,970	20	1,360	7,460
Mahogany, African	Sawed	.212	.53	10.0	10	7,930	10	2,000	10	7,220	10	4,200	20	1,770	8,380
Do	Sliced	.170	.52	11.0	6	8,360	6	2,110	6	6,690	6	4,270	72	1,180	6,920
Maple, sugar	Sawed	.196	.67	11.2	9	13,670	9	2,750	9	11,810	9	6,810	18	1,230	6,250
Do	Sliced	.176	.67	10.2	10	13,400	10	2,480	10	11,340	10	6,770	20	1,110	6,290
Do	Rotary cut	.134	.67	10.8	10	12,650	10	2,300	10	10,140	10	5,750	20	1,250	6,450
Poplar, yellow	Sawed	.194	.51	8.7	10	8,630	10	1,950	10	9,610	10	4,780	20	1,150	5,960
Do	Sliced	.172	.50	8.5	10	9,060	10	1,790	10	8,140	10	4,720	20	890	5,180
Do	Rotary cut	.179	.50	8.8	10	7,710	10	1,580	10	8,540	10	4,180	20	805	4,500

* Parallel and perpendicular refer to direction of grain of faces relative to direction of application of force.
† Specific gravity based on oven-dry weight and volume at test.

A special series of tests was made to determine the effect of the method of cutting veneer on the strength of plywood panels made from it. Detailed results are presented in table 7, and the general conclusions drawn follow:

(*a*) The effect of the method of cutting veneer on the strength of plywood depends on the species cut, although in general, the effect, as shown by the bending and tension tests, is not great.

(*b*) Of the three methods of cutting, the sawed and sliced material, for the species tested, gave the more similar results. The commercial white ash, sugar maple, and yellow poplar pannels cut by these methods were slightly superior in bending and tensile strength to the rotary-cut panels.

(*c*) For birch the panels of rotary-cut veneer were slightly superior in bending and tensile strength to panels of either sawed or sliced veneer.

(*d*) For the species tested, with the possible exception of the African mahogany, panels of sawed veneer twist less than panels of either sliced or rotary-cut veneer.

(*e*) With the exception of birch the results show little difference in the twisting of panels of sliced or rotary-cut veneer.

For the convenient calculation of the weight of veneer and plywood, table 8 has been prepared. This table presents the weights, per square foot, of veneer of various thicknesses and species, at the average air-dry moisture condition shown in the second column. The weight of blood albumen glue per square foot and the weight of a typical casein glue (Certus) per square foot are also given, so that it is possible to calculate the average weight of any plywood made up of the species listed and using blood or casein glue. This is done simply by adding together the weights of the individual plies and the weight of the glue, which is obtained by multiplying the weight of the glue per square foot by the number of glue lines in the plywood. This number is always one less than the number of plies.

While it is usually not necessary to know the tensile strength of single-ply veneer as such, this figure is very convenient in computing the probable strength in tension of plywood made up in various manners. The last column of table 9 presents computed tensile strengths of single-ply veneer. Reference to the other data in this table will be found in the text under the discussion of plywood.

PLYWOOD.

In general plywood consists of a number of layers of wood veneer glued together by some suitable glue or adhesive. Occasionally the term is applied to material in which one or more of the layers are composed of some other material than wood.

The weight of plywood has already been discussed in connection with the weight of veneer (see table 8).

Until recently little information was available on the mechanical properties of plywood. Within the last year and a half, however, about 50,000 tests have been made and tabulated. Since the subject is rather new, a full discussion is presented, followed by tables of strength properties.

PROPERTIES OF WOOD PARALLEL AND PERPENDICULAR TO THE GRAIN.

Wood, as is well known, is a nonhomegenous material with widely different properties in the various directions relative to grain. This difference must be recognized in all wood construction, and the size and form of parts and placement of wood should be such as to utilize to the best advantage the difference in properties along and across the grain. It is the strength of the fibers in the direction of the grain that gives wood its relatively high modulus of rupture, and tensile and compressive strength parallel to the grain. Were it a homogenous material, such as cast iron, having the same strength properties in all directions that it has parallel to the grain, it would be unexcelled for all structural parts where strength with small weight is desired. As it is the tensile strength of wood may be 20 times as high parallel to the grain as perpendicular to the grain and its modulus of elasticity from 15 to 20 times as high.

TABLE 8.—*Weights of veneer.*

[In ounces per square foot of one-ply, oven-dry; veneer thicknesses in inches.]

Species.	Specific gravity oven-dry based on volume air-dry.	Air-dry moisture content, per cent.	$\frac{1}{100}$	$\frac{1}{80}$	$\frac{1}{64}$	$\frac{1}{60}$	$\frac{1}{55}$	$\frac{1}{48}$	$\frac{1}{40}$	$\frac{1}{32}$	$\frac{1}{28}$	$\frac{1}{24}$	$\frac{1}{20}$	$\frac{1}{16}$	$\frac{1}{12}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{3}{16}$	$\frac{1}{4}$
Ash, black	0.50	10.4	0.42	0.52	0.65	0.69	0.76	0.87	1.04	1.30	1.49	1.74	2.08	2.60	3.47	4.16	5.20	6.94	7.81	10.41
Ash, commercial white	.58	8.9	.48	.60	.75	.80	.88	1.00	1.21	1.51	1.72	2.01	2.41	3.02	4.02	4.82	6.04	8.05	9.05	12.06
Basswood	.38	8.4	.32	.40	.49	.53	.58	.66	.79	.99	1.13	1.32	1.58	1.98	2.64	3.16	3.96	5.28	5.94	7.92
Beech	.63	11.2	.52	.66	.82	.87	.95	1.09	1.31	1.64	1.87	2.19	2.62	3.28	4.37	5.24	6.56	8.74	9.84	13.12
Birch, yellow	.63	9.6	.52	.66	.82	.87	.95	1.09	1.31	1.64	1.87	2.19	2.62	3.28	4.37	5.24	6.56	8.74	9.84	13.12
Butternut	.39	7.6	.32	.41	.51	.54	.59	.68	.81	1.02	1.16	1.35	1.62	2.03	2.71	3.25	4.06	5.42	6.09	8.12
Cedar, Spanish	.37	7.3	.31	.38	.48	.51	.56	.64	.77	.96	1.10	1.28	1.54	1.92	2.57	3.08	3.85	5.13	5.78	7.70
Cherry, black	.51	9.2	.42	.53	.66	.71	.77	.88	1.06	1.33	1.52	1.77	2.12	2.65	3.54	4.25	5.31	7.08	7.96	10.62
Chestnut	.44	8.6	.37	.46	.57	.61	.67	.76	.92	1.14	1.31	1.53	1.83	2.29	3.05	3.66	4.58	6.10	6.87	9.16
Cottonwood (common)	.43	4.7	.36	.45	.56	.60	.65	.75	.90	1.12	1.28	1.49	1.79	2.24	2.98	3.58	4.47	5.97	6.71	8.96
Cypress, bald	.44	9.0	.37	.46	.57	.61	.67	.76	.92	1.14	1.31	1.53	1.83	2.29	3.05	3.66	4.58	6.10	6.87	9.16
Douglas fir (coast type)	.51	6.2	.42	.53	.66	.71	.77	.88	1.06	1.33	1.52	1.77	2.12	2.65	3.54	4.25	5.31	7.08	7.96	10.62
Douglas fir (mountain type)	.44	9.4	.37	.46	.57	.61	.67	.76	.92	1.14	1.31	1.53	1.83	2.29	3.05	3.66	4.58	6.10	6.87	9.16
Elm, white	.51	8.8	.42	.53	.66	.71	.77	.88	1.06	1.33	1.52	1.77	2.12	2.65	3.54	4.25	5.31	7.08	7.96	10.62
Gum, black	.52	7.2	.43	.54	.68	.72	.79	.90	1.08	1.35	1.55	1.80	2.17	2.71	3.61	4.35	5.42	7.32	8.12	10.82
Gum, cotton	.52	6.1	.43	.54	.68	.72	.79	.90	1.08	1.35	1.55	1.80	2.17	2.71	3.61	4.35	5.42	7.32	8.12	10.82
Gum, red	.49	11.3	.41	.51	.64	.68	.74	.85	1.02	1.28	1.46	1.70	2.04	2.55	3.40	4.08	5.10	6.80	7.66	10.20
Hackberry	.54	9.2	.45	.56	.70	.75	.82	.94	1.12	1.40	1.61	1.87	2.25	2.81	3.75	4.49	5.62	7.49	8.43	11.24
Hemlock, western	.42	8.6	.35	.44	.55	.58	.64	.73	.87	1.09	1.25	1.46	1.75	2.18	2.91	3.50	4.37	5.83	6.56	8.74
Magnolia (evergreen)	.51	8.8	.42	.53	.66	.71	.77	.88	1.06	1.33	1.52	1.77	2.12	2.65	3.54	4.25	5.31	7.08	7.96	10.62
Mahogany (Central American)	.49	7.9	.41	.51	.64	.68	.74	.85	1.02	1.28	1.46	1.70	2.04	2.55	3.40	4.08	5.10	6.80	7.66	10.20
Mahogany (African)	.46	8.0	.38	.48	.60	.64	.70	.80	.96	1.19	1.37	1.59	1.91	2.39	3.19	3.83	4.78	6.38	7.17	9.57
Maple (silver)	.48	8.2	.40	.50	.62	.67	.73	.83	1.00	1.25	1.43	1.67	2.00	2.50	3.33	4.00	5.00	6.66	7.50	7.00
Maple (sugar)	.62	10.5	.52	.65	.81	.86	.94	1.08	1.29	1.61	1.85	2.15	2.58	3.23	4.30	5.16	6.46	8.60	9.69	12.91
Oak, commercial red	.64	10.7	.53	.67	.83	.89	.97	1.11	1.33	1.66	1.90	2.22	2.66	3.33	4.44	5.32	6.66	8.88	9.99	13.3
Oak, commercial white	.68	11.0	.57	.71	.88	.94	1.03	1.18	1.41	1.77	2.02	2.36	2.83	3.54	4.72	5.66	7.08	9.43	10.61	14.1
Pine, long-leaf	.66	9.2	.55	.69	.86	.92	1.00	1.15	1.37	1.72	1.96	2.29	2.75	3.44	4.58	5.50	6.88	9.16	10.32	13.75
Pine, sugar	.37	11.4	.31	.38	.48	.51	.56	.64	.77	.96	1.10	1.28	1.54	1.92	2.57	3.08	3.85	5.13	5.78	7.70
Pine, short-leaf	.54	11.0	.45	.56	.70	.75	.82	.94	1.12	1.40	1.61	1.87	2.25	2.81	3.75	4.49	5.62	7.49	8.43	11.24
Pine, western yellow	.41	10.8	.34	.43	.53	.57	.62	.71	.85	1.07	1.22	1.42	1.71	2.13	2.84	3.41	4.27	5.69	6.40	8.54
Pine, white	.39	9.9	.32	.41	.51	.54	.59	.68	.81	1.02	1.16	1.35	1.62	2.03	2.71	3.25	4.06	5.42	6.09	8.12
Poplar, yellow	.41	6.1	.34	.43	.53	.57	.62	.71	.85	1.07	1.22	1.42	1.71	2.13	2.84	3.41	4.27	5.69	6.40	8.54
Spruce, Sitka	.38	8.9	.32	.40	.49	.53	.58	.66	.79	.99	1.13	1.32	1.58	1.98	2.64	3.16	3.96	5.28	5.94	7.92
Sycamore	.50	9.2	.42	.52	.65	.69	.76	.87	1.04	1.30	1.49	1.74	2.08	2.60	3.47	4.16	5.20	6.94	7.81	10.41
Tanguile (Philippine mahogany)	.54	11.8	.45	.56	.70	.75	.82	.94	1.12	1.40	1.61	1.87	2.25	2.81	3.75	4.49	5.62	7.49	8.43	11.24
Walnut, black	.57	4.8	.47	.59	.74	.79	.86	.99	1.19	1.48	1.70	1.98	2.37	2.97	3.96	4.75	5.94	7.92	8.91	11.87

Weight of glue per square foot: Blood albumen, about 0.3 ounce; Certus, about 0.4 ounce.

Sample.—To get the weight of a square foot of 5-ply wood consisting of 1 ply of $\frac{1}{12}$-inch basswood, 2 plies of $\frac{1}{16}$-inch basswood, and 2 plies of $\frac{1}{20}$-inch yellow birch for faces, at 12 per cent moisture, glued with Certus glue:

$$\text{Weight} = (1\times 2.64 + 2\times 1.98 + 2\times 2.62)\ 1.12 + 4\times 0.4 = 14.68 \text{ ounces.}$$

The weight of wood is quite variable, so that while the table given represents the average weights of material tested, large variations from these figures may be expected in individual pieces of veneer.

The example presented is slightly in error through neglecting the change in volume between the moisture content at 12 per cent and the moisture listed in the table.

In the case of shear the strength is reversed, the shearing strength perpendicular to the grain being much greater than the strength parallel to the grain. The low parallel-to-the-grain shearing strength makes the utilization of the tensile strength of wood along the grain difficult since failure will usually occur through shear at the fastening before the maximum tensile strength of the member is reached.

The large shrinkage of wood across the grain with changing moisture content may introduce distortion in a board that decreases its uses where a broad flat surface is desired. The shrinkage from the green to the oven-dry condition across the grain for a flat sawn board as determined by the average of 150 species is about 8 per cent, and for a quarter-sawed board about 4½ per cent, while the shrinkage parallel to the grain is practically negligible for most species.

PLYWOOD PANELS *v.* SOLID PANELS.

It is not always possible in a given use so to proportion a board or solid panel as to develop the necessary strength in every direction and at the same time to utilize the full strength of the wood in all directions of the grain. In such cases it is the purpose of plywood to meet this deficiency by crossbanding, which results in a redistribution of the material.

In building up plywood a step is made in obtaining equality of properites in two directions—parallel and perpendicular to the edge of a board. The greater the number of plies used for a given panel thickness, the more nearly homogeneous in properties is the finished panel. Thus, in an airplane engine mounting made of 15-ply veneer the mechanical properties of the panel in the direction parallel to the grain of the faces are almost the same as those in the direction at right angles to this. However, an increase in such properties as bending strength and modulus of elasticity at right angles to the grain of the faces is accompanied by a decrease of the values parallel to the grain of the faces with an increase of the number of plies. For a very large number of plies (of the same species and thickness) we may assume that the tensile strength in the two directions is the same and that it is equal to the average of the parallel-to-the-grain and perpendicular-to-the-grain values of an ordinary solid board or panel. This is not always exactly true, since the maximum stress of the plies with the grain at right angles to the force may not be reached at the same time as the maximum of the plies with the grain parallel to the force. Internal stresses due to change of moisture content may also tend to unbalance the strength ratio.

SYMMETRICAL CONSTRUCTION IN PLYWOOD.

On account of the great difference in shrinkage of wood in the direction parallel to the grain and perpendicular to it, a change in moisture content of plywood will inevitably either introduce or release internal stresses. Consider, for example, a three-ply construction and subject it to low-humidity conditions, so that the moisture content of the plywood is lowered. Because the grain of the core is at right angles to the grain of the faces, the core will tend to shrink a great deal more than the faces in the direction of the grain of the faces. This shrinkage subjects the faces to compression stresses and the core to tensile stresses. If the faces are of exactly the same thickness and of like density, the stresses are symmetrically distributed and no cupping should ensue.

Now consider that one face of a three-ply panel has been glued with the grain in the same direction as the core and that the moisture content of the panel is reduced. It is obvious that the internal stresses are now no longer symmetrically distributed, inasmuch as the compressive stress in one face has been removed. This face now shrinks a great deal more than the other face in the direction of the grain of the latter. The result is that cupping takes place. Figure 29a shows the effect of drying on a three-ply construction (unsymmetrical) in which the grain of two adjacent plies was parallel. The panel has curled up into a cylindrical surface with the parallel plies on the inner side. By adding another ply at right angles to the core we see that symmetry could again be established and that while we would have a four-ply panel in reality it gives a three-ply construction with a core of double the face thickness and would be regarded as such.

The necessity for exercising care in sanding the faces of a panel is obvious, inasmuch as different thicknesses on the faces would introduce unequal forces with changing moisture content.

In order to obtain symmetry, it is also necessary that both faces or symmetrical plies be of the same species.

To summarize: A veneer panel must be symmetrically constructed in order to retain its form with changes of moisture. Symmetry is obtained by using an odd number of plies. The

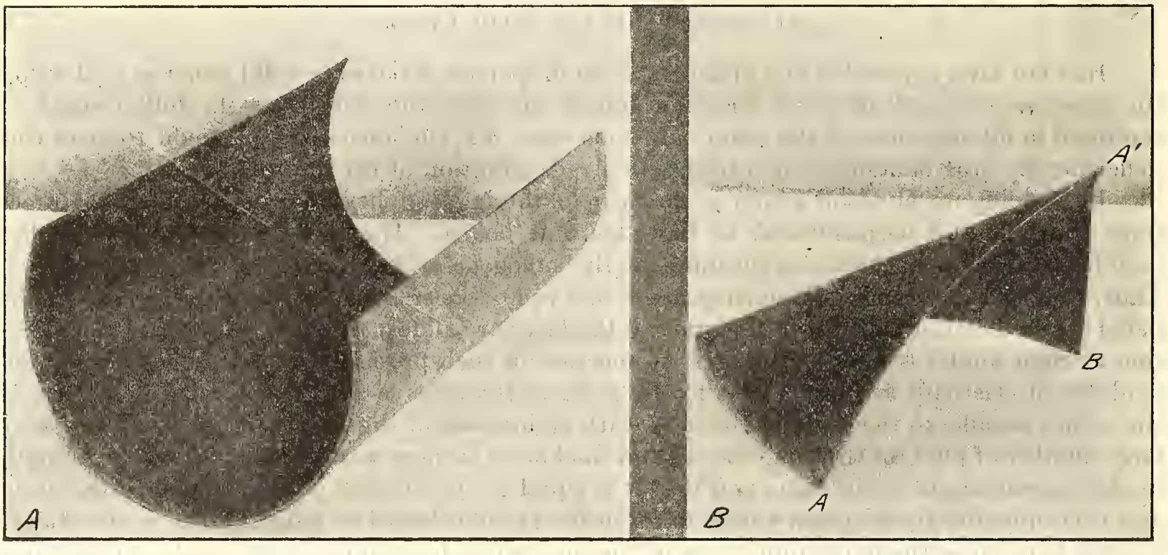

Fig. 29.—(a) Cupping resulting from unsymmetetrical construction in plywood. (b) Twisting resulting from plywood construction with grain of faces at 45 degrees with grain of core.

plies should be so arranged that for any ply of a particular thickness there is a parallel ply of the same thickness and of the same species on the opposite side of the core and equally removed from the core.

DIRECTION OF THE GRAIN OF ADJOINING PLIES.

In the discussion of symmetry of construction it was understood that the adjoining plies were always glued with the grain either parallel to or exactly at right angles to the core. In careless construction this may not always be the case. An extreme case of this kind is shown in figure 29b, in which the plies were glued so that the grain of each face of the panel was at 45 degrees with the grain of the core and so that the two faces were at 90 degrees with respect to each other. Whereas the unsymmetrical construction introduces cupping, a construction involving angles other than 0 and 90 degrees introduces twisting.

In building up a three-ply veneer panel the core should be glued with the grain at 90 degrees with the faces or as close to this as feasible.

EFFECT OF MOISTURE CONTENT.

The previous discussion has brought out the fact that a change in moisture content of a panel may introduce cupping and twisting in the panel if the panel is not carefully constructed. Hence it is highly desirable that the moisture content of the veneer before gluing be controlled so as to make the moisture content of the finished panel when it leaves the clamps about the same as it will average when in use and that all plies be at the same moisture content before gluing. The limits of from 10 to 15 per cent moisture in the finished panel will usually give satisfactory results when the panel is in service in the open air.

SHRINKAGE OF PLYWOOD.

The shrinkage of plywood will vary with the species, the ratio of ply thickness, the number of plies, and the combination of species. The average shrinkage obtained in 54 tests on a variety of combinations of species and thicknesses in bringing three-ply wood from the soaked to the oven-dry condition was 0.45 per cent parallel to the face grain and 0.67 per cent perpendicular to the face grain, with the ranges of from 0.2 to 1 per cent and 0.3 to 1.2 per cent, respectively. Other combinations and thicknesses may extend these limits and change the average somewhat. The species included in the tests made were mahogany, birch, poplar, basswood, red gum, chestnut, cotton gum, elm, and pine.

EFFECT OF VARYING THE NUMBER OF PLIES.

The question frequently arises, Should three or more plies be used for a panel of a given thickness? The particular use to which the panel is to be put must answer this question. Commercial considerations will also enter. Veneer of most species less than $\frac{1}{48}$ inch thick can not be cut by the rotary process with uniform success, and while a number of species may be cut by slicing to $\frac{1}{64}$ inch and less, such material is limited in width.

In general it may be said that the greater the number of plies the flatter the plywood will remain when subjected to moisture variations.

If the same bending or tensile strength is desired in the two directions, parallel and perpendicular to the grain of the faces, the greater the number of plies the more nearly the desired result is obtained. This same result may be obtained by a proper selection of ratio of core to total plywood thickness in three-ply construction. It must be borne in mind, however, that a plywood with a large number of plies, while stronger at right angles to the grain of the faces, can not be as strong parallel to the grain of the faces as three-ply wood, and hence a three-ply panel is preferable where greater strength is desired in one direction than in the other. Table 11 gives strength values for three-ply, five-ply, and seven-ply yellow birch plywood.

Where great resistance to splitting is desired, such as in plywood that is fastened along the edges with screws and bolts and is subject to forces through the fastenings, a large number of plies affords a better fastening.

It is a common experience that a glued joint is weakened when two heavy laminations are glued with the grain crossed. The same weakness exists in plywood when thick plies are glued together. When plywood is subject to moisture changes, stresses in the glued joint due to shrinkage are greater for the thick plies than for the thin plies. Hence in plywood constructed with many thin plies the glued joints will not be as likely to fail as in plywood constructed of a smaller number of thick plies.

EFFECT OF VARYING THE RATIO OF CORE TO TOTAL THICKNESS.

At first thought it may seem that the proper selection of the ratio of core to total plywood thickness in three-ply construction may enable the designer to get the same strength in both directions as is possible with many-plied panels. While this is true in general, it is not true that the same ratio will serve for both tension and bending. In birch, for example, a ratio of core to total plywood thickness of 5 to 10 gives the same strength in tension in both directions, but a ratio of about 7 to 10 gives the same strength in bending. For either ratio the plywood is not nearly as resistant to splitting as plywood of a greater number of plies totaling the same thickness.

SPECIES OF LOW DENSITY FOR CORES.

Where column strength and a flat panel are desired, full advantage of a strong species, such as birch, in the faces is best attained by using a thick core of a species, such as basswood or yellow poplar, rather than a thinner core of the same weight but of a species of geater density. A combination of strong faces and a thick light wood core has the advantage of greater separation of the faces than when using the thinner core of a heavier species, giving a marked increase in the internal resistance to forces that tend to bend the panel and a correspondingly great strength in bending with the same weight.

Consider, for example, that a certain panel contains a core of the same weight but of a specific gravity of one-half that of another core. This means that the core of lighter species is twice as thick as the core of high density and that the panel faces are spaced twice as far apart. In a long column, for instance, this is very desirable, for the maximum load a column can carry varies as the cube of the thickness. It is evident that a marked superiority in the load sustained might be expected in the low-density core panel over the high-density core panel of the same weight when the load is applied parallel to the grain of the faces.

The same line of reasoning applied to column strength may also be applied to resistance to cupping. A panel with a core of low density will cup less than a panel of the same weight with a core of high density. The load to produce failure in bending would likewise be greater for the former case.

PLYWOOD TEST DATA.

The column-bending modulus is obtained by loading a piece of plywood 5 inches by 12 inches as a column with the 12-inch length vertical. It is computed by the following formula:

$$S=\frac{P}{A}+\frac{6M}{bd^2},\ \text{where}$$

S = Column-bending modulus.
A = Area of cross section.
P = Load at maximum moment.
M = Maximum bending moment.
b = Width of test piece.
d = Thickness of test piece.

Like the modulus of rupture in the standard static bending test, the column-bending modulus is not a true stress existing in the fibers at the instant of failure. It is merely a measure of the magnitude of the external bending moment that a piece of plywood can withstand before it fails.

If a piece of plywood is subjected to forces that tend to bend it, as would be the case either in a long column or in a beam, the designer confronted with the problem of determining its proper thickness may use the column-bending modulus in exactly the same way that the modulus of rupture is used. It will be noted, of course, that the column-bending modulus must be used which applies to the particular plywood construction desired. The total plywood thickness is to be used in all equations involving the column-bending modulus.

The use of the tensile strength data is obvious. The strength values given are based on the total plywood thickness. (Table 9.)

TABLE NO. 9.—*Tensile strength of plywood and veneer.*

Species.	Number of tests.	Moisture at test (per cent).	Specific gravity * of plywood.	Tensile strength † of 3-ply wood parallel to grain of faces (pounds per square inch).	Tensile strength ‡ of single-ply veneer, 1½ (d) (pounds per square inch).
	(a)	(b)	(c)	(d)	(e)
Ash, black	120	9.1	0.49	6,180	9,270
Ash, commercial white	200	10.2	.60	6,510	9,770
Basswood	200	9.2	.42	6,880	10,320
Beech	120	8.6	.67	13,000	19,500
Birch, yellow	200	8.5	.67	13,200	19,800
Cedar, Spanish	115	13.3	.41	5,200	7,800
Cherry [1]	115	9.1	.56	8,460	12,690
Chestnut	40	11.7	.43	4,430	6,645
Cottonwood	120	8.8	.46	7,280	10,920
Cypress, bald	35	10.3	.47	6,560	9,840
Douglas fir	174	8.7	.49	6,230	9,340
Elm, cork	65	9.4	.62	8,440	12,660
Elm, white	160	8.9	.52	5,860	8,790
Gum, black	35	10.6	.54	6,960	10,445
Gum, cotton	80	10.3	.50	6,260	9,390
Gum, red	182	8.7	.54	7,850	11,775
Hackberry	80	10.2	.54	6,920	10,380
Hemlock, western	119	9.7	.47	6,800	10,200
Magnolia [2]	40	9.9	.59	10,000	15,000
Mahogany, African [3]	20	12.7	.52	5,370	8,060
Mahogany, Philippine [4]	25	10.7	.53	10,670	16,010
Mahogany, true	35	11.4	.48	6,390	9,585
Maple, soft [5]	120	8.9	.57	8,180	12,270
Maple, sugar	202	8.0	.68	10,190	15,290
Oak, commercial red	115	9.3	.59	5,480	8,220
Oak, commercial white	195	9.5	.64	6,730	10,095
Pine, white	40	10.2	.43	5,640	8,460
Poplar, yellow	165	9.4	.50	7,390	11,080
Redwood	65	11.2	.41	5,100	7,650
Spruce, Sitka	103	8.4	.43	5,600	8,400
Sycamore	163	9.2	.56	8,030	12,045
Walnut, black	110	9.1	.59	8,250	12,375

* Specific gravity based on oven-dry weight and volume at test.
† Based on total cross-sectional area.
‡ Based on assumption that center ply carries no load.
Data based on tests of 3-ply panels with all plies in any one panel same thickness and species.
[1] Probably black cherry. [2] Probably evergreen magnolia. [3] Probably khaya sp. [4] Probably tanguile. [5] Probably silver maple.

SAMPLE COMPUTATION.

To obtain the tensile strength of 3-ply wood consisting of two $\frac{1}{20}$-inch birch faces and a $\frac{1}{16}$-inch basswood core.

Parallel to face grain $=2\times\frac{1}{20}\times 19,860=1,986$ pounds per inch of width.

Perpendicular to face grain $=1\times\frac{1}{16}\times 9,450=591$ pounds per inch of width.

This computation neglects the tensile strength of the ply or plies perpendicular to the grain, which is comparatively small. The results are therefore slightly in error.

The resistance to splitting is of considerable importance in panels when these are to be fastened with screws or bolts and are subject to forces at the fastenings. The numerical value of the work required to split a panel of a given thickness has no direct application in design. It is only in comparison with other panels of other species or construction that work in splitting has any significance. The work done is, of course, a measure of resistance to splitting. It is not entirely a property of the wood, as it depends very largely upon the strength of the glue.

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TABLE 10.—*Strength of various species of 3-ply panels.*

All plies in any one panel of the same thickness and of the same species; grain of successive plies at right angles. All material rotary cut. Perkins glue used throughout. Eight thicknesses of plywood, ranging from $\frac{3}{30}$ inch to $\frac{3}{8}$ inch, were tested.

Species.	Average specific gravity of plywood based on oven-dry weight and volume at test.	Average per cent moisture.	Column-bending modulus.				Tensile strength.				Splitting resistance.		Modulus of elasticity.	
			Parallel.*		Perpendicular.*		Parallel.*		Perpendicular.*					
			Number of tests.	Pounds per square inch.	Number of tests.	Pounds per square inch.	Number of tests.	Pounds per square inch.	Number of tests.	Pounds per square inch.	Number of tests.	Per cent of birch.†	Parallel (1,000 pounds per square inch).*	Perpendicular (1,000 pounds per square inch.
Ash, black	0.49	9.1	120	7,760	120	1,779	120	6,180	120	3,940	240	73	1,073	96
Ash, commercial white	.60	10.2	200	9,930	200	2,620	200	6,510	200	4,350	400	71	1,420	143
Basswood	.42	9.2	200	7,120	200	1,670	200	6,880	200	4,300	400	63	1,213	85
Beech	.67	8.6	120	15,390	120	2,950	120	13,000	120	7,290	240	94	2,149	167
Birch, yellow	.67	8.5	195	16,000	200	3,200	200	13,200	200	7,700	400	100	2,259	197
Cedar, Spanish	.41	13.3	115	6,460	115	1,480	115	5,200	115	3,340	230	60	1,032	84
Cherry [a]	.56	9.1	115	12,260	115	2,620	115	8,460	115	5,920	230	80	1,627	152
Chestnut	.43	11.7	40	5,160	40	1,110	40	4,430	40	2,600	80	74	744	75
Cottonwood	.46	8.8	120	8,460	120	1,870	120	7,280	120	4,240	240	85	1,437	109
Cypress, bald	.47	10.3	35	7,830	35	1,820	35	6,560	35	4,390	70	69	1,144	91
Douglas fir	.49	8.7	150	9,460	174	1,950	174	6,230	174	4,000	348	84	1,566	129
Elm, cork	.62	9.4	65	12,710	65	2,500	65	8,440	65	5,500	130	99	1,982	136
Elm, white	.52	8.9	160	8,680	160	1,970	160	5,860	160	3,990	320	75	1,224	109
Gum, black	.54	10.6	40	8,090	40	1,920	35	6,960	35	4,320	70	55	1,275	113
Gum, cotton	.50	10.3	80	7,760	80	1,580	80	6,260	80	3,760	160	60	1,300	111
Gum, red	.54	8.7	182	9,970	182	2,070	182	7,850	182	4,930	364	80	1,592	120
Hackberry	.54	10.2	80	8,100	80	1,880	80	6,920	80	4,020	160	84	1,154	99
Hemlock, western	.47	9.7	119	9,250	119	1,960	119	6,800	119	4,580	238	63	1,581	112
Magnolia [b]	.59	9.9	40	9,830	40	2,340	40	10,000	40	5,740	80	98	1,704	135
Mahogany, African [c]	.52	12.7	20	8,070	20	2,000	20	5,370	20	3,770	......	......	1,261	144
Mahogany, Philippine [d]	.53	10.7	25	10,160	25	2,310	25	10,670	25	5,990	50	90	1,820	169
Mahogany, true	.48	11.4	35	8,500	35	1,940	35	6,390	35	3,780	......	......	1,252	117
Maple, soft [e]	.57	8.9	120	11,540	120	2,420	120	8,180	120	5,380	240	106	1,752	145
Maple, sugar	.68	8.0	202	15,600	202	3,340	192	10,190	202	6,530	404	114	2,112	189
Oak, commercial red	.59	9.3	115	8,500	115	2,070	115	5,480	115	3,610	230	70	1,289	120
Oak, commercial white	.64	9.5	195	10,490	195	2,310	195	6,730	195	4,200	390	85	1,343	118
Pine, white	.43	10.2	35	7,920	40	1,770	40	5,640	40	3,870	80	52	1,274	99
Poplar, yellow	.50	9.4	165	8,860	165	1,920	165	7,390	165	4,720	330	51	1,544	115
Redwood	.41	11.2	65	7,900	65	1,500	65	5,100	65	3,000	130	48	1,211	118
Spruce, Sitka	.43	8.4	103	7,640	103	1,689	103	5,600	103	3,250	206	82	1,395	105
Sycamore	.56	9.2	163	11,040	163	2,340	163	8,030	163	5,220	326	77	1,628	130
Walnut, black	.59	9.1	110	12,660	110	2,770	110	8,250	110	5,260	220	77	1,736	141

* Parallel and perpendicular refer to the direction of the grain of the faces relative to the direction of the application of the force.
† The relative splitting resistance of the various panels tested depends largely on the holding strength of the glue.
a Probably black cherry.
b Probably evergreen magnolia.
c Probably khaya sp.
d Probably tanguile.
e Probably silver maple.

The results of strength tests on plywood of various common veneer species are given in table 10. Except for birch all tests are on only one shipment of the species, so that the results will in all probability be changed somewhat by the addition of future test data. The mahogany results are on thin plywood ranging in thickness from $\frac{3}{30}$ inch to $\frac{3}{16}$ inch, while the sizes of the plywood for all other species ranged from $\frac{3}{30}$ inch to $\frac{3}{6}$ inch.

In most cases it was found that the column-bending modulus of thin plywood was slightly less than the column-bending modulus of the thick plywood.

TABLE 11.—*Comparison of strength of 3, 5, and 7 ply yellow birch plywood, all plies of same thickness in any one panel.*

Number of plies.	Average specific gravity.*	Average per cent moisture.	Number of tests.	Column-bending modulus, in pounds per square inch.		Tension, in pounds per square inch.		Average splitting resistance compared to 3-ply birch, for the same plywood thickness, in per cent of 3-ply.
				Parallel.†	Perpendicular.†	Parallel.†	Perpendicular.†	
3	0.67	8.5	195	16,000	3,200	13,200	7,700	100
5	.67	6.6	25	14,700	6,800	13,100	8,600	129
7	.70	7.1	25	14,300	7,900	12,900	9,300	191

* Specific gravity, based on oven-dry weight and volume at test.
† Parallel and perpendicular refer to direction of grain of faces relative to direction of application of force.

Table 11 shows the decrease in the unit strength of plywood in the direction of the grain of the faces when the number of plies is increased, and the increase in the unit strength of plywood perpendicular to the grain of the faces when the number of plies is increased.

TABLE 12.—*Comparison of strength of three-ply wood having a core of high density with similar plywood having a core of low density of the same thickness; each ply $\frac{1}{20}$ inch thick.*

Number of tests very limited. Results tabulated will probably be changed by further tests.

Species.			Number of tests.	Plywood thickness	Per cent moisture at test.	Specific gravity, based on oven-dry weight and volume at test.	Column-bending modulus in pounds per square inch.		Tension in pounds per square inch.		Maximum unit load in pounds per square inch, 5 by 12 inch specimen tested as a column.	
Face.	Core.	Face.					Parallel.*	Perpendicular.*	Parallel.*	Perpendicular.*	Parallel.*	Perpendicular.*
				Inches.								
Birch	Birch	Birch	30	0.15	9.4	0.68	14,200	3,170	11,900	7,290	258	21
Do	Basswood	...do	10	.14	8.2	.61	15,200	1,600	12,900	3,800	250	12
Sugar maple	Sugar maple	Sugar maple	33	.15	6.9	.69	16,100	3,210	9,910	6,540	265	45
Do	Basswood	...do	5	.15	7.0	.62	17,700	2,600	12,000	3,700	247	15
Red gum	Red gum	Red gum	20	.14	9.5	.55	9,550	2,060	8,410	4,720	193	35
Do	Basswood	...do	5	.15	8.3	.44	7,200	1,400	4,900	3,000	115	11
Do	Yellow poplar	...do	5	.14	6.5	.51	10,100	....	6,200	4,500	149	17

* Directions refer to direction of application of the force relative to the grain of the faces.

Table 12 shows that the strength values of plywood parallel to the grain of the faces are practically the same for three-ply wood having a core of dense wood as for plywood having a core of light wood. The strength values across the grain of the faces are, however, very much less for the plywood with core of low density. In other words, the strength values of three-ply wood parallel to the grain of the faces are almost entirely determined by the strength values of the face material, and the strength values across the grain of the faces are very largely determined by the strength values of the core species.

Table 13 gives a number of factors that are of value in selecting the thickness and species of the plies for a three-ply panel.

TABLE 13.—*Thickness factors for veneer.*

Giving: (1) Veneer thickness for the same total bending strength as birch; (2) veneer thickness for the same weight as birch.

Species.	D. Average specific gravity of species * based on oven-dry weight and air-dry volume.	Specific gravity of glued plywood as tested.	Per cent moisture of plywood as tested.	S. Per cent unit bending strength compared with birch.†	K_s. Thickness factor for the same total bending strength as birch, $\sqrt{\frac{100}{S}}$.	K_w. Thickness factor for the same weight as birch, $\frac{0.63}{D}$.
Ash, black	0.50	0.49	9.1	52	1.39	1.26
Ash, commercial white	.58	.60	10.2	72	1.18	1.09
Basswood	.38	.42	9.2	48	1.44	1.66
Beech	.63	.67	8.6	94	1.03	1.00
Birch, yellow	.63	.67	8.5	100	1.00	1.00
Cedar, Spanish	[a].34	.41	13.3	43	1.52	1.85
Cherry [b]	.51	.56	9.1	80	1.12	1.24
Chestnut	.44	.43	11.7	34	1.72	1.43
Cottonwood	.43	.46	8.8	56	1.34	1.47
Cypress, bald	.44	.47	10.3	53	1.37	1.43
Elm, cork	.66	.62	9.4	78	1.13	.95
Elm, white	.51	.52	8.9	58	1.31	1.24
Fir, Douglas	[c].51	.49	8.7	60	1.29	1.24
Gum, black	.52	.54	10.6	56	1.34	1.21
Gum, cotton	.52	.50	10.3	48	1.44	1.21
Gum, red	.49	.54	8.7	64	1.25	1.29
Hackberry	.54	.54	10.2	55	1.35	1.17
Hemlock, western	.42	.47	9.7	60	1.29	1.50
Magnolia	.51	.58	9.9	67	1.22	1.24
Mahogany, African	[a].46	.52	12.7	56	1.34	1.37
Mahogany, Philippine [d]	[a].57	.53	10.7	68	1.21	1.10
Mahogany, true	[a].49	.48	11.4	57	1.32	1.29
Maple, soft [e]	.48	.57	8.9	74	1.16	1.31
Maple, sugar	.62	.68	8.0	100	1.00	1.02
Oak, commercial red	.63	.59	9.3	59	1.30	1.00
Oak, commercial white	.69	.64	9.5	69	1.20	.91
Pine, white	.39	.43	10.2	52	1.38	1.61
Poplar, yellow	.41	.50	9.4	58	1.31	1.54
Redwood	[a].36	.41	11.2	49	1.43	1.75
Sycamore	.50	.56	9.2	71	1.09	1.26
Spruce, Sitka	.38	.43	8.4	50	1.41	1.66
Walnut, black	.57	.59	9.1	83	1.10	1.10

* Taken from Bulletin 556 of the U. S. Department of Agriculture.
† Average of the column-bending moduli parallel and perpendicular to grain compared to birch.
[a] Based on subsequent tests.
[b] Probably black cherry.
[c] Coast type Douglas fir.
[d] Probably tanguile.
[e] Probably silver maple.

The thickness factor (K_s) is used to obtain the thickness of a ply of any species having the same total bending strength as a given ply of birch. It is arrived at as follows:

The strength of any structural member is determined either by the direct load it can sustain or the bending moment it can resist without failure. In plywood the latter factor is the better criterion of strength. If we denote the maximum bending moment of a strip of three-ply wood 1 inch wide and of thickness d_1 by M_1 and the stress at failure by S_1 (column-bending modulus), then $M_1=\frac{S_1 d_1^2}{6}$.

Similarly, the strength of another strip of a different species will be denoted by M_2, its stress at failure S_2, and thickness d_2. By a proper selection of thickness d_2 the second strip may be made to withstand the same maximum bending moment, so that $M_2=M_1$ or $S_2 d_2^2=S_1 d_1^2$. From this the desired thickness $d_2=d_1\sqrt{\frac{S_1}{S_2}}$. Taking d_1 as the unit of thickness of a birch plywood strip and expressing the maximum stresses in percentage of birch, we have $d_2=\sqrt{\frac{100}{S_2}}$, or, in general, $K_s=\sqrt{\frac{100}{S}}$, where K_s is the thickness of the plywood, whose column-bending modulus corresponds to S and whose total bending strength, given by the bending moment, is the same as that of birch plywood of thickness unity.

The same reasoning also applies to single plies, so that K_s may be used to get the thickness of a single ply, which will give the same total bending strength as a birch ply of thickness unity. For example, for yellow poplar $K_s=1.46$, and a ply of this species, $1.46\times\frac{1}{16}=0.091$ inch, is equivalent in strength in bending to a birch ply $\frac{1}{16}$ inch thick.

By way of explanation it must be understood that unit bending strength refers to a maximum stress such as the modulus of rupture, or the column-bending modulus, while total bending strength refers to the load or bending moment a beam can sustain or the bending moment a column can sustain.

It should be kept in mind that these factors will doubtless be modified somewhat by further tests.

The thickness factor (K_w) is used to obtain the thickness of a ply of any species equal in weight to a ply of yellow birch of given thickness. It is obtained by simply dividing the density of birch by the density of the species for which the thickness is desired. The density data used in computing K_w are the same as that given in United States Department of Agriculture Bulletin 556, "Mechanical Properties of Woods Grown in the United States." The weight of the glue in the plywood is neglected.

For yellow poplar, for example, the thickness of a ply equal in weight to a $\frac{1}{16}$-inch ply of birch is $1.54\times\frac{1}{16}=0.096$ inches.

The column-bending tests, upon which the data in table 10 are based, were all made on specimens of the same lengths, and it was felt desirable to determine what effect, if any, the change in length of the column might have upon the maximum unit load, the slenderness ratio remaining constant. Special panels of three-ply birch, all plies of the same thickness in each panel, were made up from veneer of the following thicknesses: $\frac{1}{30}$, $\frac{1}{24}$, $\frac{1}{20}$, $\frac{1}{16}$, $\frac{1}{10}$, $\frac{1}{8}$, and test columns varying in length from 20 inches to 6 inches were cut from them and tested. The conclusion drawn from these tests is that for a given slenderness ratio the length of the column has little, if any, effect on the maximum unit load which a three-ply birch column will sustain. It is assumed that the same conclusion will apply to panels of other species.

Table 9, to which reference has already been made, presents data by which it is possible to calculate the strength in tension of plywood composed of various kinds of veneer. Column (*d*) of this table is identical with the corresponding column in table 10. Column (*e*) is to be used in calculating the strength in tension of plywood made up of different species. The method of calculation is based upon the fact that the tensile strength of wood in a direction perpendicular to the grain is very small in comparison with that parallel to the grain and may, therefore, for purposes of approximation, be neglected. To obtain the tensile strength in any direction, simply add together the tensile strength, parallel to the grain, of the individual plies the grain of which lies parallel to the direction in which the strength is desired. The sample computation will make this entirely clear.

The shearing strength of plywood is of importance in connection with the design of box beams having plywood cheek pieces and for similar construction. Several series of tests are under way to determine the shearing strength of plywood of various thicknesses when unsupported for various distances. While these tests are not as yet completed, it is evident that it will not be possible to use a shearing strength in calculating these members much greater than that of solid wood of the same species. There is much more residual strength in plywood after the first failure than in solid wood, and for this reason a somewhat higher working stress would be justified. Until more data are available the shear allowed in plywood should not be over 25 per cent greater than that allowed in solid wood of the same species. This assumes that in the cheeks of horizontal beams the face plies will be vertical, a condition dictated by experience to be best practice.

RIVETED JOINTS IN PLYWOOD.

The matter of joints in plywood is of the greatest importance in connection with the construction of various types of built-up structures such as fuselages, boat hulls, pontoons,

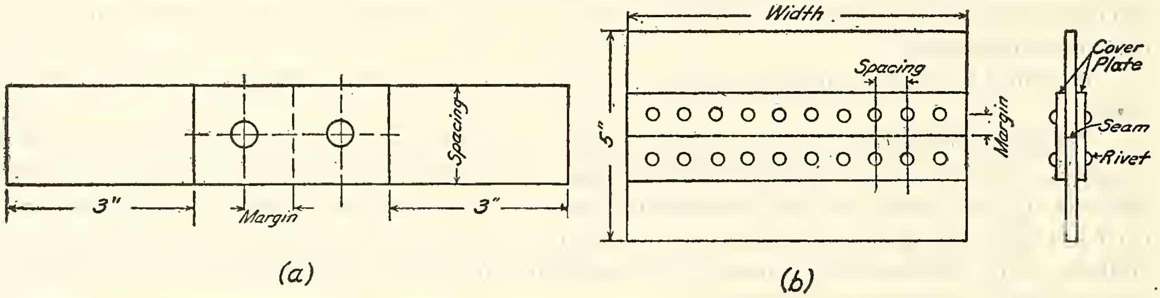

Fig. 30.—(a) Test specimen for single-rivet tests. (b) Test specimen for multiple-rivet tests.

and beams and girders. Several series of tests have been made to determine the efficiency of various types of joint for different kinds of loading.

The first series of tests was made upon riveted joints designed for tension and compression. The tests were all made in tension; both solid and hollow rivets were used. Two types of test were run; most of the tests were made on specimens only wide enough to accommodate one rivet (fig. 30a), and later enough wide specimens were tested (fig. 30b) to verify the assumption that the data on the narrow specimens could be applied without correction to wider ones.

In general, most of the tests were made on butt joints, with straps on each side. In some cases the straps were of plywood and in others of galvanized sheet metal about 0.02 inches thick. The nomenclature used will become clear upon examination of figure 30.

The first tests were made upon red gum plywood composed of three plies of $\frac{1}{16}$ material, riveted with solid copper rivets through sheet-metal cover plates. The grain of the face plies was perpendicular to the seam. Figure 31 shows the strength of the joint with varying margins and spacing. It is apparent that the best conditions are obtained with a 1-inch margin and a one-half inch spacing.

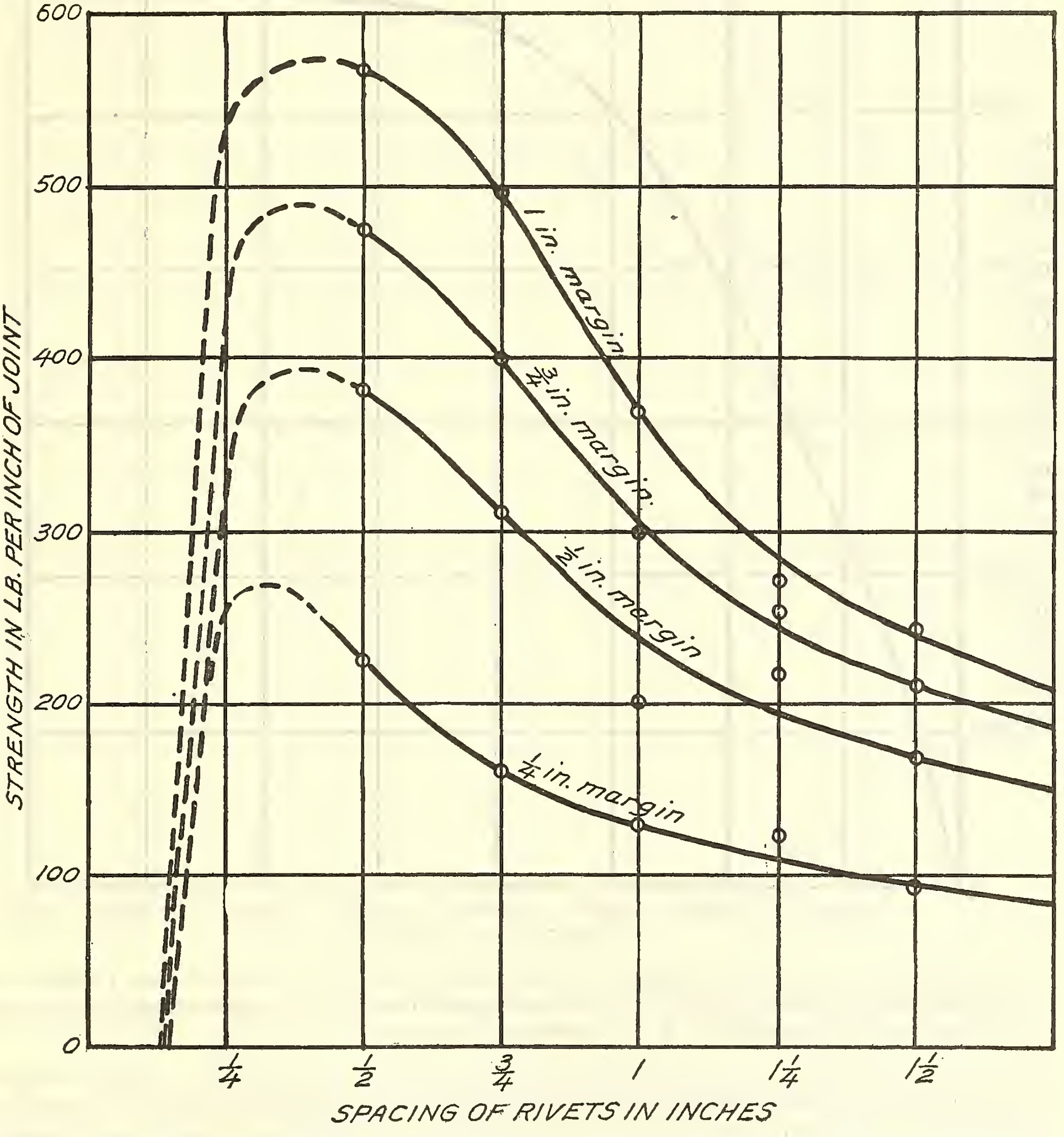

Fig. 31.—Single-riveted butt joints in plywood. Relations among strength, margin, and spacing: Red gum plywood, plies 1/16 by 1/16 by 1/16 inch; solid copper rivets, 0.15 inch diameter; sheet-metal cover plates; grain of faces perpendicular to seam; moisture, 7.4 per cent.

Figure 32 shows the variation of strength when using a constant spacing of one-half inch and margins varying from one-quarter inch to 2 inches. This figure shows very clearly that no appreciable additional strength can be obtained by increasing the margin above 1 inch.

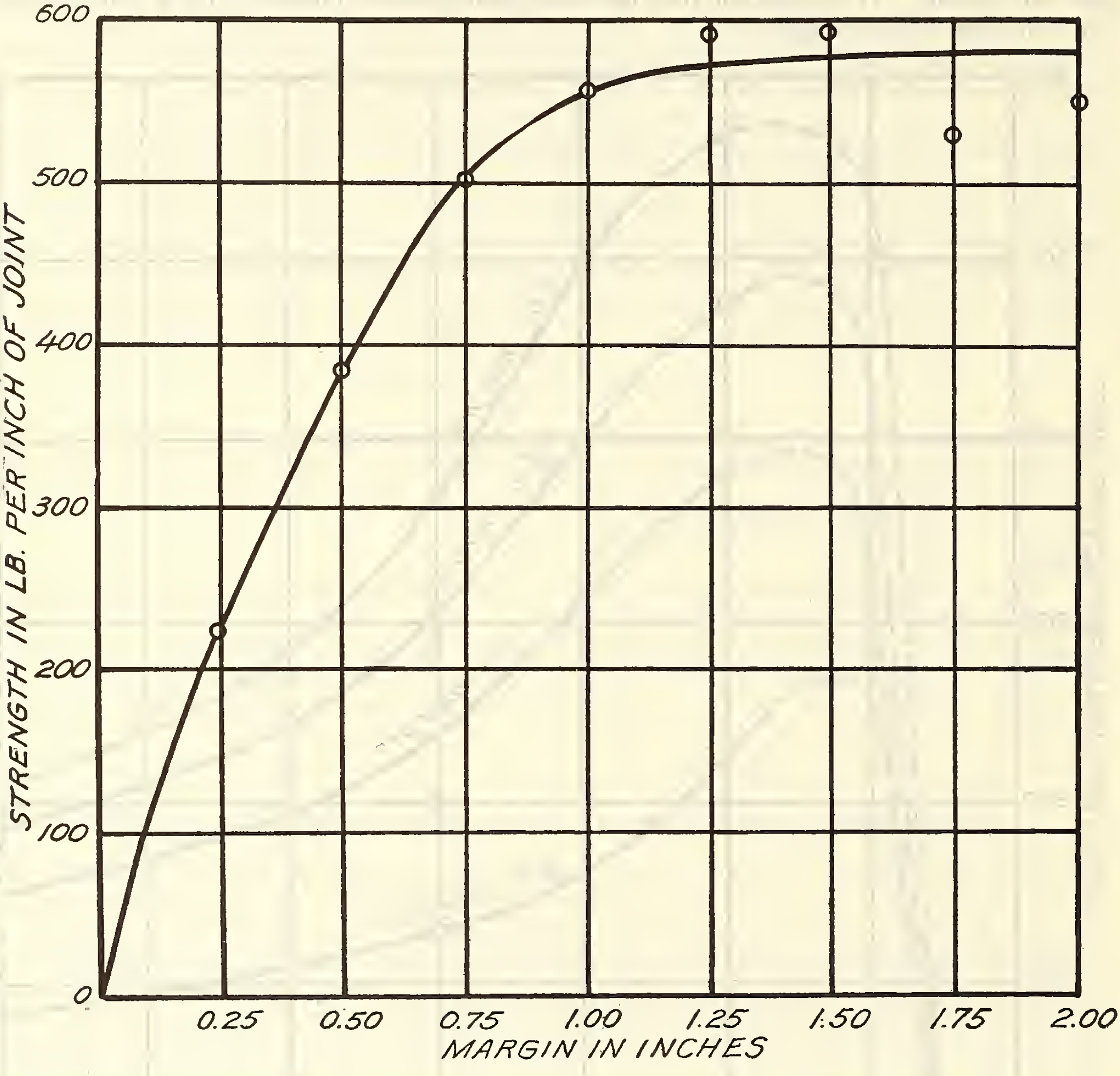

Fig. 32.—Single-riveted butt joints in plywood. Relation between strength and margin: Spacing 1/2 inch; red gum plywood, plies 1/16 by 1/16 by 1/16 inch; solid copper rivets, 0.15 inch diameter; sheet-metal cover plates; grain on faces perpendicular to seam; moisture, 7.4 per cent.

In fact, it was found that in case the grain of the face plies was parallel to the seam, the margin could be reduced to three-quarters inch without sacrificing an appreciable amount of strength.

Similar tests made on three-ply birch, each ply one-sixteenth inch, gave similar results, as shown in figures 33 and 34. With a margin of $1\frac{1}{2}$ inches, the maximum strength was secured with a spacing of one-half inch.

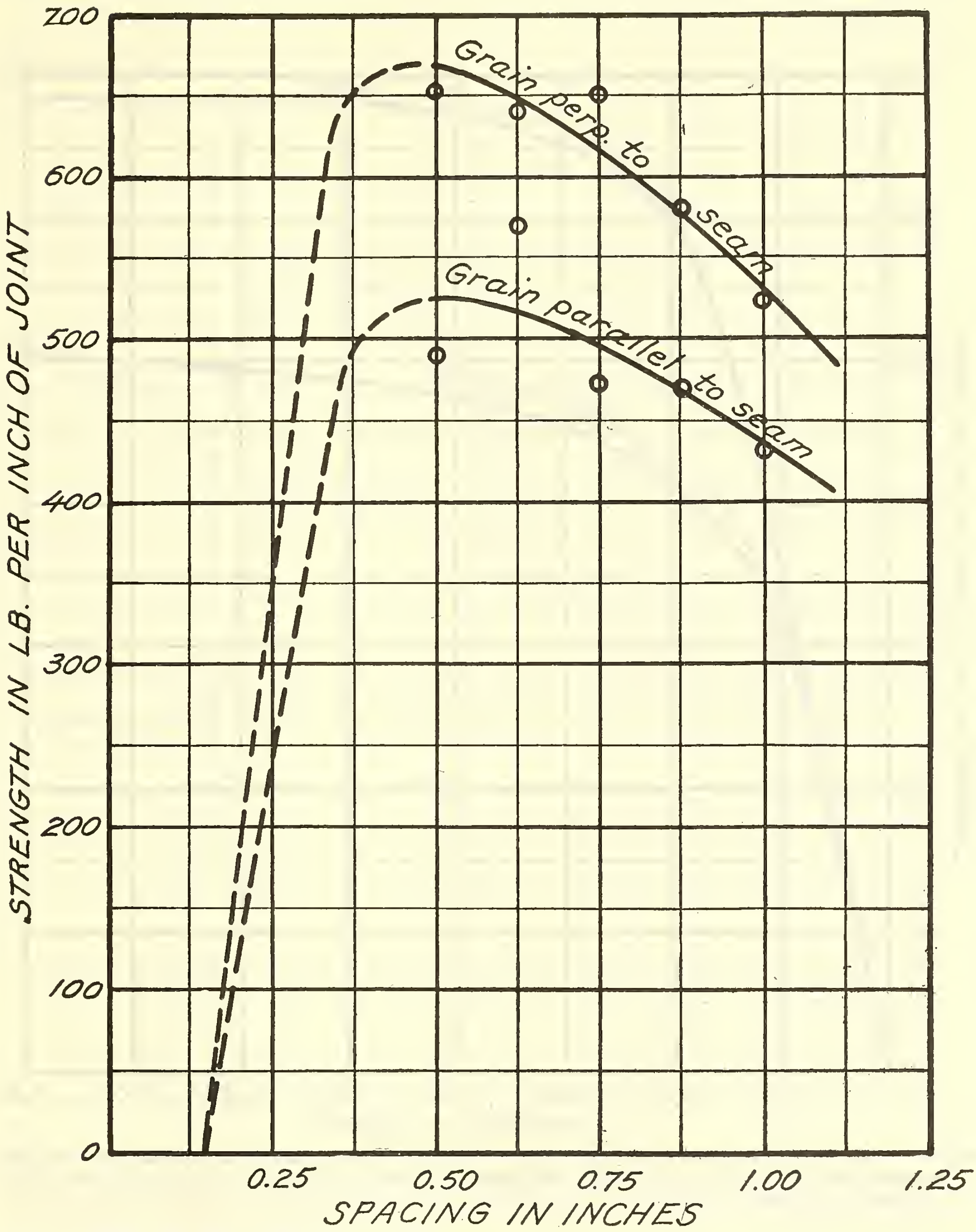

Fig. 33.—Single-riveted butt joints in plywood. Relation between strength and spacing: Margin, 1 1/2 inches; birch plywood, plies 1/16 by 1/16 by 1/16 inch; solid copper rivets, 0.15 inch diameter; sheet-metal cover plates; moisture, 6.6 per cent.

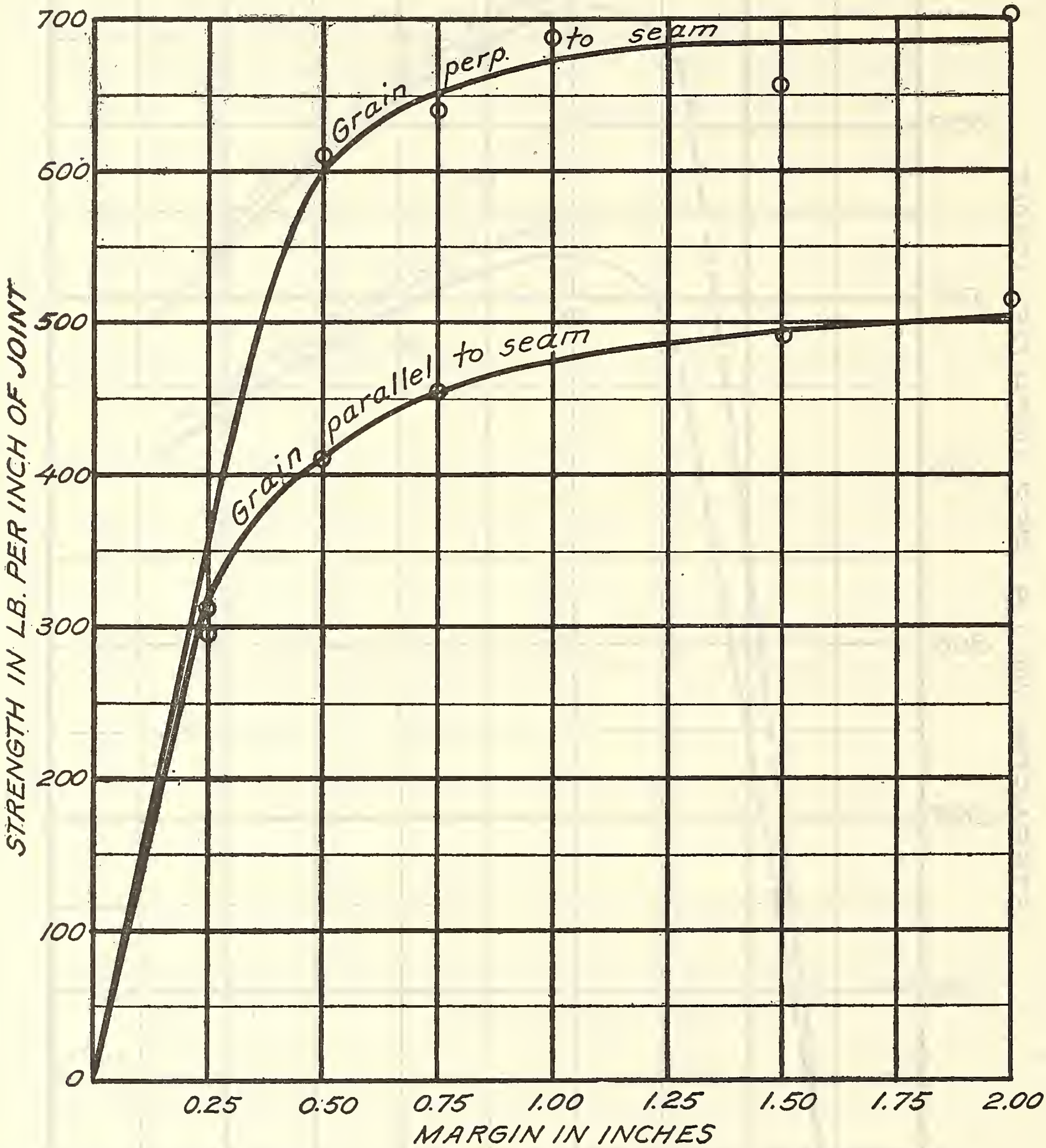

Fig. 34.—Single-riveted butt joints in plywood. Relation between strength and margin; spacing, 1/2 inch; birch plywood, plies 1/16 by 1/16 by 1/16 inch; solid copper rivets, 0.15 inch diameter; sheet-metal cover plates; moisture, 6.6 per cent.

The margin could have been reduced to 1 inch or even less without a great falling off in efficiency. Figure 35 indicates that a spacing of one-half inch is the best with thinner birch (each ply $\frac{1}{20}$ inch).

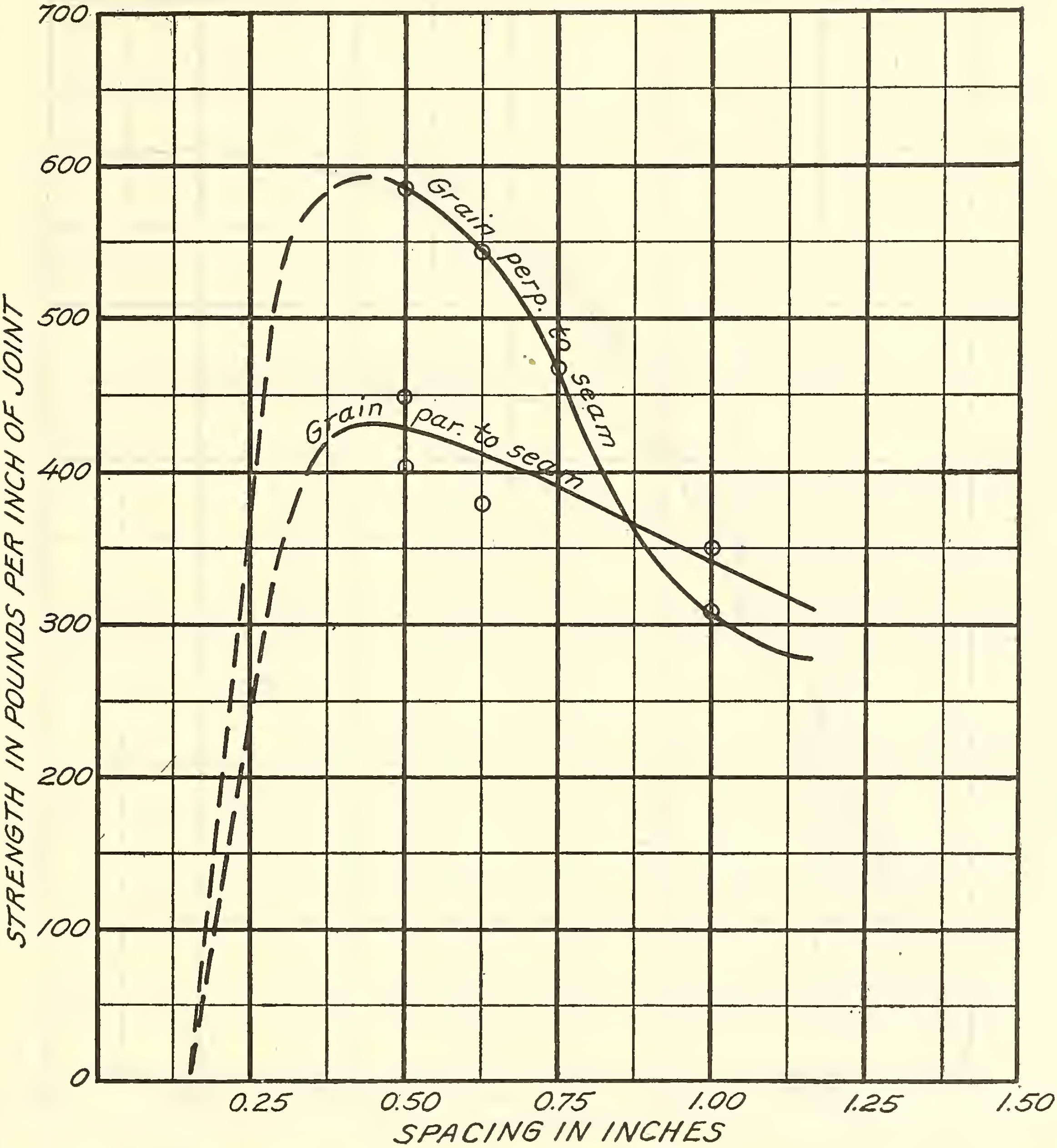

Fig. 35.—Multiple-riveted butt joints in plywood; relation between strength and spacing; test joint, 5 to 5 1/2 inches wide; margin, 1 inch; birch plywood, plies 1/20 by 1/20 by 1/20 inch; solid copper rivets, 0.15 inch diameter; sheet-metal cover plates; moisture, 5.6 per cent.

Figures 36 and 37 show the strength of joints made in three-ply birch (each ply one-twentieth of an inch) with five-eighths-inch hollow aluminum rivets and plywood cover plates. A spacing of 1¼ inches gave the best efficiency with a margin of 2 inches. It is possible that

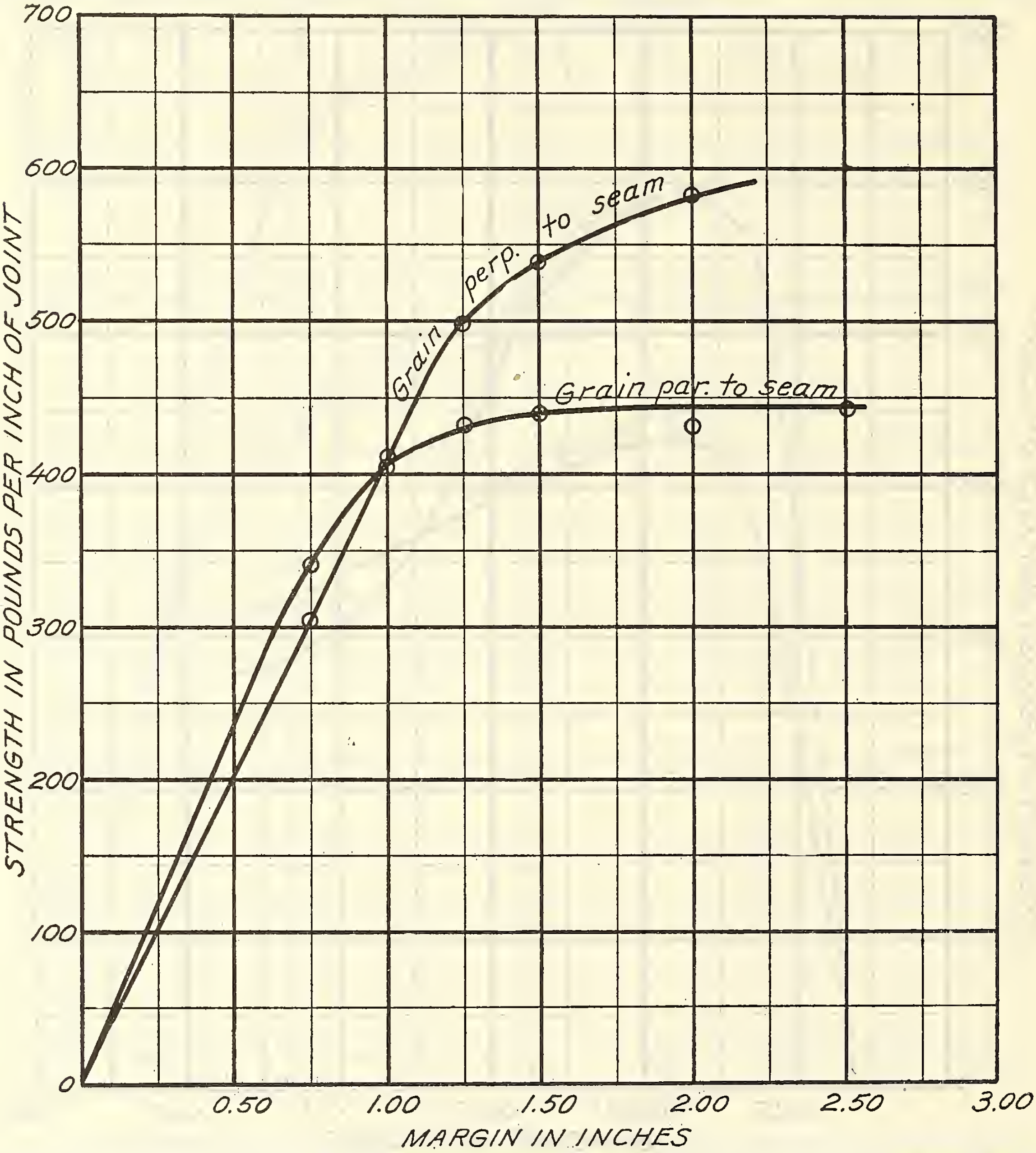

Fig. 36.—Single-riveted butt joints in plywood; relation between strength and margin; spacing, 1.25 inches; birch plywood, plies 1/20 by 1/20 by 1/20 inch; hollow aluminum rivets, 5/8 inch outside diameter; plywood cover plates; moisture, 5.6 per cent.

greater strength could have been secured in the case of the specimens with the grain of the faces perpendicular to the seam had a greater margin than 2 inches been used. In the case of the specimens with the grain of the faces parallel to the seam a margin of 1¼ inches could have been used without any great reduction in strength.

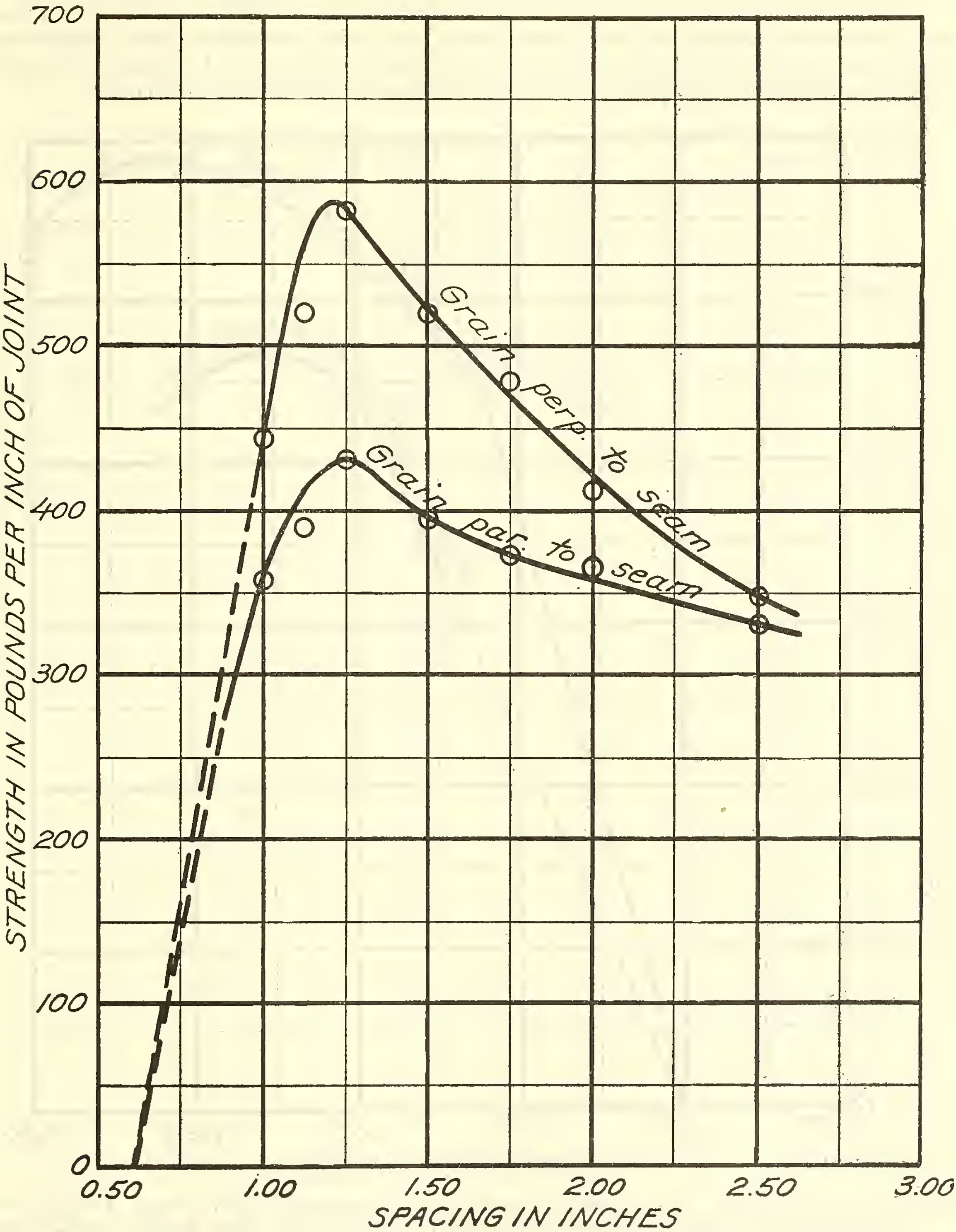

Fig. 37.—Single-riveted butt joints in plywood; relation between strength and spacing; margin, 2 inches; birch plywood, plies 1/20 by 1/20 by 1/20 inch; hollow aluminum rivets, 5/8 inch outside diameter; plywood cover plates; moisture, 5.6 per cent.

The results of tests upon three-ply birch (each ply one-twentieth inch) with plywood cover plates and one-half inch and three-eighths inch hollow aluminum rivets, respectively, are plotted in figures 38 and 39. These tests were made with margins of 2 inches. However, smaller margins could no doubt have been used without appreciable loss in strength.

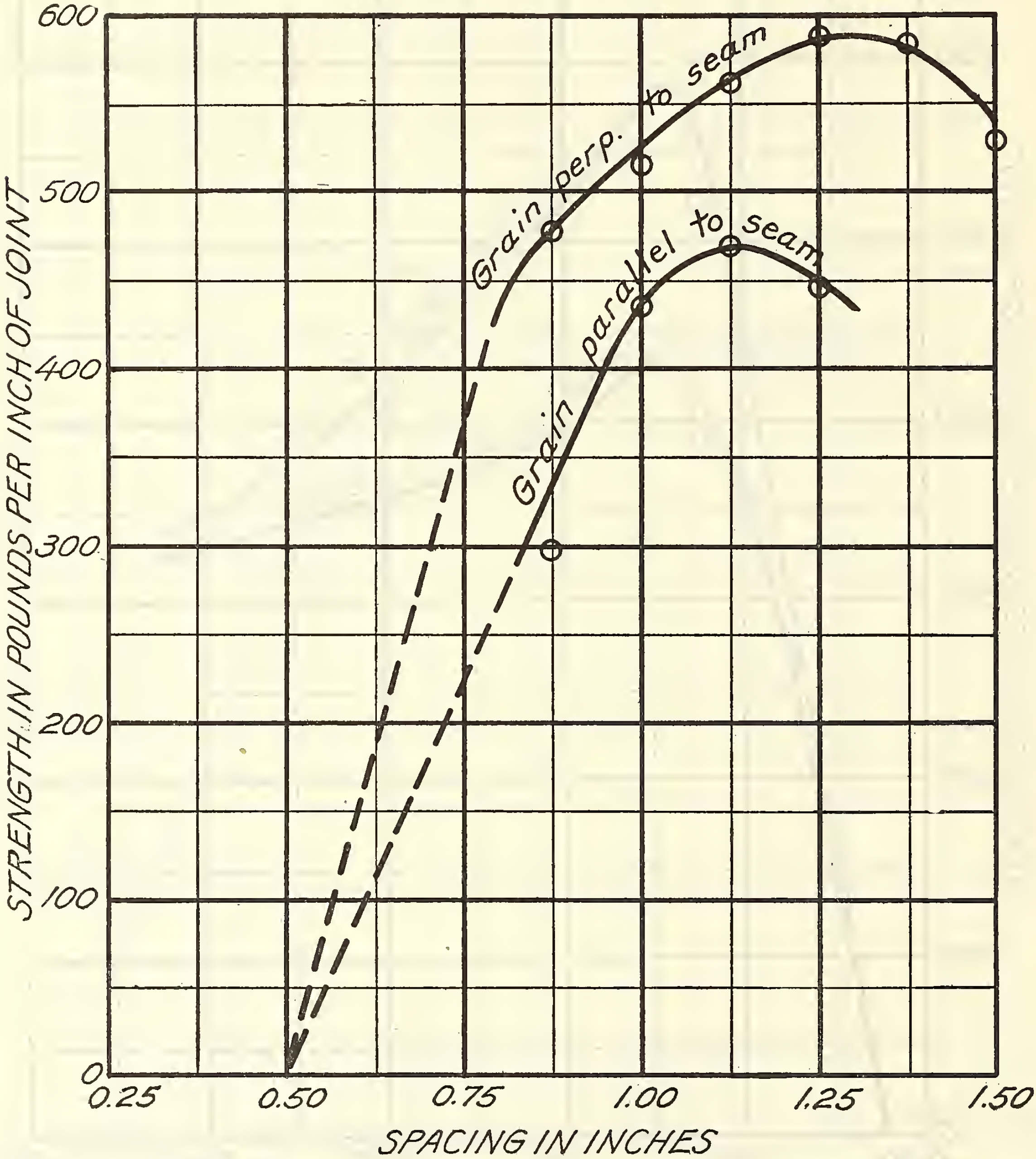

Fig. 38.—Single-riveted butt joints in plywood; relation between strength and spacing; margin, 2 inches; birch plywood, plies 1/20 by 1/20 by 1/20 inch; hollow aluminum rivets, 1/2 inch outside diameter; plywood cover plates; moisture, 5.6 per cent.

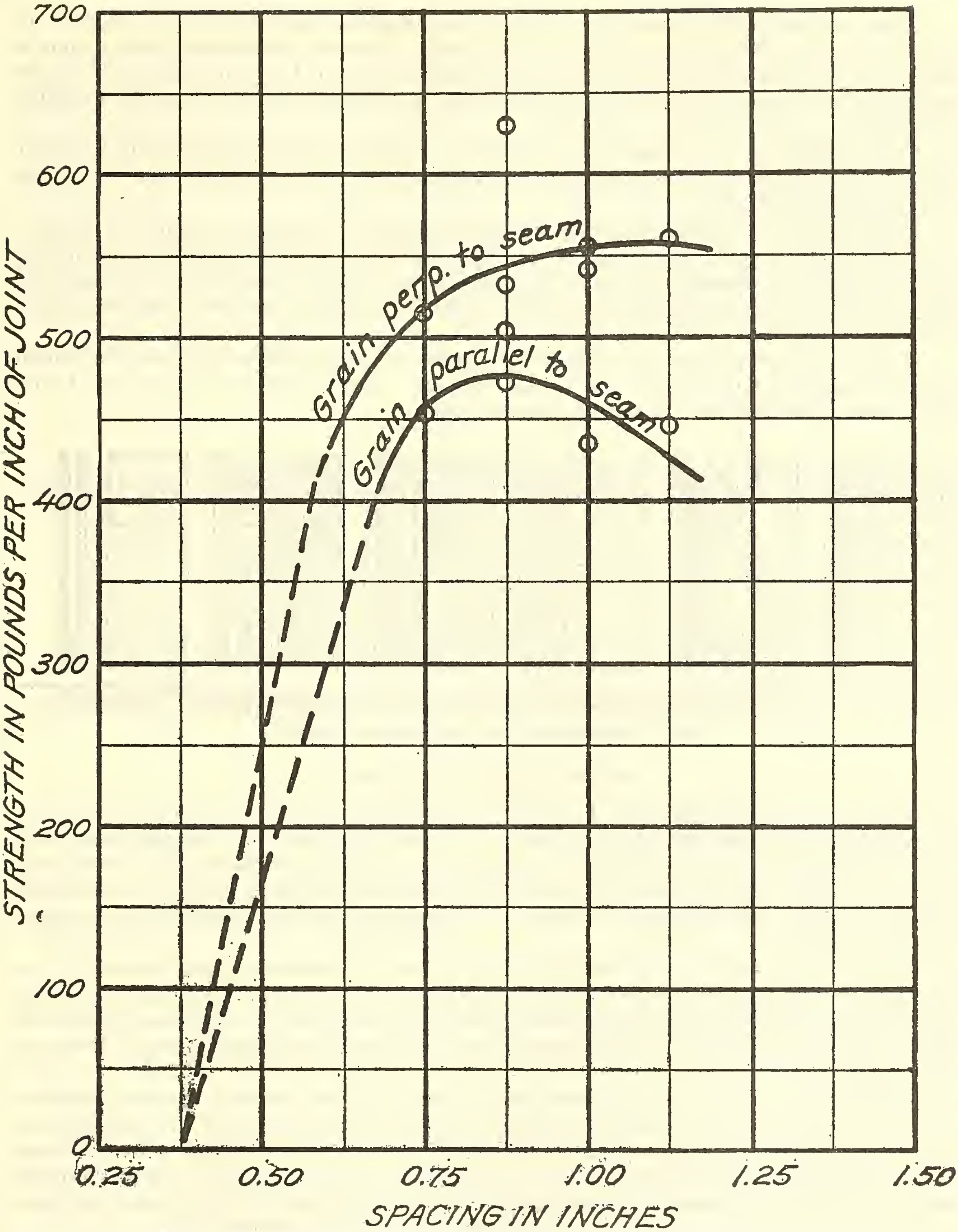

Fig. 39.—Single-riveted butt joints in plywood; relation between strength and spacing; margin, 2 inches; birch plywood, plies 1/20 by 1/20 by 1/20 inch; hollow aluminum rivets, 3/8 inch outside diameter; plywood cover plates; moisture, 5.6 per cent.

When the most efficient spacing and margin are used, there is practically no difference in strength for the different sizes of rivets investigated. However, the smaller rivets require a smaller spacing and therefore more labor in manufacture. On the other hand, the margin required is less than in the case of the larger rivets, and this may in some cases be a decided advantage.

Cover plates may be of metal or plywood, as preferred. If of metal, aluminum sheet about three-sixty-fourths inch or one-sixteenth inch thick is recommended for the thicknesses of plywood investigated.

The efficiency of the joints was determined by testing a number of samples of the plywood, both parallel and perpendicular to the face plies, and it was determined that under the best conditions the efficiency of the joints with the face plies perpendicular to the seam was about 30 per cent, while with the face plies parallel to the seam the maximum efficiency was a little over 50 per cent.

While riveted joints may be satisfactory under certain circumstances, they can not be used where an efficiency much over 50 per cent is required. In these cases it is necessary to use glued joints, of which there are several different types.

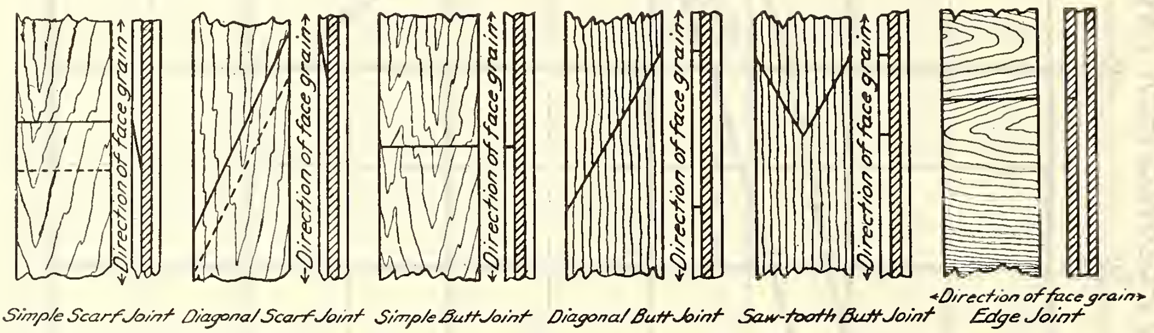

Fig. 40.—Joints in the face veneer of three-ply wood.

JOINTS IN INDIVIDUAL PLIES.

Joints in individual plies may be made in a variety of ways. Figure 40 shows several possible methods for joining pieces of veneer. A considerable number of strength tests upon several of these joints have been made. The simple scarf joint has been tested for a long range of slopes of scarf. The diagonal scarf joint, as well as the diogonal butt joint, have been tested for various slopes of the diagonal. The saw-tooth butt joint has been tested for various angles of the saw tooth.

In balancing up the various factors of strength, ease of manufacture, and efficiency it was decided that the simple scarf joint is the most desirable of the group. The simple butt joint should not be used where strength is important. The edge joint is satisfactory if carefully made. The slope of the scarf in the simple scarf joint should be within the range of from 1 to 20 to 1 to 30.

In comparison with the use of rivets, joints in individual plies are probably more practical. They have an advantage, too, in that the joints in the plies of a given panel may be staggered, so that any defect that may occur in any particular joint only partially weakens the entire panel. The time and labor involved in the preparation of this type of joint, while probably less than the time and labor involved in the preparation of riveted joints, is greater than that in preparing the scarf joint extending through the entire thickness of the panel.

JOINTS EXTENDING THROUGH THE ENTIRE THICKNESS OF PLYWOOD.

Many tests have been made upon scarf joints extending through the entire thickness of a panel. Such joints were prepared by various manufacturers using different glues, different combinations of veneer thicknesses and species, and various slopes of scarf. Two types of scarf joints extending through the entire plywood thickness have been tested and are here described as the straight scarf joint and the Albatros scarf joint. The two types are shown in figure 41. The tests indicate quite conclusively that the straight scarf joint is the superior joint of the two. An examination of the Albatros joint will show that the face ply of the one panel does not meet the face ply of the second panel or only partially meets it. In place of being glued to wood that has the grain running in the same direction, the face ply of one panel is glued to the core of the second panel, in which the grain runs at right angles to the grain in the face. Joints in which the grain of the two pieces joined is at right angles are not as strong as joints in which the grain of the two pieces is parallel.

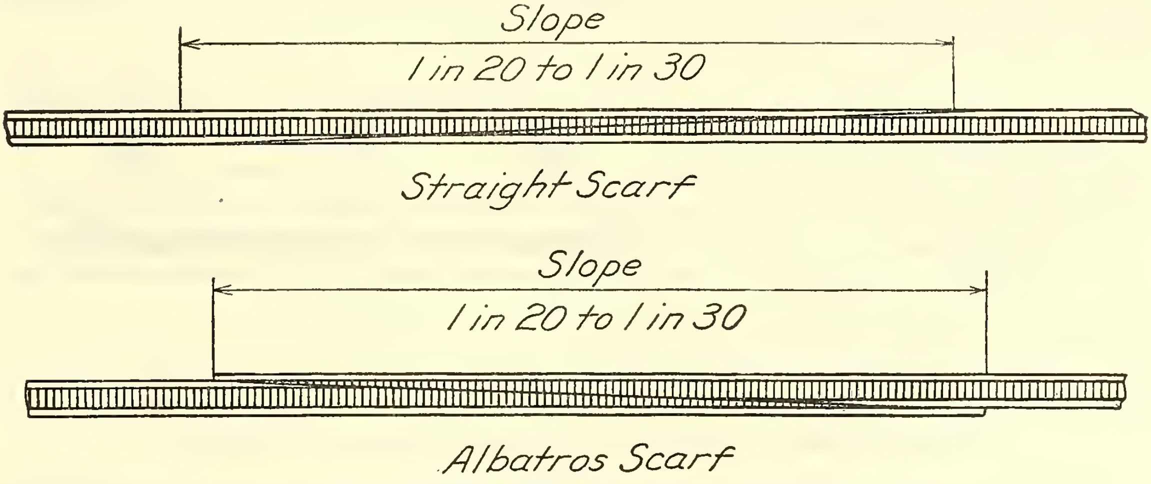

Fig. 41.—Joints in plywood extending through the entire thickness.

Tension tests on the straight scarf joint show that an efficiency of over 90 per cent may be obtained with this type of joint for a slope of scarf as low as 1 in 10. On account of the variations in the effectiveness of the gluing by different manufacturers, it is recommended that a slope of scarf greater than this be used. A slope in the neighborhood of 1 in 25, with a range of from 1 in 20 to 1 in 30, is recommended.

Severe weakening of scarf joints is often due to sanding of the face plies at the joint. Observations on joints of this kind that were sanded showed that at times more than half of the face ply is ground away. Inasmuch as the strength of a panel lies almost entirely in the face plies (in case of three-ply panels parallel to the direction of the grain of the faces), it is obvious that a reduction in the thickness of the face plies will materially affect the strength of a panel. Consequently it is recommended that if the scarf joint is sanded at all that it be only lightly sanded by hand, so as not to decrease the thickness of the face veneer.

Figure 42 shows the method used for cutting the scarf and for gluing the two pieces of plywood together. The board above the panel should be relatively massive and flat so as to distribute the pressure from the screws. Two or three layers of blotting paper furnish sufficient padding to accommodate irregularities in the surface.

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THIN PLYWOOD.

In an effort to develop a substitute for linen for wing covering which could be used on present types of wing framework, several different kinds of thin plywood have been developed. Among these are plywoods composed of three plies of wood, each ply as thin as one one-hundred-and-tenth inch, plywoods with veneer faces and fabric cores, plywoods with veneer faces and metal wire core, plywoods with veneer core and cloth faces, and several other types. A method was developed which made it commercially possible to glue up very thin plywood without undue loss, although the losses in making thin plywood are naturally much greater than in making comparatively thick plywood on account of the fragile nature of the thin sheets and their tendency to warp and twist when glue is applied to them. It was not found possible to produce a plywood having all the requisite properties which was as light as doped linen. The general conclusions drawn from the investigation follow:

1. Spanish cedar, mahogany, birch, sugar maple, red gum, yellow poplar, black walnut, and basswood may be cut into veneer sufficiently thin for consideration in plywood air-plane wing covering as substitutes for linen.

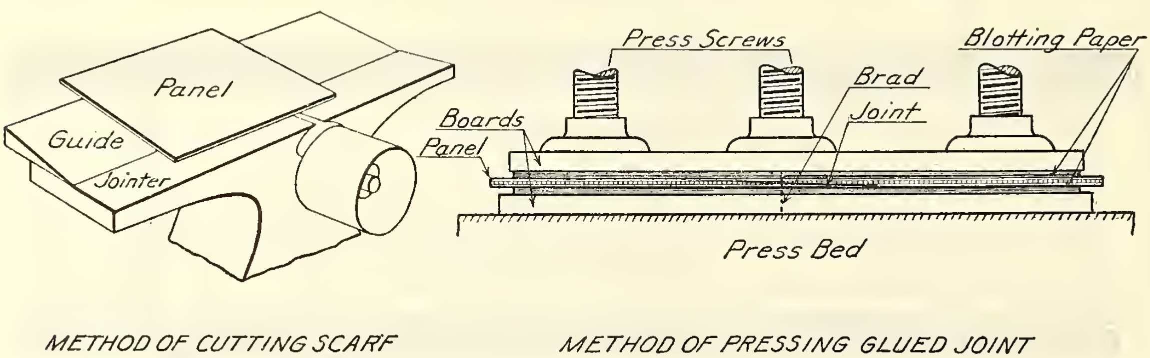

Fig. 42.—Method of making plywood joints extending through entire thickness.

2. These species may be glued satisfactorily by the method of introducing the glue between the plies by means of tissue paper previously coated with glue.

3. It does not seem that plywood sheets of the same weight per square foot as doped linen can be prepared on a practical scale.

4. Covering made either of veneer or of a combination of veneer with fabric, such as linen, cotton, wire screening, or kraft paper, in order to be both practical from the point of view of manufacture and satisfactory in mechanical properties as shown by test, weighs from two to three times as much as doped linen.

5. Plywood that might be considered practical from the point of view of manufacture possesses from two to three times the tensile strength of doped linen.

6. The thinnest ply-wood that can be manufactured at present with any degree of facility (3 plies of one one-hundred-and-tenth inch Spanish cedar) lacks toughness and tearing strength.

7. In general the tearing strength of a practical thin plywood covering is considerably higher than that of doped linen, while its resistance to blows as indicated by the toughness test is lower.

8. In order to obtain the requisite degree of toughness, it is necessary to introduce a cloth fabric into the construction. Grade A cotton now in use in airplane construction is satisfactory for this purpose.

9. Combinations of veneer with kraft paper developed satisfactory tensile strength, but are low in toughness. They compared favorably with linen in tearing resistance.

10. Combinations of veneer with light wire screening, thus far tested, are heavy and unsatisfactory from the point of view of tensile strength per unit weight. Their toughness and tearing resistance are not superior to cloth when used in combination with veneer.

11. Thin plywood or a combination of veneer with cloth is more rigid than linen.

12. Thin plywood unprotected by a finish changes moisture content rapidly and shrinks or expands with a change in atmospheric humidity to the extent of either showing an appreciable loosening or assuming a drum-head tightness when fastened along the edges. A finish of three coats of spar varnish very largely eliminates rapid change in moisture content.

WOVEN PLYWOOD.

Tests have been conducted upon plywood made up with basket-weave faces and corrugated core. The faces are woven out of splints of spruce veneer $1\frac{7}{32}$ inches wide and 0.017 inch thick, while the core is made of spruce $1\frac{7}{8}$ inches wide and 0.018 inch thick. The total thickness over all is almost 0.2 inch.

The following conclusion is drawn from the tests: The high rigidity at low loads, the high tearing strength, stability under varying humidities, and comparatively high toughness indicate that the woven plywood tested may be a very desirable material for construction in airplanes.

Data concerning glues for ply-wood will be found in the text under the general heading "Glues."

The following specification for waterproof plywood is based upon the strength tests just described and upon the glue tests presented farther on.

SPECIFICATION FOR WATER-RESISTANT VENEER PANELS OR PLYWOOD.

GENERAL.

1. General specifications for inspection of material, issued by the Bureau of Construction and Repair, in effect at date of opening of bids, shall form part of these specifications.

2. This specification covers the requirements for veneer panels for use in aircraft where a water-resistant ply-wood is specified.

MATERIALS.

3. The following species of wood may be used in plywood construction:

Basswood.	Mahogany (true and African).	Walnut.
Beech.	Maple (hard and soft.)	Western hemlock.
Birch.	Redwood.	White elm.
Cherry.	Spanish cedar.	White pine.
Fir (grand, noble, or silver).	Spruce.	Yellow poplar.

4. Other species of wood shall not be used without the written approval of the Bureau of Construction and Repair.

5. *Veneer.*—The veneer must be sound, clear, smooth, well-manufactured stock, of uniform thickness and free from injurious defects. Sap streaks and sound pin knots will not be considered defects. Discoloration will be allowed.

6. The veneer may be rotary cut, sliced, or sawed.

7. *Thickness.*—Unless otherwise specified, no single ply of veneer shall be thicker than $\frac{1}{12}$ inch. In three-ply stock the thickness of the core must be between 40 and 75 per cent of the total thickness of the plywood, except for panels one-sixteenth inch or less in thickness.

8. *Glue and cement.*—Any glue or cement may be used which will meet the tests specified in paragraphs 20 and 21.

MANUFACTURE.

9. *Grain.*—The grain in each ply shall run at right angles to the grain in the adjacent plies unless otherwise stated in the order.

10. *Manufacture.*—The plywood must have a core of soft or low-density wood and faces of hard or high-density wood unless otherwise specifically stated in the order. The core may be made of several plies, in which case the grain of the adjacent plies must be perpendicular. The plies must be securely glued together, after which the plywood must remain flat and free from blisters, wrinkles, lapping, checks, and other defects. Plywood manufactured with cold glue must remain in the press or retaining clamps not less than three hours.

11. *Joints.*—Plywood 10 inches wide or less shall have faces made of one-piece stock. In order to conserve the narrow widths of veneer, accurately made edge joints will be allowed in the faces and cores of wider stock, but the number of joints permitted in any ply shall not exceed the width of the panel, in inches, divided by eight. Edge joints are joints running parallel to the grain of the plies joined. All plywood built of jointed stock must be so constructed that all joints are staggered at least 1 inch.

12. In panels over 8 feet long scarf joints will be permitted; the smaller angle of the scarf shall have a slope of less than 1 in 25. Scarf joints in adjacent plies must be staggered. Scarf joints are joints in which the seam runs across the ply at right angles to the grain.

13. Butt joints will not be permitted.

14. In case the core or crossbanding is taped at joints only unsized perforated cloth tape or open-mesh unsized cloth tape applied with waterproof glue or cement shall be used.

15. *Moisture content.*—The finished plywood shall be dried to a moisture content of 9 to 11 per cent, with a tolerance of plus or minus 2 per cent, before it is shipped from the manufacturer's plant. The equalization of moisture shall be effected by kiln drying, followed by conditioning.

16. *Kiln drying.*—The panels must be piled and placed in dry kilns as soon as possible after being released from the press. The method of piling must be approved by the Bureau of Construction and Repair. After the stacking is completed the panels shall be properly weighted to prevent warping during the drying process. The best results in the kiln are obtained with a temperature of from 95° to 115° F. and a humidity ranging from 50 to 60 per cent, depending upon the thickness of plywood and number of plies. The circulation must be maintained at all times.

17. *Conditioning.*—All panels must be conditioned before fabrication or shipment. The conditioning shall be done indoors under temperature and humidity conditions existing in the factory for a period of not less than 24 hours for three-ply panels one-eighth inch thick and proportionately longer for thicker stock. The piling and weighting shall be the same as specified for dry-kiln stacks.

18. *Cutting.*—Cutting for length and width shall be full and true. The veneer shall be cut to the thickness desired in the finished plywood and any overallowance on this thickness for the sanding operation is very undesirable.

19. *Finish.*—In all cases the tape must be removed from the faces of the panel, and, unless otherwise specified in the order, the plywood shall be lightly sanded to a smooth finish free from defects.

TESTS.

20. *Submission of samples for test.*—The manufacturer shall submit to the Bureau of Construction and Repair for test 20 samples, each 1 foot square, of the plywood which he proposes to furnish to airplane manufacturers.

21. *Boiling or soaking test.*—The waterproof quality of the glue shall be tested either by boiling in water for a period of eight hours or by soaking in water at room temperature for a period of 10 days. After boiling or soaking the samples shall be dried at a temperature not exceeding 150° F. to a 10 per cent moisture content. The plies must not separate when the sample panels are subjected to this test.

22. *Shear test.*—The strength of the glue shall be tested in five test specimens cut from a sample panel. The form of the test specimen is shown in figure 43. The ends of the specimen shall be gripped in the jaws of a tension-testing machine and the load applied at a speed of less than one-half inch per minute. The glued surface must not fail at a load of less than 150 pounds per square inch.

23. *Approved list.*—Manufacturers whose plywood does not comply with these specifications will not be considered in awarding of contracts. The list of manufacturers whose product has satisfactorily passed the tests outlined in paragraphs 20 and 21 may be procured from the Bureau of Construction and Repair, Navy Department, Washington, D. C.

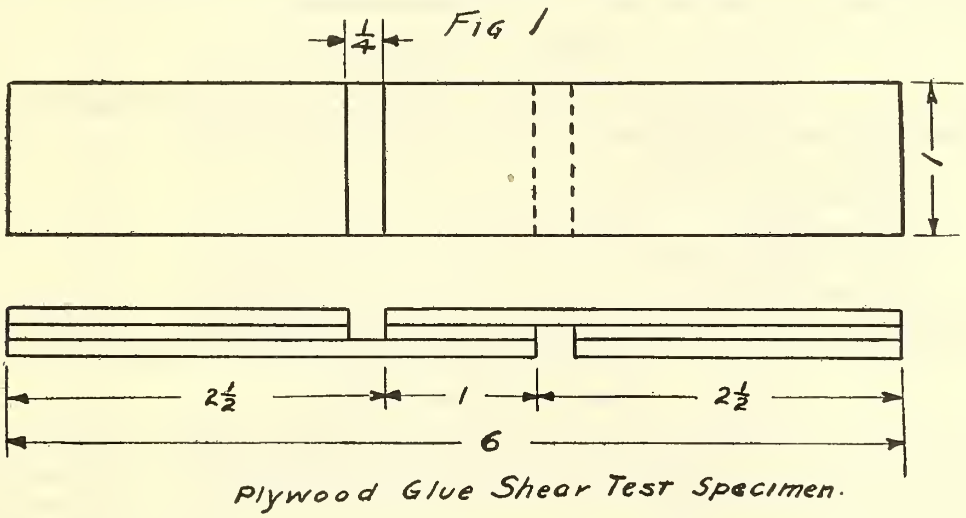

Fig. 43.—Plywood glue shear test specimen.

INSPECTION.

25. Unless otherwise stated, all veneer and plywood shall be inspected at the plywood manufacturer's plant.

26. The inspector shall make the tests specified in paragraphs 21 and 22 on at least one sample panel from each press for each eight-hours' run.

27. In case the plywood fails to meet the soaking and shear tests it shall be rejected. If the glue fails to meet one of these tests but passes the other, the test in which it fails must be repeated on not less than twice the original number of specimens selected taken from two or more panels. If the glue fails to pass the second test, the plywood represented by the samples must be rejected.

28. In case of consistent failure or lack of uniformity in product, the manufacturer will be required to submit a detailed written statement giving the following information:

(*a*) The composition of the glue and the correct practice in mixing it.

(*b*) The maximum time between mixing and applying the glue.

(*c*) The exact procedure in applying the glue and in pressing and curing the plywood and such other details as the Inspection Department may direct.

The inspector shall see that thereafter this schedule is observed.

29. The inspector shall have free access to all parts of the plants where the plywood is being manufactured and shall be afforded every reasonable facility for inspecting the materials used, the methods of manufacture, and the finished plywood.

PACKING AND SHIPPING.

30. Plywood which has passed inspection shall be packed in crates which will protect all edges and surfaces from injury during shipment.

ORDERING.

31. To facilitate the execution of contracts the order will state any special requirements which this material must meet. The order shall state the number of pieces, the width across the grain in inches, the length with the grain in inches, the thickness of the plywood and the individual plies, the number of plies, and the species of wood to be used for faces (to be marked "Faces"), for core (to be marked "Core"), and for cross-banding (to be marked "Crossband"). Sizes given shall be finished sizes and shall conform to commercial sizes when practicable. The order shall also bear the specification number.

GLUES AND GLUING.

There are a number of distinct kinds of glue commonly used in aircraft manufacture. The more important of these are as follows:

1. Hide and bone glues.
2. Liquid glues.
3. Marine glues.
4. Blood albumen glues.
5. Casein glues.

In addition to these there are many kinds of glue and cement used in the arts which are not well adapted to aircraft uses and which, consequently, need not be mentioned here.

HIDE AND BONE GLUES.

In general only the better grades of these glues are used in aircraft, and these are made from hides and are known simply as hide glues. Occasionally nonwater-resistant plywood panels made up with bone glue are used in unimportant parts of aircraft. The principal uses of hide glues in aircraft have been in laminated and spliced construction of various kinds, principally in propeller manufacture. Hide glue is still the standard propeller glue, though it has been replaced to an important extent in other laminated work.

In order to secure a very good grade of glue for propeller and similar work, suitable methods of testing were developed and certain specifications prepared. The Bureau of Aircraft Production regularly inspects lots of glue at the request of manufacturers, and glue passing the required tests is sealed and certified. It is then made available for purchase by aircraft manufacturers, who are thus assured of uniform glue of proper quality. The methods of test developed and used are given in detail in the following statement. The shearing test forms the basis for the certification of casein glue also.

TESTING OF HIDE GLUE.

Chemical analysis has been found practically useless as a means of testing glues because of the lack of knowledge of their chemical composition. Physical tests must, therefore, be relied upon. A considerable number of physical tests have been devised, some of which are important for one class of work and some for another. For judging the suitability of glue for high-grade joint work the tests considered most important are strength, adhesiveness, viscosity, jelly strength, odor, keeping qualities, grease, foam, and reaction to litmus. In the subsequent discussion of these tests their application to joint glue will be especially kept in mind.

Strength tests are made by gluing together two or more pieces of wood and noting the pressure or pull required to break them apart. Many different methods of making the test specimens and breaking them have been devised. These depend to a certain extent upon the character of work expected of the glue and the nature of the testing apparatus available. The simplest and most convenient strength test is to glue two blocks together, as shown in figures 44 and 45b, and shear them apart in a timber-testing machine (see fig. 45 a and c). It will

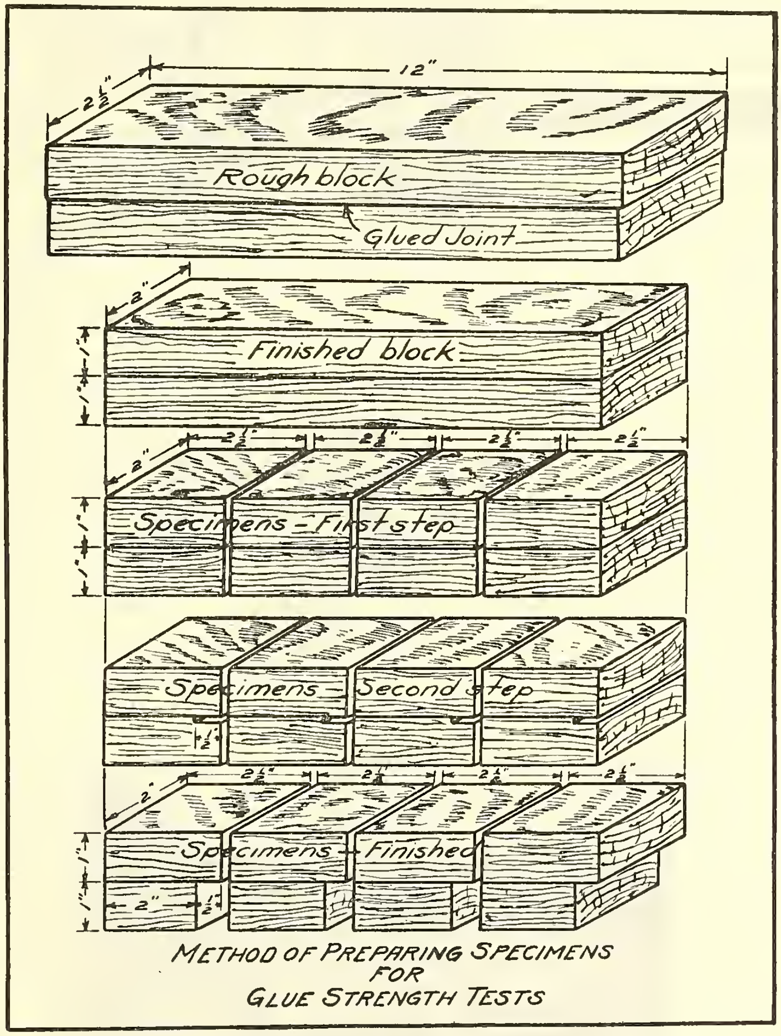

Fig. 44.—Method of preparing specimens for glue-strength tests.

usually be found that there is considerable difference in the values obtained for the individual specimens. The amount of difference, however, can be kept at a minimum by using care to see that the specimens are selected, prepared, and tested under as nearly the same conditions as possible. In making strength tests the selection of the wood is a very important factor. The species selected should be the one upon which it is proposed to use the glue or one fully as strong. Care should be taken also that the wood is above the average strength of the species, in order that there may be less opportunity for the wood to fail before the glue. If the wood is too weak, the full strength of the glue is not determined.

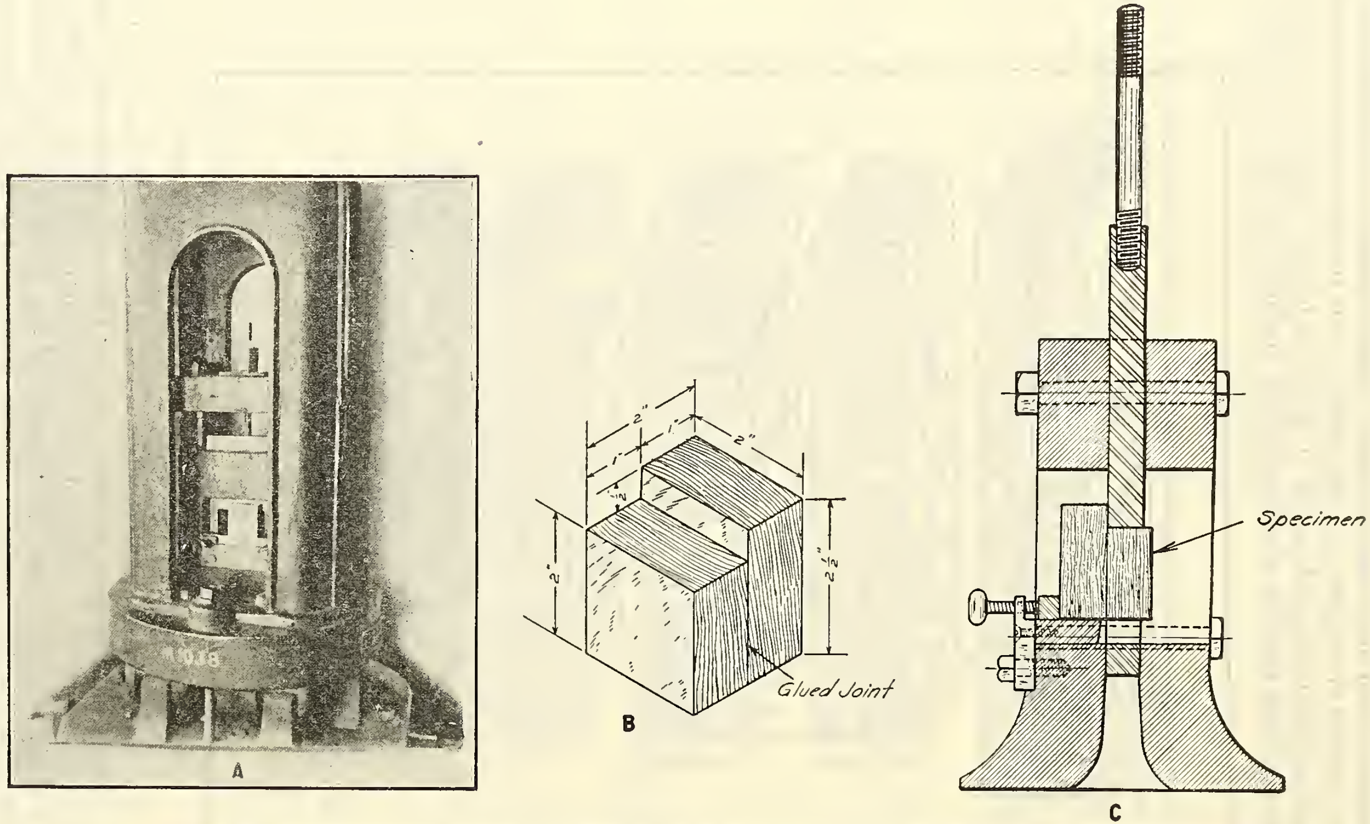

Fig. 45.—A. Testing machine with shearing tool in place. B. Finished specimen. C. Sectional view of shearing tool with test block in position.

No block should fail below 2,200 pounds per square inch, and the average shearing strength for a propeller glue should be at least 2,400 pounds per square inch.

The viscosity of a glue is determined by allowing a specified amount at a given temperature to flow through an orifice. The time required is a measure of the viscosity. The time required for water to flow through is taken as the standard. In general it is found that a glue with high viscosity is stronger than one with a low viscostiy and will absorb more water, although there are exceptions. Hide glues, as a rule, have higher viscosities than bone glues.

A number of different shaped viscosimeters have been devised. In the glue manufacturer's laboratory, where many tests must be made each day, an instrument must be used which will give results quickly. This can be done with a pipette cut off at one end or with a straight glass tube contracted at one end. These instruments are not always arranged so the temperature of the glue within them can be controlled, and for a number of other reasons they are not entirely accurate. Better control of temperature and greater accuracy can be had with the Engler viscosimeter. This is more complicated and more expensive than the glass tubes and also slower to operate, but it has the advantage, in addition to greater accuracy, of being an instrument which is in general use for testing many kinds of materials. The values obtained by its use are readily understood by laboratory men and can be readily checked. The instrument can be purchased standardized and ready for use.

The term "jelly strength" refers to the firmness or strength of the jelly formed by a glue solution of specified strength upon cooling. Strong glues usually have high jelly strength. There is no standard instrument for determining jelly strength and no standard unit for expessing it. In some laboratories the pressure required to break the surface of the jelly is measured. In others the depth to which a weight of special shape will sink is observed. Sometimes the jelly is cast in a conical shape, and the weight required to press the point of the cone a certain distance is taken. More common, however, is the finger test, in which the relative strength of two or more jellies is compared by pressing the jelly witn the fingers. In making this test with any apparatus it is important that the conditions be very carefully controlled in order that comparative results may be obtained. The temperature of the jelly when tested is particularly important, as the relative strength of a number of jellies is not the same at different temperatures. In other words, the jelly strength of the different glues is not affected to the same extent by changes in temperature. The ideal condition is to cool and test the jellies in a room constantly maintained at the proper temperature. This is seldom practicable, however, and the jellies must be cooled in a refrigerator and tested in a warmer room. When this is done it is important that the test be made as quickly as possible after removing the jelly from the refrigerator, so that the temperature will be practically the same as it was in the refrigerator. The strength of the glue solution must always be the same once a standard is adopted. For high-strength glues weaker solutions can be used than for low-strength glues.

The odor of a glue is determined by smelling a hot solution and gives some indication of its source or its condition. Glue which has an offensive odor is not considered of the highest grade. The bad odor may be due to the fact that partly decomposed stock was used or that the glue itself is decaying. For high-grade work it is usually specified that the glue be sweet; that is, it must not have an offensive odor. The odor of different glues varies considerably, and it is difficult or impossible to express the different "shades." It is usually not difficult, however, to determine whether or not the odor is clean, or, as it is commonly called, sweet. The temperature and strength of solution are not usually specified.

The keeping quality of a glue is determined by allowing the jelly left from the jelly-strength test to stand in the laboratory at room temperature for a number of days. The odor and condition of the glue are noted at intervals. Glues with good keeping qualities will stand several days without developing an offensive odor or showing any appearance of decomposition.

For joint work a small amount of grease in glue is not a serious objection. Too much grease, however, is objectionable, as grease has no adhesive properties. The grease can be determined by chemical means, if desired, but this is not necessary unless the exact amount of grease must be determined. The common method of testing for grease is to mix a little dye with the glue solution and paint it upon a piece of unsized white paper. If grease is present, the painted streak will have a mottled or spotted appearance. If there is no grease present, the streak will have a uniform appearance.

Glue which foams badly is objectionable because air bubbles are apt to get into the joint and thus reduce the area over which the glue is in contact with both faces. Foamy glue is especially undesirable for use in gluing machines, as in them the glue is agitated much more than when it is used by hand, and the danger of incorporating air bubbles is greater. The amount of foam is tested by beating the glue solution for a specified time with an egg beater or similar instrument and then noting the height to which the foam rises and the quickness with which it subsides. Different laboratories do not make the test in exactly the same way, but in any laboratory after a method is once adopted it should be strictly adhered to thereafter. It is common to determine the foam on the solution used in the viscosity test.

By its reaction to litmus a glue shows whether it is acid, alkaline, or neutral. The test is made by dipping strips of red and blue litmus paper in the glue solution remaining after the viscosity test or some other test and noting the color change. An acid glue turns blue litmus red, an alkaline glue turns red litmus blue, and a neutral glue will not change the color of either red or blue litmus. A glue containing a slight amount of acid is slightly preferable to one which is neutral or alkaline, because it is not quite so favorable a medium for the growth of the organisms which cause the decay of glue.

From the above description of the various glue tests it is apparent that, for the most part, they give comparative rather than absolute results. It is rather difficult to compare the results of tests made by one laboratory with those of another, as the strength of solution, temperature, and manipulation are often different. For this reason the most satisfactory method of purchasing glues is to specify that they must be equal to a standard sample which is furnished the bidder to test in any way he sees fit. The bidder should also be informed as to the methods the purchaser proposes to use in testing a glue submitted to him as equal to the standard sample.

PRECAUTIONS IN USING HIDE GLUE.

In using hide glue there are a number of precautions that must be observed to obtain satisfactory results. If improperly used, a very high-grade glue may give poor joints. It is important, first, to find out the right proportion of glue and water to get the best results. This is largely a matter of experience, but it can also be determined by strength tests. When the right proportions are decided upon, they should be strictly adhered to thereafter, and the glue and water should be weighed out when making up a new batch of glue rather than measured or guessed at. Clean cold water should be put on the glue, which should be allowed to stand in a cool place until it is thoroughly water soaked and softened. This may take only an hour or it may take all night, depending upon the size of the glue particles. When the glue is soft, it should be melted over a water bath and the temperature not allowed to go higher than about 150° F. High temperatures and long-continued heating reduce the strength of the glue solution and are to be avoided. The glue pot should be kept covered as much as possible in order to prevent the formation of a skin or scum over the surface of the glue.

The room in which the glue is used should be as warm as possible without causing too much discomfort to the workmen, and it should be free from drafts. In a cold, drafty room the glue cools too quickly and is apt to set before the joint has been put into the clamps. This results in weak joints. It is also considered good practice to warm the wood before applying the glue. Wood should never be glued when it is cold, and of course only thoroughly seasoned wood should be used. Since high-strength animal glues set so quickly on cooling, they should be applied and the joints clamped as quickly as consistent with good workmanship.

In clamping glued joints the pressure should be evenly distributed over the joint, so that the faces will be in contact at all points. The amount of pressure which will give the best results is a question which has never been definitely settled. One experimenter found that a pressure of about 30 pounds per square inch gave better results on end joints than higher or lower pressures. Apparently no tests have yet been made to show the best pressure to use on edge or flat grain joints. In gluing veneers it is necessary to use high pressure in order to flatten out the irregularities of the laminations. Pressures as high as 150 or 200 pounds per square inch are sometimes used.

Strict cleanliness of glue pots and apparatus and of the floors and tables of the glue room should be observed. Old glue soon becomes foul and affords a breeding place for the bacteria which decompose glue. The fresh glue is therefore in constant danger of becoming contaminated. Glue pots should be washed after every day's run in hot weather and two or three times a week in cooler weather. Only enough glue for a day's run should be mixed at a time, so that mixed glue will not have to be held over from one day to another. If these sanitary precautions are not observed, poor joints are apt to be the result.

LIQUID GLUES.

Liquid glues, frequently known as fish glues, have been used to quite an extent for the smaller work such as gluing cap strips, tape, blocks, moldings, etc. They are being replaced gradually by casein glues, which have the advantage of water resistance. In general liquid glues are not as strong as certified hide glue, although the shearing strength of several which have been tested has been as high as 2,400 pounds per square inch.

MARINE GLUES.

These glues are used mainly to apply muslin between the inner and outer skins of floats and flying boat hulls. They are required to be of a sticky, viscous nature and relatively nondrying and elastic. They are usually composed of pine tar, rosin, manila resin, and alcohol. On account of their nondrying nature, these glues have comparatively low strength. They are readily soluble in gasoline, and it is necessary, therefore, to make provision to prevent gasoline from getting into the bilge water. In general, marine glues are not used to make joints in wood construction where high strength is required.

BLOOD ALBUMEN GLUES.

These glues, which are made from blood albumen secured from packing houses, are the strongest and most water resistant of all so-called "waterproof glues" in common use to-day. In general, it is necessary to use heat (about 225° F.) to set them, and consequently their usefulness is limited largely to plywood and similar thin material, although it is possible to glue thicker material in cases where the proper heat can be applied successfully. Practically all plywood glued with blood glues is glued between steam-heated plates, which furnish a convenient source of heat.

Properly manufactured blood albumen plywood will pass all the tests prescribed in the plywood specification without difficulty. Not only does the shearing strength average far above that required, but the resistance to boiling and soaking is generally much greater than the specification requires. Further, the residual strength of the glue after boiling and soaking is, in general, decidedly superior to that of casein glues.

A method has recently been developed for the gluing of very thin plywood, in which fine tissue paper is impregnated with blood albumen glue and then dried. This tissue is then used just as ordinary mending tissue. A sheet is placed between the layers of veneer to be glued and the whole put under pressure between steam-heated plates. Since the process is a dry one, the troubles due to swelling and warping are eliminated.

In general, it is anticipated that the use of blood albumen glues will be confined to manufacturers of plywood for some time to come and that the only contact which the aircraft manufacturer will have with it will be in the plywood which he purchases.

CASEIN GLUES.

The major ingredient of these glues is casein, a product secured from the souring of milk. Until a year ago casein glues were hardly known in this country, but they have been developed commercially by several concerns, and their use in aircraft has increased rapidly. They have the advantage that they may be used quite cold and that no heat is used either in mixing or in setting them. Further, they set up quickly but have the disadvantage of taking a comparatively long time to develop their maximum strength.

Casein glue is widely used in making water-resistant plywood and its use in laminated construction (except propellers) is steadily increasing. It is also being used in places where formerly fish glues were mostly used.

The best grades of casein glue are fully as strong as certified hide glue in shear, and their resistance to high humidities and to soaking is much greater. Tests now under way indicate that the shock resistance of casein glues is as great as that of certified hide glue. The technical use of casein glue is very simple, but it is necessary to follow instructions carefully in order to secure best results. The instructions which follow represent the best practice and are based upon experience both in laboratory and in the shop.

INSTRUCTIONS FOR USE.

Equipment.—In using waterproof casein glues the mixers used ordinarily for animal glue and vegetable glue are generally not very successful, as a more rapid and thorough stirring than these mixers give is usually necessary. It is possible that some types of ordinary glue mixers can be speeded up enough to give good results with casein glues, but they have additional disadvantages in being rather difficult to keep clean. The most successful mixer so far found for these glues is the power cake mixer, such as is used by bakers, or machines constructed on a similar plan. These machines have several speeds and mix the glue in a detachable kettle which is easily cleaned. They can also mix relatively small quantities, so that no batch of glue needs to stand very long before being used up. Copper, brass, or aluminum vessels should not be used for mixing casein glues, as the alkali in the glues attacks these metals. It is advisable also to equip the glue pot with a metal hood fitted with a feed hopper in order to prevent spattering outside of the glue pot during the course of mixing.

Preparation of glue.—It is advisable, in all cases, to thoroughly mix the contents of a freshly opened barrel of prepared glue, and preferably several barrels should be mixed at once before any of the dry powder is withdrawn for use, in order to counteract the segregation of ingredients of varying specific gravities which may have occurred during shipment from the factory to the point of consumption. This mixing may be accomplished by transferring the contents of the barrels to a box of suitable size in which the dry glue is turned over a sufficient number of times and thoroughly mixed with a clean shovel.

It is necessary to caution against the practice observed in some plants of sifting the powdered glue and discarding from it the coarse matter which remains upon the screen. This may remove from the glue an essential ingredient and thus defeat the purpose for which the glue is intended.

Proportions of dry glue and water.—The proportion of water to mix with the dry glue should be as directed by the glue manufacturer. It is to be borne in mind, however, that fixed proportions, satisfactory for each and every barrel of glue received, can not be specified because of a slight lack of uniformity which may exist in the product. Hence, only average proportions can be stipulated by the manufacturer, and the operator, in order to obtain satisfactory consistencies, may find it necessary at times to vary from the average proportions specified. It has been found in some cases that using exactly the same proportions of glue and water, the glue from one barrel may be thinner than that from another. It is hoped that this difficulty will be overcome before long by improved manufacturing methods, but until it is much will have to depend upon the judgment of the operator. It should also be remembered that some classes of work require thicker glue than others.

Mixing the glue.—The correct quantity of water is placed in the glue pot and the mixing blade is brought into action at proper speed. A high speed is necessary at first, especially if the glue is not added to the water very slowly, in order to avoid the formation of lumps in the glue. There is a considerable range of speed, however, which will give satisfactory results. In some cases a speed of 140 revolutions per minute of the shaft which carries the mixing blade (about 350 revolutions per minute of the blade itself) is used satisfactorily. By adding the glue carefully, however, a speed as low as 80 revolutions per minute of the vertical shaft (180 revolutions per minute of the blade) can be successfully used. The powdered glue is now slowly introduced through the feed hopper and the agitation is allowed to continue for about five minutes and then stopped.

The sides of the glue pot should now be scraped in order to direct any of the spattered material into the mixture, whereupon the blade is again brought into action at reduced speed (60 to 90 revolutions per minute) for a period of at least ten minutes. The object of reducing the speed after the first stage of mixing is to prevent the incorporation of an excess of air. At the end of this stirring period the glue is ready for use, provided all the fine casein particles are dissolved and no appreciable amount of air has been whipped in. If the glue still contains fine particles of undissolved casein and has the appearance of "cream of wheat" mush, however, the mixture should be continued. It was formerly considered necessary to allow the glue to stand without stirring for a short period before using it. The object of this was to allow all the casein to dissolve. It has now been found, however, that it is better practice to accomplish this solution by continued mixing than by standing. If, however, it is found that air bubbles have been whipped into the glue during mixing, it is desirable to let it stand awhile so the air can separate.

In mixing casein glues which may require the addition of different ingredients singly the above practice should be varied from to conform with the directions of the manufacturer.

Consistency of glue.—It may be found that the proportions used do not always give exactly the same consistency. So long as the glue is neither too thick nor too thin to spread well, however, slight differences in consistency between individual batches or shipments of glue need not be considered serious. Good results may be expected if the glue spreads properly. Other things being equal, thick mixtures develop higher strength than thin mixtures, and when great strength is desired it is desirable to use the thickest mixtures practicable.

If in mixing up a batch of glue from a new barrel or shipment of some kinds of glue it is found that the proper consistency is not obtained, it is possible to alter it if attended to immediately and before the glue has been removed from the mixing pot. This should not be attempted on important work unless the operator fully understands his glue, and it should be entirely avoided if possible.

If the glue mixture obtained is seen, before it is taken from the mixing pot, to be too thick to spread properly, it can be thinned by adding an extra part or two of water, as may be required, and stirring at slow speed until the water is thoroughly incorporated. This holds for any casein glue. Under no circumstances, however, should water be added to glue which has thickened on standing or after being used awhile.

If the glue mixture is found, before removing from the mixing pot, to be too thin, it may be thickened by carefully adding a proper amount of dry glue with continued stirring. This is practicable only for glue in which all the ingredients are mixed together dry, and is not suitable for glues in which the various ingredients are added separately. The stirring should then be continued long enough to dissolve all the casein of the added glue. Another method which might be used is to mix a thicker batch of glue and then mix the two batches together. It is far preferable to avoid using either method, and with proper care it should seldom be found necessary.

Application and use of glue.—The glue in any batch should be used up completely before it begins to thicken materially. The length of time during which the mixed glue can be successfully used may vary with different shipments. The operator must judge whether or not the glue is fit to use at any time by its consistency. Tests have shown that good results may be expected from a normal glue at any time during its working life up to the time when it becomes too thick to spread properly.

In spreading the glue it is important that enough be applied to coat all the surface of both faces of the joint. An appreciable amount of glue should squeeze out of the joints when pressure is applied. As little time as possible should elapse between the spreading of the glue and the pressing. The exact time which can safely elapse will vary with the kind of wood being used, the consistency of the glue, the amount of glue applied, the temperature, and other factors. In making veneer panels it is considered best practice to get the stack under pressure within ten minutes or less from the time the first ply is spread.

The minimum time the joints must be left under pressure is not known. It is considered safest and best practice, however, to leave the joints in the press or in retaining clamps for at least three hours. After the glued material is taken from the press it should be dried either artificially or naturally to remove the moisture added by the glue. It is best also to allow the material to stand a week or two to develop the full strength and water resistance of the glue. The panels should, of course, be piled properly during the drying period to prevent warping.

The above discussion is applicable in general to casein glues, whether of the prepared type, such as Certus, Napco, Casco, or Perkins waterproof glue, or of the type which is mixed by the user directly from the raw materials.

The following points should be kept in mind in preparing and using casein glues:

(1) Thoroughly mix each barrel of glue before using.
(2) Weigh the glue and water; do not measure it.
(3) Avoid lumpy mixtures.
(4) Avoid mixtures which are too thick or too thin.
(5) Mix until all the fine particles dissolve and a smooth mixture is obtained.
(6) Do not use glue after it becomes too thick to spread properly.
(7) Do not attempt to thin or thicken glue after it leaves the mixer.

DIRECTIONS FOR MIXING CERTUS GLUE.

In general use about 10 parts of glue and 17 to 20 parts of water. Both water and glue should be weighed, not measured. With the water in the mixing can, start the mixing blade at high speed (80 to 140 revolutions per minute of the vertical shaft is about right) and add the dry glue rather slowly. Continue this rapid stirring for about 3 to 5 minutes after the last dry glue is added; then stop the mixer, scrape down the sides of the can, and start mixing at slow speed (40 to 60 revolutions per minute of the vertical shaft is about right). After 10 to 15 minutes at slow speed the glue should be ready for use. If it has a granular appearance at the end of this time, however, the casein is not all dissolved, and mixing should be continued long enough to get casein particles into solution. The glue is then ready to use.

DIRECTIONS FOR MIXING NAPCO GLUE.

In general use about 10 parts of glue and 17 to 20 parts of water. Both water and glue should be weighed, not measured. With the water in the mixing can, start the mixing blade at high speed (80 to 140 revolutions per minute of the vertical shaft is about right) and add the dry glue rather slowly. Continue this rapid stirring for about 3 to 5 minutes after the last dry glue is added, then stop the mixer, scrape down the sides of the can, and start mixing at slow speed (40 to 60 revolutions per minute of the vertical shaft is about right). After about 30 minutes at slow speed the glue should be ready for use. If it has a granular appearance at the end of this time, however, the casein is not all dissolved, and mixing should be continued long enough to get the casein particles into solution. The glue is then ready to use.

DIRECTIONS FOR MIXING CASCO GLUE.

Before starting any mixing weigh out all ingredients, using the following proportions:

A { Water, $22\frac{1}{2}$ parts.
Prepared Casco casein, 10 parts.

B { Water, 1 part.
Caustic soda, $\frac{1}{2}$ part.

C { Water, 5 parts.
Hydrated lime, 5 parts.

With the water of A in the mixer and paddle operating at an intermediate speed (in the neighborhood of 60 to 90 revolutions per minute of the vertical shaft of a cake and dough mixer) slowly add the casein and continue stirring till the mass is free from lumps. This should require about 3 or 4 minutes.

Now slowly add the one-half part of caustic soda which has been previously completely dissolved in the 1 part of water, and continue stirring for about 3 minutes.

Next add the 5 parts of hydrated lime which has previously been worked into a smooth paste with the 5 parts of water, and continue stirring until a smooth mixture free from lumps and undissolved particles of casein is obtained. This should require about 15 minutes, possibly a little longer. The glue is now ready for use. If it is found that any appreciable quantity of air has been incorporated into the glue by the stirring, the glue should be allowed to stand 10 to 20 minutes before using to allow the air to escape.

Glue mixed according to the above procedure is ordinarily considered satisfactory for gluing veneer one-twelfth inch thick or thinner. If the glue appears too thin, however, it can be made thicker by using less water, as suggested below. For joint work or thicker veneer also a somewhat thicker consistency is desirable. This can be obtained by using 17 to 20 parts of water under A instead of $22\frac{1}{2}$ parts.

DIRECTIONS FOR MIXING PERKINS WATERPROOF CASEIN GLUE.

(As recommended by the manufacturer September, 1918.)

When the paddle itself is running about 400 revolutions per minute, the following method is highly satisfactory for making up "P. W. G." into finished glue:

Dissolve 1 pound of 76 per cent caustic soda in 30 pounds of water contained in the large bowl. Add 14 pounds of "P. W. G." slowly to the caustic solution with thorough and brisk agitation. Continue agitation for about 5 minutes. Allow the glue to stand 20 to 30 minutes after mixing before using.

When the speed of the paddle itself is less than 400 revolutions per minute the following method will give a smooth, fine flowing batch:

Add 14 pounds of "P. W. G." to 27 pounds of water. Agitate to smooth consistency. Continue agitation and add in small portions a solution made by dissolving 1 pound of caustic soda in 3 pounds of water. Continue agitation for about 5 minutes after ingredients are all in. Allow to stand 20 or 30 minutes after mixing before using.

Neither casein nor blood albumen glues seem to be affected by gasoline in the slightest degree. A number of panels made up by representative manufacturers were soaked for a long period (several months) in gasoline without any sign of deterioration. Similar panels were also soaked for a like period in gas engine oil (Polarine) without any apparent deterioration. These tests indicate that both blood albumen and casein plywoods can be used around the engine without fear of damage by gasoline and oil.

Frequently it becomes desirable to fill, shellac, or varnish parts which are later to be glued. Tests made to determine the strength of joints made on wood treated in this manner show that they are very weak and absolutely unreliable. No joints in aircraft work should be made except with the bare wood.

AIRCRAFT PARTS.

On account of the impossibility of computing, with any degree of accuracy, the strength of many aircraft parts and assemblies, it has been found necessary to supplement the designs and calculations with actual test to destruction. The tests have frequently shown unexpected weak points, which have been strengthened and the parts retested. Through development of this character some very remarkable results have been achieved, and the way has been opened for similar work along allied lines.

LAMINATED CONSTRUCTION.

One of the first problems to come up in this connection was a study of laminated wood construction. Opinion concerning the merits of this type of construction have been divided for a long time, and designers have allowed their fancy free reign in devising widely varying styles of built-up members. Until about a year ago designers were allowed to use either solid or built-up construction, in accordance with their individual needs or desires, but during the present year there has been a very decided trend toward official insistence upon laminated construction in preference to solid, especially in the case of wing spars. There are several reasons back of this trend, not the least important of which is the increasing difficulty of securing large sizes in the desired grades. In the case of propellers lamination has been practically universal for many years.

While lamination undoubtedly does promote the use of smaller and shorter material, with the consequent better utilization of lumber and does insure the elimination of large hidden defects, it requires the exercise of a great deal of care to insure satisfactory results. The principal difficulties encountered lie in the warping and twisting of the finished part. The relations existing between shrinkage and moisture, density, and direction of grain have already been discussed in detail. Let it suffice to say that unequal shrinkage, with consequent twisting or warping, will result in a laminated structure if the various laminations differ materially from each other in any of the three factors mentioned, namely, moisture, density, and direction of grain.

Propellers probably need as much care in their manufacture as any aircraft parts in order to insure permanence of pitch, balance, etc. The following rules for the selection of wood for laminated construction have been prepared especially for propeller manufacture, though they apply in general to all laminated construction:

(1) All material should be quarter-sawed if possible.
(2) Quarter and flat-sawed laminæ should not be used in the same propeller.
(3) All laminæ should be brought to the same moisture content before gluing up.
(4) All laminæ in the same propeller should have approximately the same specific gravity.
(5) All laminæ in the same propeller should be of the same species.

Dry wood when exposed to very humid air absorbs moisture and swells. Wood dried in a normally dry atmosphere till its moisture content becomes practically constant loses moisture, and shrinks when exposed to extremely dry conditions. Two pieces of wood when exposed continuously to the same environment will eventually come to practically the same moisture content, irrespective of their relative moisture contents when first exposed to this environment.

Individual pieces of wood, even those of the same species, vary greatly in their rate of drying. Quarter-sawed pieces have a different drying rate from plain-sawed pieces. Dense pieces dry more slowly than those which are less dense.

Suppose that a flat-sawed board is glued between two quarter-sawed boards, all three having the same moisture content, say, 15 per cent, when glued up; or, suppose, that under similar conditions a very dense piece is glued between two pieces which are less dense; or, suppose that a board containing 15 per cent moisture is glued between two others, each containing 10 per cent but all three being of the same density and cut in the same manner. Then suppose the finished product to be dried to, say, 8 per cent moisture. Every piece will shrink, but in each instance the center piece will tend to shrink more than the outside ones. The glued joint will be under a shearing stress, since the center piece has a tendency to move with respect to those on the outside. Under this condition the glued joint may give way entirely, it may partially hold, or it may hold perfectly. In either of the latter cases the center piece will be under stress in tension across the grain, and consequently will have a tendency to split. This tendency may become localized and result in visible splitting or it may remain distributed and cause a lessening of the cohesion between the wood fibers, but without visible effect.

If a combination of these three cases occurs, it may be much more serious in its effect than any one alone. For instance, suppose that in a propeller alternate laminations are of flat-sawed, dense boards, glued at a relatively high moisture content, while the others are quarter-sawed, less dense, and at a much lower moisture when glued. The tendency of the flat-sawed laminations to shrink will be very much greater than that of the others, with the result that internal stresses of considerable magnitude will be set up.

It is not difficult to see how these internal stresses may combine with the stresses from external causes and with the continual vibration to produce failure under external loads which are considerably smaller than the propeller would safely resist if manufactured with proper care.

In the case of laminated struts and beams the laminations should be matched as to direction of annual rings, as they appear on the end section, to balance shrinkage as much as possible.

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Quarter-sawed and flat-sawed material should never be used in the same member. Neither should either quartered or flat stock be used with stock cut at intermediate angles. In laminating together pieces cut with the annual rings at an angle of about 45 degrees with the faces the rings in the adjacent laminations should be approximately perpendicular to each other instead of approximately parallel to each other.

WING BEAMS.

In order to determine the general principles underlying the design of built-up wing beams, to develop the best forms from the standpoints of efficiency, utilization of low-grade stock, and ease of manufacture, and to study problems connected with manufacture, a series of 300 beams of various types and designs were built and tested. These types included only those which gave promise of strength efficiency combined with utilization of smaller material than that needed for the manufacture of solid beams, since the problem at the time was primarily one of shortage of material. The types selected besides the solid ones used for comparison are shown in figure 46.

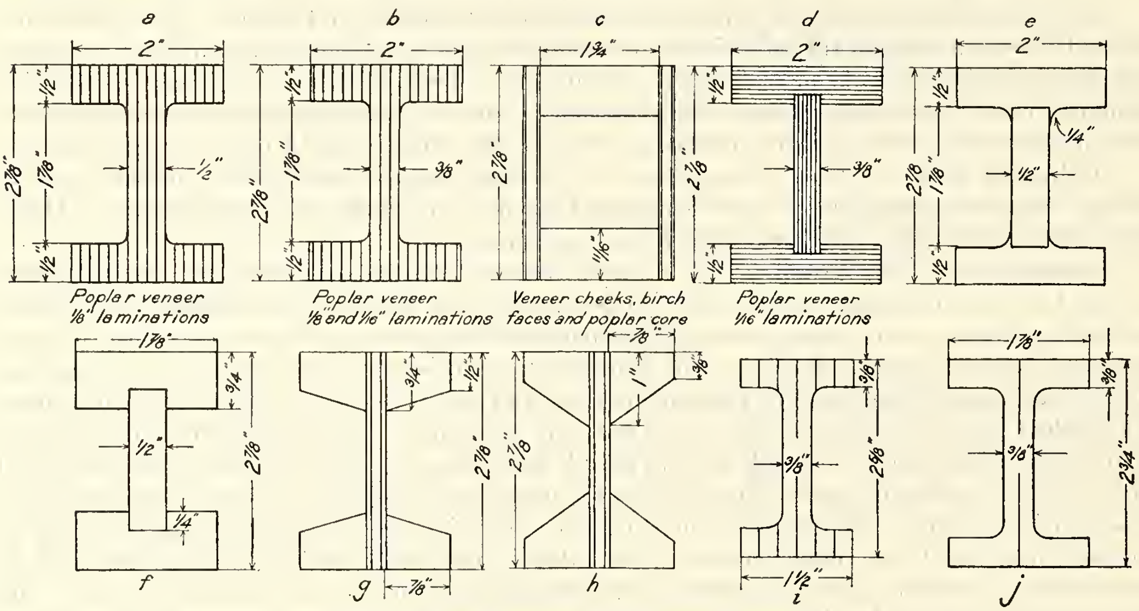


Fig. 46.—Cross sections of built-up test beams.

While it is impossible at the present time to present detailed analyses of these tests, the general conclusions drawn from them are given in the following statements:

The tests were divided into various series for ease in reference, each series representing different conditions from the others. The conclusions from each series are first given, with the general conclusions at the end.

RESULTS OF VARIOUS BEAM TESTS.

Series 1 and 12 (fig. 46e and f): These consisted of one-piece spruce beams of acceptable material compared with three-piece beams of similar matched material. The results compared favorably, although with the built-up beam without filleted joints (Fig. 46f), the work to the maximum load was approximately two-thirds of that of the single-piece beam. On the other hand, the work to maximum work in the other series (fig. 46e) was 20 per cent higher than for the single-piece beams. Consideration of the results as a whole indicate that this type of beam, properly glued, will compare favorably with the single-piece construction.

Series 2 and 13 (fig. 46i): This series included single-piece spruce beams made from rejected material compared with laminated spruce beams made from similar matched material. The laminated material gave 5 to 10 per cent lower values in modulus of rupture and 5 to 10 per cent greater values in work to maximum load. The results show that not only will the glue hold satisfactorily but that higher values would not be secured by laminating defective material than by using it in solid form.

Series 3 and 10 (fig. 46b): Series 3 is made from one-eighth-inch poplar with the grain of all plies longitudinal compared to similar material and construction with the grain of the center ply vertical.

Series 10 consists of beams of one-sixteenth-inch poplar laminations with vertical joints.

Three types were made up as follows: (*a*) The grain of the center ply vertical; the grain of all other plies horizontal. (*b*) The grain of all plies having a slope of one in five from the horizontal, the slope in adjacent plies being in opposite directions. (*c*) The grain of the six center plies having a slope of one in five from the horizontal, the slope in adjacent plies being in opposite directions; the grain of all other plies (namely the flange plies) being horizontal.

The tests showed (1) a 5 to 10 per cent reduction in the mechanical properties where the grain of the center ply was vertical, with no reduction made in the thickness of the web due to using this form of construction; (2) a reduction of approximately 20 per cent in total load and stiffness where a slope of one in five was used in alternate directions in adjacent laminations throughout the whole beam; (3) a reduction midway between the foregoing where a slope of one in five in alternate directions was used only in the web.

The conclusions to be drawn from this series are that if cross-grained material must be used, better results would be secured by laminating and placing the grain of adjacent laminations in opposite directions than to use solid beams of similar material, but that it would not be possible to secure a strength equivalent to beams of satisfactory grain throughout.

Series 4 (fig. 46g): A plywood web with Douglas fir flanges was used in this series, and included beams with the grain in the outside plies of the web longitudinal, vertical, and at 45 degrees. Hide glue was used in making the beams, and failures of the glued joints developed in the tests presumably due to faulty control in the application of the glue. These tests are being repeated, using casein glue.

Series 5 and 6 (figs. 46g and h): These series included spruce flanges with plywood webs. The face plies were one-thirty-second inch with vertical grain, while the thickness of the core was varied from one-eighth to one-sixteenth inch. The results indicate the desirability of making a web of this construction somewhat thicker than required for shear stresses only.

Series 7 (fig. 46c): An acceptable grade of spruce with plywood sides was used in these tests. Four thicknesses of plywood were used, as follows:

Outside plies $\frac{1}{32}$ inch, core $\frac{1}{8}$ inch.
Outside plies $\frac{1}{24}$ inch, core $\frac{1}{8}$ inch.
Outside plies $\frac{1}{32}$ inch, core $\frac{1}{16}$ inch.
Outside plies $\frac{1}{100}$ inch, core $\frac{1}{64}$ inch.

This type of beam gave very satisfactory results but the very thin plywood proved entirely inadequate. The results indicate that plywood with a one-sixteenth-inch core and one-thirty-second-inch faces would be suficient and that possibly a lighter construction would prove satisfactory.

Series 18 (fig. 46d): This series included beams made up of one-sixteenth-inch poplar veneer with the center ply of the web vertical. The results were satisfactory and showed that the glue held sufficiently to develop the strength of the section. This type of beam, however, would probably be increased in weight about 10 per cent above that of a solid beam of similar material due to the large quantity of glue which would be required.

Series 11 and 19 (fig. 46j): This series used white pine, with one-half of each beam of quarter-sawed and the other half of plain-sawed material and with moisture content 5 per cent higher in one-half of the beam than in the other. It is planned to subject different beams from this series to varying conditions of humidity in order to determine the effect of such conditions where the grain of the two faces of the beam are of a different character and in different directions. The greater part of this series has not yet been run, but the variations in results not due to the gluing indicate that greater defects can not be allowed in either piece than are now allowed in solid beams.

GENERAL CONCLUSIONS.

In general, practically all types of beams so far tested have given values commensurate with what might be expected of the section under test. In other words, the tests have shown that waterproof glue properly applied enables the full value of the section to be developed.

Since the success of the laminated type of construction is primarily dependent upon the efficiency of the glue, it is of the utmost importance that means be provided to insure the satisfactory supervision of the technique of gluing.

The types of beam illustrated in figure 46e and f and in figure 46c seems to offer the most immediate opportunity for effectively increasing production from the class of material now on hand and being received by the airplane manufacturers. Since in the types indicated in figure 46e and f spiral grain material can be used in the webs, these types would have the particular advantage of permitting utilization of material now rejected. The tests thus far made indicate that these beams properly made are no more variable in their strength properties than solid beams.

All of the beams of the foregoing series were made under laboratory conditions. In order to determine just what might be expected under factory conditions, several hundred of the types shown in figure 46c and e were ordered from various aircraft manufacturers and tested. The results of these tests, while not yet completely analyzed, show that, with proper supervision, it is possible for the average aircraft manufacturer to produce satisfactory built-up beams. They also show, however, that the need for thorough, intelligent supervision is imperative.

In addition to these series, numerous miscellaneous types of beams have been tested. Several of these types were similar to the types which have become more or less standard, while others may be considered freak designs. So far none of these freak designs have shown up satisfactorily. Several of the designs had some form of plywood in the flanges. In no instance have beams of this type proven as strong as beams with solid flanges or flanges in which all the grain was parallel to the longitudinal axis. Figure 47 shows various types of wing beam construction which have been used in machines or approved for use.

BEAM SPLICES.

Until the present year the matter of beam splices had not received a great deal of attention. There were in use, and embodied in specifications, many different kinds of splices, some of which were obviously very inefficient. The growing shortage of full-length material made the matter of increasing importance, and several series of tests were run both in this country and in Great Britain.*

The following report is based on tests on about 150 spliced beams and 150 unspliced beams, each spliced beam being matched to an unspliced one by being cut alongside of it out of the same plank. The beams were all Douglas fir, kiln dried, and of good quality, $1\frac{5}{8}$ by $2\frac{3}{4}$ inches in cross section, and the splices were made up by hand, using certified hide glue. The dowels

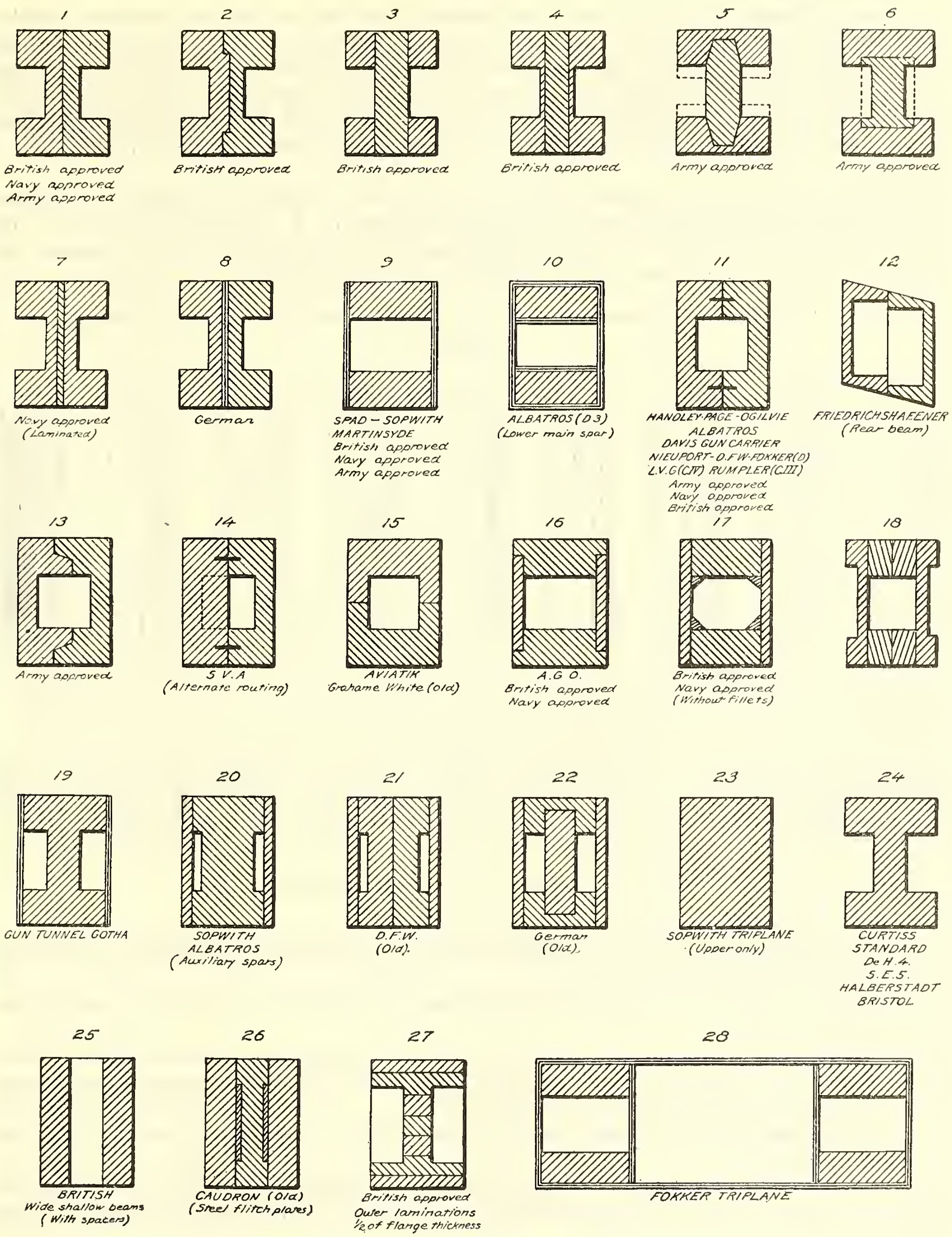
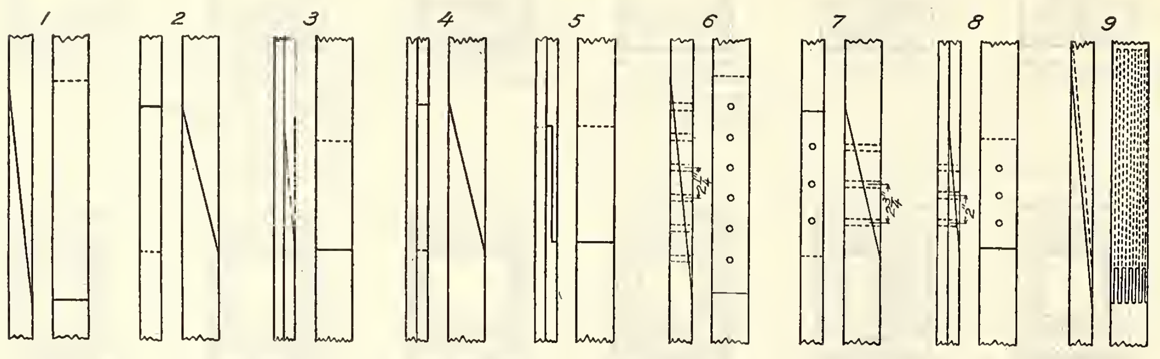


Fig. 47.—Typical built-up wing spars.

were also of Douglas fir. In no cases were clamps or tape used to reinforce the splices; neither were any of the splices bolted. The beams were all tested over a 60-inch span under third-point loading, thus producing uniform bending moment, without shear, in the central third of the span, in which the splices were all located. In order to eliminate as many variables as possible, the efficiency of each splice was calculated in per cent of the strength of the unspliced beam matched with it. The efficiencies thus obtained were then averaged for each type of splice.

Table 14 presents in condensed form the data secured and shows the average, maximum, and minimum efficiencies of each of the nine types tested. A number of these types were selected for test, not because it was thought that they would develop high efficiencies but because they had already been used or included in some specification.

TABLE 14.—*Strength of wing beam splices—spars, $1\frac{5}{8}$ by $2\frac{3}{4}$ inches in cross section; dowels, $\frac{1}{2}$ inch in diameter.*



Wing beam splice No.	1	2	3	4	5	6	7	8	9
Length of splice, inches	16. 25	11. 00	8. 125	5. 50	8. 50	16. 25	11. 0	8. 125	16. 25
Slope of splice	1 in 10	1 in 4	1 in 10	1 in 4	....	1 in 10	1 in 4	1 in 10	1 in 10
Glued area, square inches	44. 8	18. 43	22. 4	9. 22	* 23. 40	44. 80	18. 43	22. 40	85. 60
Minimum efficiency	49. 5	17. 5	77. 7	39. 1	74. 1	74. 9	19. 6	72. 0	50. 0
Maximum efficiency	88. 0	53. 2	105. 5	86. 5	91. 0	107. 0	61. 4	123. 5	100. 7
Average efficiency	73. 0	34. 0	90. 0	66. 7	81. 0	86. 4	38. 7	100. 0	75. 9

* Does not include two end areas, 2×(2.75×0.406), 2.23 square inches.

The conclusions drawn from the tests are as follows:

(1) A laminated beam spliced in one lamination is stronger than a solid beam spliced with the same type and slope.

(2) Dowels add to the strength of splices from 10 to 20 per cent on the average for the spliced beams tested.

(3) Plain scarf joints, with the plane of the scarf vertical, are the most satisfactory from all points of view. Dowels or bolts provide a great deal of residual strength in case of glue failure, while adding to the maximum strength as well.

(4) In general, a slope of scarf of one in ten will provide a satisfactory joint in either solid or laminated beams.

It is very interesting to note that the British have arrived at practically the same conclusions and that the standard British splice has a slope of one in nine, with dowels or bolts and dowels.

STRUTS.

The discussion and conclusions presented in the following paragraphs are based upon strength tests conducted on about 400 struts of various types, some of which were made of accepted material and others of material rejected by airplane inspectors for one reason or another.

Among the principal objects of these tests are the following:

(*a*) To check the individual designs and the factors of safety developed.

(*b*) To determine the variability of the material.

(*c*) To study the effect of spiral grain and other defects upon the properties of the finished struts and to develop methods of inspection.

Tests have been made upon the following kinds of struts:

Standard J–1 inners, accepted and rejected, spruce.
Standard J–1 outers, accepted and rejected, spruce.
Standard J–1 center, accepted and rejected, spruce.
DH–4 inners, accepted and rejected, spruce and fir.
DH–4 outers, accepted and rejected, spruce and fir.
F5–L outers, accepted, spruce (laminated, $2\frac{1}{4}$ by $6\frac{3}{4}$ inches).

All of these except the F5–L struts were solid. The F5–L struts are laminated, with three laminations, of which the center one is lightened by means of two oblong lightening holes.

METHODS OF TEST.

The following kinds of test were made:

(1) Standard-screw testing machine, used for making column tests on struts with the regular end fittings supplied by the manufacturer. Slow, uniform speed of compression. A number of these struts were tested up to the maximum load repeatedly without any injury.

(2) Standard-screw testing machine, used for making column tests on struts, with special knife-edge and fittings, which provided practically perfect "pin ends." Slow, uniform speed of compression. Many of the struts tested repeatedly to maximum load without injury.

(3) Dead-load tests on struts carried nearly to the maximum load.

(4) Special tests in hand machines designed for use in the inspection of struts. These machines show the maximum load direct or allow it to be calculated from the stiffness in bending.

The results secured are presented according to groups of struts as tested, and the conclusions drawn are presented at the end of the discussion for each group.

TESTS ON STANDARD J–1 STRUTS.

The first series tested consisted of 60 J–1 struts, outers, inners, and centers, all spruce. These were accepted stock and were tested principally to check the designs and determine the quality of the spruce. The following general conclusions were drawn:

(1) The quality of the spruce was satisfactory, except that 10 struts had a specific gravity less than 0.36.

(2) The struts were all slender enough to enable the maximum load to be determined without injury to the strut. In fact it was found possible to load the struts repeatedly to maximum load without injury.

(3) It was found that the ball-and-socket joints provided by the manufacturer offered some resistance to the free deflection of the struts. This resistance would probably not be present in actual flight, due to vibration. The knife-edge fittings were found to obviate this source of error and were adopted as the standard fitting for future tests.

The loads sustained by the various classes are as follows:

	Minimum.	Maximum.	Average.
Front outers	935	1,510	1,203
Front inners	1,620	2,980	2,325
Rear outers	830	1,505	1,148
Rear inners	1,610	2,965	2,067

The average moisture content for the outers was 8.2 per cent and for the inners, 8.3 per cent.

In order to form a basis for comparing the variations in the individual struts with normal variations in the spruce itself, an analysis of the stiffness of 500 specimens of spruce was made, and it was found that the average variation of the individual moduli of elasticity from the average of them all was 15 per cent. This average variation was secured as follows: The difference between each individual modulus of elasticity and the average modulus was expressed in per cent of the latter, and these percentage differences were then averaged to secure the average variation.

The average variation from the average strengths for the struts compares favorably with this figure of 15 per cent and is tabulated by strut classes:

	Per cent.
Outers	13
Inners	16
Centers	12

The individual variation of the maximum and minimum from the average strengths is as follows, again by classes of struts:

	Per cent.
Outers:	
Minimum	30
Maximum	30
Inners:	
Minimum	27
Maximum	40
Centers:	
Minimum	34
Maximum	14

In general, the struts followed Euler's law as well as could be expected, except that the ideal load deflection curve, OABC, figure 48, was modified in the actual tests to a curve more nearly represented by ODBC. This was in all probability due to unavoidable eccentricity of fittings and loading. According to the Euler theory the elastic curve of a slender column is a sine curve. The actual curve, as determined by direct measurement, approaches very near to the theoretical curve of Euler.

TESTS ON REJECTED J–1 STRUTS.

This group of struts, spruce outers and inners, was rejected by Government inspectors, and tested primarily to determine the effect of defects upon the strength of the struts and to study means of inspection. Standard methods of test were followed. The general conclusions drawn are as follows:

(1) As a group, these struts were not as good as the 40 accepted struts previously tested. A larger portion of the rejects broke suddenly and a larger proportion broke without preliminary compression failure.

(2) Forty-one of the rejected struts appeared to the laboratory staff making the tests as satisfactory regarding both direction of grain and specific gravity. With one exception, the weakest of these 41 was as strong as the weakest of the 40 accepted struts previously tested. Further, the average of these 41 rejected struts was nearly as good as that of the 40 accepted ones.

(3) There were 18 struts whose diagonal or spiral grain was between 1 in 15 and 1 in 20. Of these, 16 compared favorably with the 41 discussed in the two preceding paragraphs.

(4) A total of 57 (16 plus 41) of the 100 rejected struts compared favorably with the 40 accepted struts previously tested.

(5) It is possible to segregate the acceptable struts from lots of rejected struts by means of simple strength tests if the passing values are appropriately chosen from preceding laboratory tests on struts like those in question.

(6) The limiting grain may safely be reduced to 1 in 15 without causing a reduction in the factor of safety, provided that strength tests and appropriate passing values are imposed. Such a plan of inspection by test would undoubtedly increase the quality and percentage of acceptance.

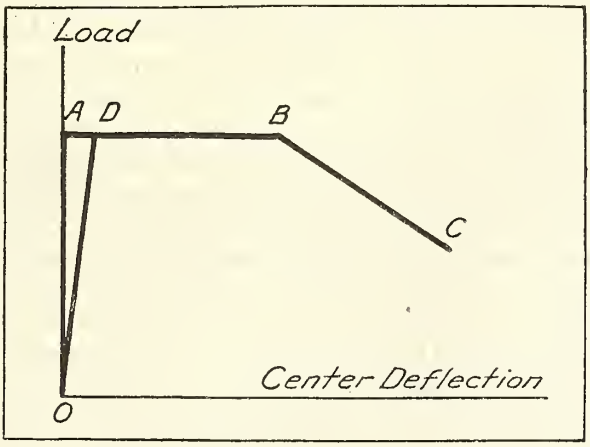


Fig. 48—Load deflection curves for slender struts.

TESTS ON STANDARD DE HAVILLAND STRUTS.

The purpose of the tests was, in general, to check the design calculations and afford a direct comparison between spruce and Douglas fir when used as struts. Half of the struts were tested in the machine in the usual manner and the other half were tested in a special dead-load apparatus. A summary of the results follows:

(1) With the exception of one strut, a spruce stick notably below specification both as to spiral grain and density, all the struts developed maximum loads greater than that for which they were designed.

(2) The weakest of the fir struts was notably low in density, but still it was considerably stronger than the calculated load.

(3) There was practically no difference in the average strengths of the spruce and the fir struts; but there was wider variation between the minimum and maximum values for spruce than for fir. Without exception, the spruce struts were lighter than the lightest fir strut. For unit weight (of strut) the spruce struts were 17½ per cent stronger on the average than the fir.

(4) In the dead-weight test all of the struts, with one exception, were stable; that is, if deflected by a side push (when under the weight of 3,200 pounds chosen for the test, which was just under the crippling load for the weakest strut), they would come back upon removal of the push. The exception was the weakest strut, which was unstable at a dead-weight of 3,030 pounds.

(5) Notwithstanding the general low specific gravity of the fir struts, the maximum loads which they sustained were high, and it would seem safe to reduce the limit from 0.47 to 0.45 for struts of the same size as spruce and to use fir interchangeably with spruce.

TESTS ON REJECTED DE HAVILLAND STRUTS.

These tests were primarily made in connection with the development of strut-testing machines and inspection by actual test. There were 70 spruce and 70 Douglas fir struts, all rejected by Government inspectors for one reason or another. One hundred had been rejected for spiral grain and the other 40 for miscellaneous defects, which, under actual test, did not influence the failures at all. The results of these tests confirmed the conclusions drawn from previous tests, both as to the need and practicability of a strength specification and test, and the limits of slope of grain and specific gravity for Douglas fir already mentioned. In addition, careful study was made of the variation of spiral grain along the length of the strut and its effect upon the maximum load, and as a result of this study the conclusion has been reached that for struts of uniform cross section, like the D-H struts, the most severe requirements for straightness of grain should be limited to the middle third and to the tapered ends and that the requirements for the balance of the strut can be more lenient.

The final recommendation concerning the slope of grain is that, assuming the determination of the maximum load for each strut and no reduction in the factor of safety, the steepest slope allowed in the center third and in the tapered ends be 1 in 15 and that the passing load for struts with a slope between 1 in 15 and 1 in 20 be set higher than for straighter-grained struts. Struts with a slope between 1 in 15 and 1 in 20 at the center third and at the tapered ends and showing the larger load specified for them are to be allowed a slope of 1 in 12 for the remainder of the strut; also, struts with straighter grain than 1 in 20, which also show the larger load specified for struts with steeper slope, may have a slope of 1 in 12 outside the middle third and the tapered ends; but struts having a grain straighter than 1 in 20 in the middle third and in the tapered ends and which meet the lower load requirements specified for them, but do not meet the higher load specified for the struts with the steeper slope, may be allowed to have grain with a slope of 1 in 15 or straighter in the remainder. The requirement for greater load in the case of the steeper slopes is put in to insure against possible greater variability in shock resistance of this material.

TESTS ON STANDARD F5–L STRUTS.

The main purpose of these tests was to determine whether or not they fall in the class of slender struts and can be loaded to their maximum loads without injury. These struts are built up of three laminations each, the center laminations being lightened by two oblong lightening holes. They are $2\frac{1}{4}$ by $6\frac{3}{4}$ by 102 inches. It was found that they could be tested up to their maximum load without injury, and it was also found possible to calculate the maximum loads by means of stiffness determinations based upon simple bending tests. The details of these methods will be described in the following paragraphs:

TWO NONINJURIOUS TEST METHODS FOR INSPECTING STRUTS.

Two noninjurious methods of test for determining the ultimate strength of interplane struts have been developed as a result of the series of tests which have been described in the preceding pages. Both methods are applicable to routine inspection tests in the factory, and the equipment needed is simple and cheap. Both methods are applicable to slender struts (all the struts so far have fallen in this class).

In order to determine the limiting slenderness ratio, $\frac{L}{r}$, governing the use of these two methods for solid spruce and Douglas fir struts, tests were made upon three spruce and three fir struts, as follows: They were first tested full length, $\frac{L}{r}$ about 165, and were then successively shortened to $\frac{L}{r}$ ratios of 140, 120, 100, 90, and 80, and tested at each length by both methods. As a result of these tests the conclusion is reached that for spruce the limiting slenderness ratio is about 100 and for Douglas fir about 90.

Three types of machine have been built and tried out satisfactorily. In the first two types the strut is actually loaded up to the maximum load. In the third type the modulus of elasticity is determined by means of a simple beam test well within the elastic limit and the maximum load calculated by a simple conversion formula.

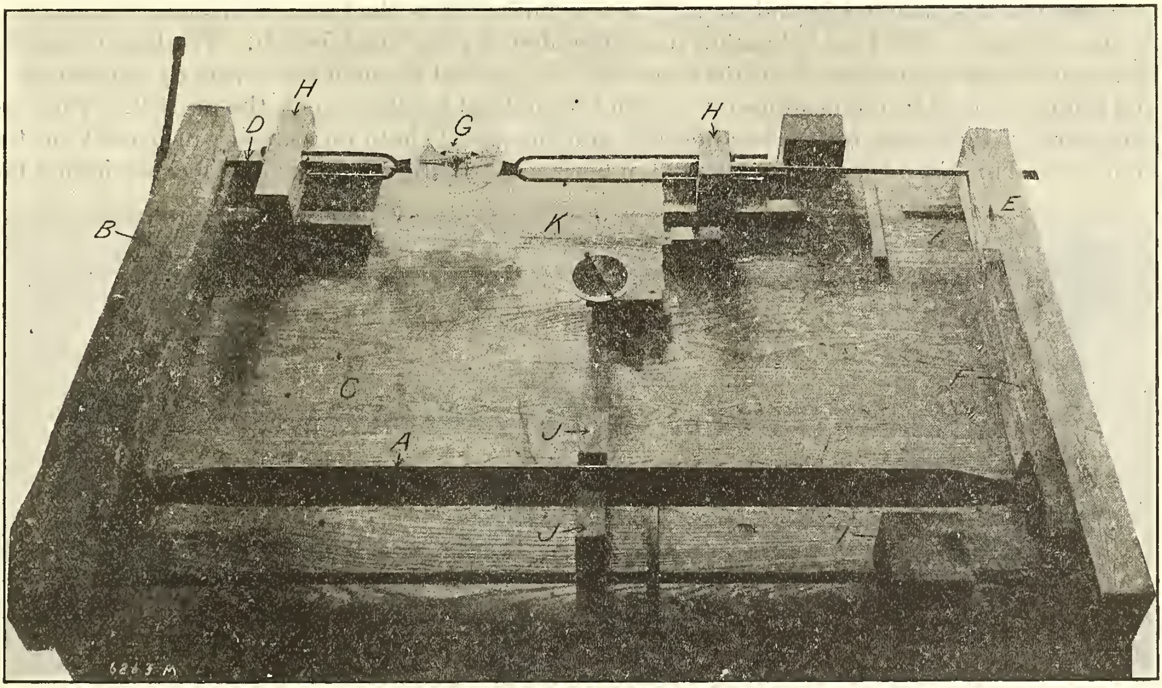


Fig. 49.—Homemade strut-testing machine, first design.

The first machine (fig. 49) employs the lever principle and is especially suitable for larger strut loads, say over 5,000 pounds. A (fig. 49) is a strut in place for testing; B is a base rigidly fastened to the top of table C; it affords support for one end of the strut and also for the pulling screw D. E is a lever, by means of which the pull (multiplied) is brought to bear on the strut as strut load. F is a knife-edge fulcrum; and G a spring dynamometer. H and I are supports for pulling rod and fulcrum rod, respectively. J–J are the stops at either side of the middle of the strut to limit excessive deflection of the strut through careless operation. The dial K is not a part of the machine for making the proposed acceptance tests. It was used for measuring strut deflections in another investigation. The dynamometer (John Chatillon & Sons, of New

York) is of 1,500 pounds capacity. It is graduated in 25-pound intervals, and 5 pounds can be estimated easily. The pulling rig is an ordinary carpenter bench vise screw, handle, etc.; the screw has eight threads to the inch.

The second machine (fig. 50) is of the direct-pull type without multiplying lever, especially suitable for the smaller strut loads, say under 5,000 pounds. It consists of a long shallow box,

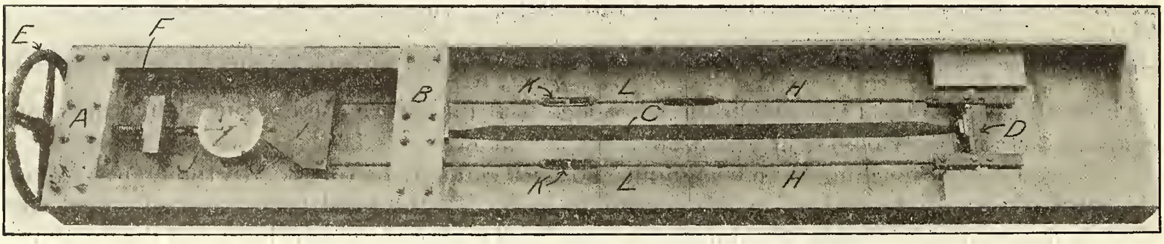


Fig. 50.—Homemade strut-testing machine, second design.

into one end of which a rigid and strong frame is built; AB is the frame mentioned; C is a strut in place for test. The load is brought upon the strut by the headpiece D. The load is applied by means of the handwheel E on the screw F; it is applied through the spring dynamometer G and pulling rods H to the headpiece D. The rods extend freely through the piece B. They are supported at their ends by the headpiece D and the part I, both on castors which track on the floor of the box when the machine is used in horizontal position. J is wood block encircling the

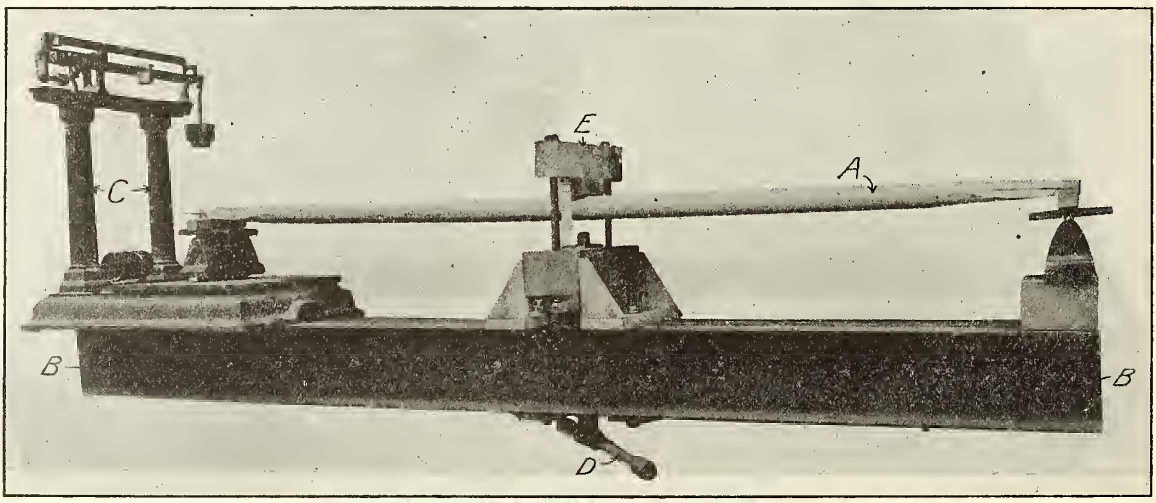


Fig. 51.—Beam machine for strut testing.

pulling nut. It prevents the nut from turning and affords attachment for the dynamometer. Adjustment for different strut lengths is afforded by the turnbuckles K and the distance rods L.

The third machine (fig. 51) is a "beam machine" for the second method of determining strut strength. A is the strut in place for testing; BB are I beams forming the base of the entire appliance; they support the weighing scale C, the loading screw D, and one end of the strut. The middle deflections of the strut are measured by means of the usual device, a thread stretched between two points on the strut just over the supports and a suitable vertical scale just behind this thread and fixed to the strut or to the loading block E.

Discussion of noninjurious test methods.—Reference has been made to a simple formula used to calculate the maximum load of a strut from a smaller load and the corresponding deflection in a bending test (the method illustrated in fig. 51). The following discussion will show how this formula is developed.

Euler's column formula seems to be in most common use for calculating the maximum strength of interplane struts, and the method under discussion is based mainly on that formula. It is:

$$Q=\frac{C\pi^2EI}{L^2} \qquad (1)$$

Where Q = Total crushing strength of column in pounds.
C = A coefficient depending on the character of the end bearings (free or fixed).
E = Modulus of elasticity in pounds per square inch.
I = Moment of inertia.
L = Length of column between bearings in inches.

The deflection in a strut supported flatwise near its ends and then subjected to a cross-bending load, such that the strut (as a beam) is not overstrained, is given by the formula:

$$d=K\frac{Pl^3}{EI} \qquad (2)$$

Where d = Deflection at center in inches.
K = A coefficient depending on loading and manner of support of the strut as a beam.
P = Any moderate (beam) load, not overstraining the beam, in pounds.
l = Span in the beam test in inches.
E = Modulus of elasticity in pounds per square inch.
I = Moment of inertia.

For any given strut equations (1) and (2) may be equated by solving for EI in both cases, thus:

$$EI=\frac{QL^2}{C\pi^2}=\frac{KPl^3}{d}$$

Solving for Q gives the formula:

$$Q=\frac{CK\pi^2Pl^3}{L^2d} \qquad (3)$$

For struts on knife-edges supports C = 1. Struts (on ball-and-socket supports, pin supports, and the like) in flying airplanes are subjected to vibration which breaks down the friction at the supports and makes the supports equivalent to knife-edges. Hence it seems wise, as in practice, to calculate the ultimate strength of airplane struts as though knife-edge supported; that is, with C = 1. In regard to the most suitable kind of loading of the strut as a beam, only center and third point were considered; others were regarded as impractical. By actual trial of 12 struts it was found, contrary to expectation, that center loading gave the better results; accordingly, that loading was finally decided upon. For such loading and simple nonrestraining supports, K = 1/48. Hence equation (3) becomes

$$Q=\frac{.206l^3}{L^2}\times\frac{P}{d} \qquad (4)$$

which is the final form. It will be noted that P and d (or their ratio) are the only quantities for which test must be made in order to furnish the value of Q for any particular strut. $\frac{P}{d}$ is the center load per inch of deflection; it is therefore a measure of the stiffness of the strut.

For struts not uniform in cross section or composition the Euler (column) formula and the beam deflection formula still hold. Appropriate mean or average values of E and of I must, of course, be used in each, but whether or not these average values in the column formula are respectively equal to those in the deflection formula, thus permitting their cancellation or elimination, can not be answered positively for all nonuniform struts. It is believed that the answer is affirmative. There is affirmative evidence from tests of 20 tapered solid struts (10 outer and 10 inner struts for the J–1 airplane), also from tests on 5 built-up struts (5 pieces, plywood covered); that is to say, the second method of test, based on formula (4), was applied to these struts and very good results were obtained.

Comparison of two test methods by actual trials.—Thirty-five struts were tested by the beam method and for comparison by the column method also. The tests by the beam method were made with the struts on knife-edge supports. The results are recorded in the columns marked Q_1 (table 15). The results by the column method are recorded in columns marked Q_2 (table 15). The per cent differences between Q_1 and Q_2 appear in the following columns. They are decidedly small, and the test verification of the theory of this second method is highly satisfactory. The table includes solid struts of spruce and Douglas fir, both of uniform and tapered section, and struts of uniform section built up of spruce and birch.

TABLE 15.—*Maximum or crippling loads for certain struts determined by measurement in column tests and by calculation from cross-bending tests.*

(*a*) Solid struts uniform in section.

No.	Species.	Q_1	Q_2		$\frac{Q_1-Q_2}{Q_1}$		Average grain.	
			l=52 inches.	l=60 inches.	l=52 inches.	l=60 inches.	Spiral.	Diagonal.
DH–4 inners		*Pounds.*			*Per cent.*	*Per cent.*		
G–41	Spruce	5,175	5,380	5,390	−4.0	−4.1	65	95
G–42	do	6,350	6,420	6,530	−1.1	−2.8	65	50
G–56	do	5,125	5,270	5,530	−2.8	−7.9	80	80
G–57	do	4,375	4,310	4,575	+1.5	−4.6	14	60
G–64	do	3,445	3,640	3,645	−5.6	−5.8	25	100
DH–4 outers:								
G–70	Fir	2,075	2,040	2,080	+1.7	−0.2	30	80
G–74	do	2,240	2,200	2,180	+1.8	+2.7	15	95
G–76	do	2,560	2,520	2,570	+1.6	−0.4	39	21
G–79	do	2,020	2,035	2,060	−0.7	−2.0	18	80
G–80	do	2,460	2,485	2,510	−1.0	−2.0	16	95
J–1 inners:								
D–1	Spruce	2,540	2,570	2,510	−1.2	+1.2		
D–13	do	1,800	1,750	1,820	+2.8	−1.1		
D–14	do	1,975	1,920	1,945	+2.8	+1.5		
D–17	do	1,950	1,920	2,030	+1.5	−4.1		
D–2	Fir	2,170	2,220	2,200	−2.3	−1.4		
J–1 outers:								
D–19	Spruce	1,450	1,425	1,430	+1.7	+1.4		
D–20	do	1,235	1,195	1,200	+3.2	+2.8		
D–21	do	1,060	1,010	1,030	+4.7	+2.8		
D–7	do	1,415	1,355	1,385	+4.2	+2.1		
D–8	do	1,390	1,385	1,360	+0.4	+2.2		
Average					2.4	2.7		

Q_1=Max. load as measured in column-bending test.
Q_2=Max. load as calculated from cross-bending test.

$$Q_2=\frac{2Pl^3}{48DL^2}$$

D=Deflection at load P in cross bending.
l=Span in cross bending.
L=Effective length in column bending.

TABLE 15.—*Maximum or crippling loads for certain struts determined by measurement in column tests and by calculation from cross-bending tests*—Continued.

(*b*) Solid struts tapered (Span=l=64 inches).

No.	Species.	Q_1	Q_2	$\frac{Q_1-Q_2}{Q_1}$
		Pounds.	*Pounds.*	*Per cent.*
J-1 inners:				
D-1	Spruce	2,275	2,450	−7.1
D-13	do	1,700	1,720	−1.2
D-14	do	1,790	1,835	−2.5
D-17	do	1,775	1,750	+1.4
D-19	do	1,400	1,430	−2.1
D-2	Fir	2,030	2,120	−4.4
J-1 outers:				
D-20	Spruce	1,165	1,210	−3.9
D-21	do	1,000	1,040	−4.0
D-7	Fir	1,300	1,330	−2.3
D-8	do	1,315	1,350	−2.6
Average				3.2

(*c*) Built-up struts,* uniform in section (span=l=60 inches).

No.	Species.	Q_1	Q_2	$\frac{Q_1-Q_2}{Q_1}$
		Pounds.	*Pounds.*	*Per cent.*
J-14	All spruce and birch	4,250	4,160	+2.1
J-15				
J-16		4,815	4,710	+2.2
J-17		3,760	3,600	+4.2
J-18		3,500	3,540	−1.1
J-19		3,425	3,440	−0.4
Average				2.0

* The core was a double box made of spruce; it was covered or stream lined with two-ply spruce; the inner ply was longitudinal, about one-eighth inch thick, the outer circumferential, about one-thirty-second inch thick. Other dimensions were as for DH-4 inners.

It will be noted that many of the struts were tested on two spans. One span was practically the maximum which the strut afforded. The two spans were tried out to ascertain whether choice of span is important. As expected, the choice was unimportant with struts of uniform cross sections, but with tapered struts the longest span gave best results. Several struts were tested twice on the same span. The second time turned over—that is, the side which was the upper in the first test was the lower in the second. The values of $\frac{P}{d}$ in the two tests were practically alike in each case.

A high degree of skill is not necessary in using the cross-bending test for inspecting struts, but for good results care should be taken about details. Both ends of the strut should be supported in such a way that bending can occur without the ends slipping on the supports. The supports should be such that there is no doubt where the points of support are, because the exact value of span is required in formula (4). The bending load P is relatively small compared with the maximum (100 to 400 pounds for struts so far tested). Hence, a weighing apparatus correct to 1 or 2 pounds should be provided. The deflection should be read with reference to points on the strut immediately over the support and not on the machine. For best results a single value of $\frac{P}{d}$ should not be relied upon. Good practice is to read loads and deflections for a load deflection graph. The mean straight line gives the best value of $\frac{P}{d}$ for use in the formula. Of course, the loadings should not be carried to the elastic limit. In the tests of J–1 and DH–4 struts deflections up to one-half inch were used. This was really more than necessary. All that is needed is enough of the (straight) load deflection graph to be certain of its slope, $\frac{P}{d}$.

MISCELLANEOUS STRUT TESTS.

Tests of struts stream lined with plywood.—Seven struts of two distinct designs were tested as square-ended columns and compared directly with solid spruce struts of the same gross area and solid spruce struts of the same weight, also tested as square-ended columns. The sections of the built-up struts are shown in figure 52a and b. The test length was 5 feet. As was to be expected, the design shown in figure 52a did not develop satisfactory strength, and after testing four struts the design shown in figure 52b was developed and three struts made up, using, respectively, birch, soft maple, and red gum plywood. These struts developed about double the strength of the other type, and appear to be rather well balanced (as square-ended columns), since one of them failed by shearing of the spruce web.

The plywood struts were naturally larger than solid spruce struts of the same strength and shape, although lighter, and consequently would create greater wind resistance or drift.

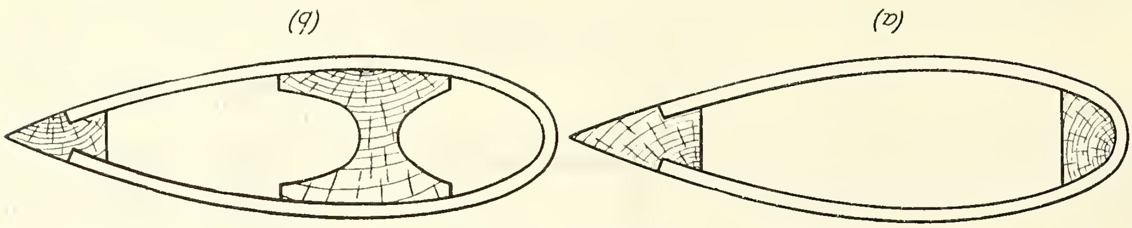


Fig. 52.—Spruce and plywood struts.

In order to reach an equitable basis of comparison it was necessary to consider both weight and drift. Assuming an air speed of 80 feet per second and that 1 pound of resistance is equivalent to 6 pounds of weight, the equivalent weight of the plywood struts was calculated to be 91 per cent of that of the solid struts (of the same strength and shape) at this speed.

Naturally at higher speeds the advantage of the plywood struts is correspondingly less, disappearing entirely long before present maximum speeds are reached. The average weight of the plywood struts was 3.91 pounds and the average actual load sustained was 9,700 pounds (as square-ended columns).

Tests on struts covered with bakelized canvas.—Tests were made on 24 spruce struts, more or less cross grained, and covered with bakelized canvas (micarta). The external dimensions of all the struts were alike, but half were covered with two layers of canvas and the other half with four layers; the former having, therefore, more wood in them than the latter. All the struts were tested in column bending for maximum load without injuring them. All were subsequently tested to failure, 16 with the canvas partially or wholly removed.

Since the modulus of elasticity of bakelized canvas is lower and its specific gravity much higher than that of spruce, one would expect this material to be a poor substitute for spruce in struts, so far as total strength and strength per unit weight of strut is concerned. All tests made verify this expectation, but the canvas covering improved the quality of the defective spruce struts in one respect, namely, the capacity of the strut to withstand severe shock. This conclusion is based on the fact that the deflection at failure for eight covered struts was considerably greater than for four struts stripped.

The struts covered with two layers of canvas were stronger than those with four because there was more wood in them and they were much stronger per unit of weight than the latter.

Struts covered with canvas were but little stronger than the same struts stripped of canvas. The covered ones were weaker than the stripped ones per unit weight of strut. Further, it is computed that the canvas-covered struts were weaker than spruce struts of the same size would have been.

Comparisons with 40 J–1 struts previously tested show that the covered struts were not as high in total strength or strength per unit weight as the plain struts.

Several struts had the outer layer of canvas removed for some distance from the ends, and these struts so stripped were to all intents and purposes as strong as they were originally.

Effect of taper on the strength of struts.—Tests were made on 40 solid struts to determine the effect of taper. These struts were of spruce and Douglas fir. Some were of the sizes and shapes corresponding to DH–4 inners and outers and the others of the sizes and shapes corresponding to the central sections of Standard J–1 inners and outers. (It will be remembered that the J–1 struts have a central section about 0.46 the length of the strut, which is of uniform section, the taper starting at the ends of this section and running in a smooth curve to the ends.) These 40 struts were all first tested for maximum load while of uniform section. They were all tapered to the geometrical form of the J–1 taper and again tested for maximum load. Finally the DH–4 struts were given a very pronounced taper and tested a third time for maximum load. The results of the series of tests are presented in condensed form in table 16.

TABLE 16.—*Effect of taper on the strength and weight of struts.*

Lot No.	Number of struts.	Change due to first taper.			Change due to second taper.		
		A.	B.	C.	A.	B.	C.
		Per cent.	*Per cent.*	*Per cent.*	*Per cent.*	*Per cent.*	*Per cent.*
1	3	− 4.9	+0.6	+5.7	−22.2	−32.3	−6.6
2	3	− 4.4	0.0	+4.4	−26.7	−23.5	−3.9
3	5	− 3.7	−1.5	+2.1	−17.8	−20.3	−3.6
4	5	− 5.1	−2.0	+3.1	−18.8	−22.1	−4.6
5	7	− 8.1	−3.1	+4.8	....	....	....
6	5	− 9.0	−3.5	+6.2	....	....	....
7	7	− 9.7	−2.4	+7.9	....	....	....
8	5	−11.2	−3.1	+9.0	....	....	....

A represents change in weight due to taper.
B represents change in maximum load due to taper.
C represents change in maximum load per unit weight of strut, due to taper.

Lots 1 to 4 were DH–4 struts, and lots 5 to 8 of J–1 size.

The maximum load per unit weight was increased by the first taper from a minimum of 2.1 per cent for lot 3 to a maximum of 9 per cent for lot 8. The weighted average increase was 5.5 per cent.

It will be noted that the second taper reduced the strength weight ratio as well as the maximum load.

DESIGN AND MANUFACTURE OF BUILT-UP STRUTS.

The following general discussion is based upon the results of several hundred thousand tests on wood in various forms, as well as upon the experience gained in the design, manufacture, and test of struts of various types. While much of the discussion is quite obvious, it is believed to be pertinent.

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Built-up struts possess a number of advantages and disadvantages as compared to the solid one-piece construction, some of which are as follows:

Advantages:

- Use of small pieces of material.
- More effective distribution of material.
  - (*a*) By routing.
  - (*b*) By using materials of different density.
- Possibility of using defective material.
- Complete failure may not occur with failure of one lamina.

Disadvantages:

- Greater warping or bowing if pieces are not rightly selected and well manufactured.
- Greater difficulty in manufacture.
- Greater time required for manufacture.

One of the main advantages of built-up struts is the possible use of smaller dimension material with its corresponding lower cost and greater availability. It is further a matter of common observation that many of the larger pieces which contain defects such as to make them unsatisfactory for use as a single unit would yield smaller pieces free from defects and suitable for built-up construction. The material near the center of a solid strut contributes but little in proportion to its weight to the maximum load the strut will carry. Struts lightened by routing at the center, therefore, have the advantage of a greater strength-weight ratio than a solid strut. Enough material at the major axis of symmetry is, of course, necessary to carry the shear, which is greatest along this axis and near the ends of the strut. A built-up strut lends itself readily to routing or lightening at the center.

The taper of solid struts is likewise meant to accomplish a reduction in weight. Weight reduction with a minimum reduction in strength, however, can probably be most effectively obtained through routing in built-up construction. This, however, is more feasible with struts of larger dimension, and probably, all things considered, should not be undertaken on struts whose minor axis is less than $1\frac{3}{4}$ inches. It is common practice in built-up struts lightened in this manner to discontinue the routing at regular intervals, thus leaving a solid cross section at these given points.

Use of materials of different density.—It may be shown that a metal column with proper distribution of material will theoretically withstand a load two or three times greater than a solid wooden section of the same total weight, length, and section boundary. This is based on the assumption that no local buckling takes place. With thin metal walls this assumption would, of course, not be strictly true, as buckling actually does occur. The conclusion is valid, however, that the denser material, with its greater stiffness, may be desirable for struts and is most effective when distributed at the greatest possible distance from the neutral axis. This points to the possible advantages of a combined wood and metal strut and demonstrates in built-up wooden struts, especially the larger sizes, that the use of denser species for the outer portions, with a lighter species for a core, would furnish a possible efficient combination. The use of a combination of species of wood of different density, however, would not be desirable in solid built-up struts of small size, and if used in the larger sizes would require special construction to distribute stresses resulting from unequal changes in dimension and unequal stiffness, as will be considered later.

Tests on combined metal and wood struts are now under investigation, and while very encouraging results have been obtained additional work along this line will be necessary before definite recommendations can be made for production consideration.

Possibility of using defective material.—But little data is available on the effect of defects such as spiral or diagonal grain in the individual pieces on the strength of built-up struts. In connection with the use of spiral grain material for struts, however, it may be noted that the modulus of elasticity is not as greatly reduced by this defect as are the other mechanical properties, and therefore the maximum load in struts which is largely dependent on the stiffness may not be greatly reduced with slopes of grain as great as 1 in 15. In built-up struts containing but one glued surface parallel to the major axis the limitations of defective material should be maintained up to the standard required for one-piece construction. Large struts, however, may be composed of three (or more) sections, as shown in figure 54. The center section, containing the major axis of symmetry, receives little other than shear stress. It is probable that a greater tolerance of grain could be permitted here than in the outer laminations or in one-piece construction. Tests to secure information on this point are necessary and are under consideration.

Possibility of warping or bowing.—One difficulty frequently encountered on the manufacture of built-up struts is the tendency to warp or bow. Practically all wood contains internal stresses to a greater or lesser extent, and failure to take into consideration the factors which influence these stresses contributes largely to the trouble mentioned. As is well known, wood changes dimensions at right angles to the grain to a considerable extent with change in moisture content. Unequal changes in the widths of various laminations causes severe stress in the glued joints and may even cause failure. Among the important factors which cause unequal changes in dimensions in the different laminations are:

(*a*) The use of plain-sawed and quarter-sawed laminations in the same strut.

(*b*) The use of laminations that differ in density.

(*c*) The use of laminations that differ in moisture content.

(*a*) In connection with the use of plain-sawed and quarter-sawed material it may be noted that the shrinkage of Sitka spruce in a radial direction is only about six-tenths of that in a tangential direction. For a given change in moisture, it will therefore be seen that a plain-sawed board would normally undergo a greater change in dimension than would quarter-sawed material. In built-up construction the best results would therefore be expected with quarter-sawed material, as shown in sections 1–a and 1–b, figures 53 and 54. The use of both plain and quarter sawed material in the same built-up part should be avoided.

(*b*) Another factor which may influence the warping of built-up struts is the density of material in adjacent laminations. It has been shown that in general the shrinkage of wood varies directly as the density, and light pieces would therefore, as a rule, retain their shape better than denser ones. The adjacent laminations should be made of pieces of approximately the same density to give the best results, as otherwise considerable stress may be introduced along the glued joints, due to the tendency of the various laminations to change dimensions unequally.

(*c*) Differences in the moisture content of the various laminations at the time of manufacture may also contribute to the warping of built-up struts or other parts. Since wood shrinks with change of moisture content and since all material stored or used under similar conditions will ultimately assume approximately the same moisture content, it follows that differences in moisture content at the time of gluing will cause unequal changes in dimensions which introduce stresses in the glued surface. The fact that all material used in a given laminated member comes from the same stock does not necessarily insure against differences in moisture content between individual pieces. The wide range in the rate of drying of individual pieces, the difference in drying between quarter-sawed and plain-sawed lumber, as well as the fact that heavy pieces usually dry more slowly than lighter ones, contribute to the differences of moisture content which may be found at any time in a given stock. The position of material in a pile while air seasoning or in the kiln while being dried may also influence the rate of drying and consequently the difference in moisture content between individual pieces at a given time.

The manufacture of built-up struts with proper attention to the various factors which may affect the quality of the product as outlined in the preceding discussion would be more difficult than the manufacture of single-piece members. The time required for inspection

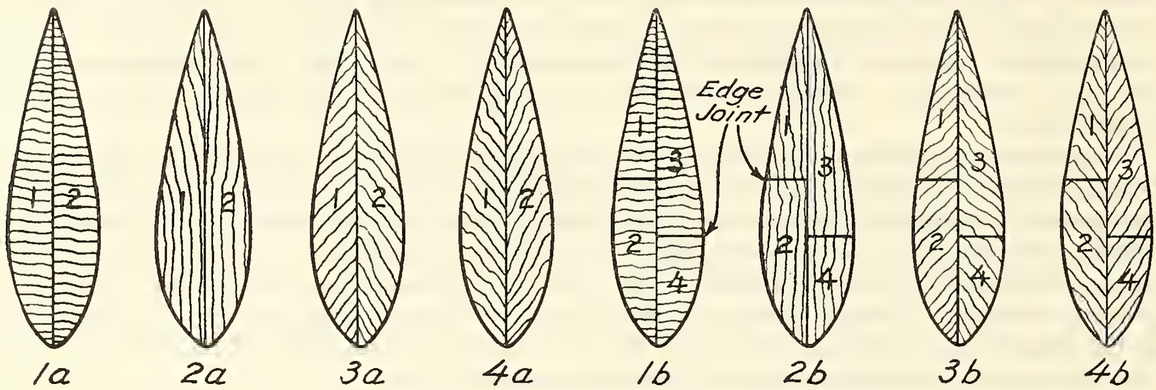

Fig. 53.—Sections of built-up struts, two and four piece construction.

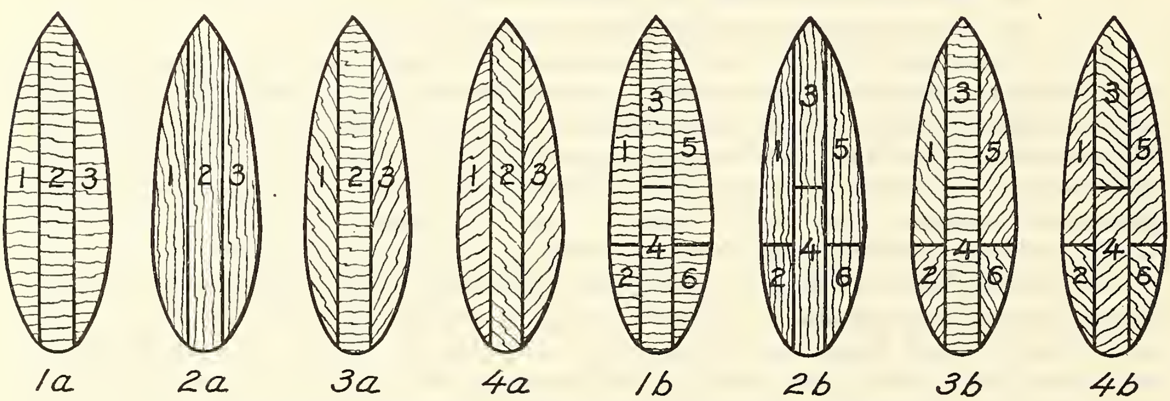

Fig. 54.—Sections of built-up struts, three and six piece construction.

would be increased on account of the greater number of pieces involved and because of the matching required. The gluing would also be an additional item to be considered in manufacture.

The additional work involved in the proper manufacture of laminated struts would probably have a tendency to reduce production, or at least would require greater facilities and more labor for a given output—particularly for struts of smaller sizes. These considerations would tend to offset the lower cost resulting from the more complete utilization of the small pieces.

Static and impact bending tests made on Sitka spruce and a few other species have shown that the position of the growth rings with respect to the faces of the test pieces does not influence the bending strength. No data, however, is available as to the effect of the position of growth rings on the strength of struts, although it is expeceted that some data along this line will be secured in the near future. From data available at present the position of growth rings in a built-up strut would be expected to affect physical properties, such as the ability to retain shape rather than strength. It is desirable in built-up members that the construction be such as to reduce the stresses to a minimum. This involves the use of material of approximately the same rate of growth, density, moisture content, and direction of growth rings in the cross section.

CONCLUSIONS.

1. The manufacture of built-up struts with a minor axis of $1\frac{3}{4}$ inches or less is not recommended.

2. (*a*) To secure the best results, the laminations of built-up strut should be approximately of the same moisture content, density, rate of growth, and, in general, except in cases of special design, of the same species.

(*b*) The construction of stream-line struts should be symmetrical about the major axis. It may be noted that symmetry and consequent balance of internal stresses can in some cases be secured without conformity to the exact requirements under (*a*) above.

3. Figures 53 and 54 show recommended sections of built-up struts.

(*a*) Sections 1–a and 1–b in both figures 53 and 54 would be expected to give the greatest freedom from internal stresses and consequent warping.

(*b*) In figure 53 but little difference in ability to retain shape would be expected between sections 2–a, 3–a, and 4–a, and also between 2–b, 3–b, and 4–b.

(*d*) There are a great number of possible combinations of material with different combinations of growth rings, and it is quite possible that other combinations giving modification of types shown should also prove satisfactory.

4. In types such as 1–b, 2–b, 3–b, and 4–b in both figures 53 and 54 it is desirable but not essential that the edge joints come under the end fittings.

5. The edge joints as shown in types 1–b, 2–b, 3–b, and 4–b in figures 53 and 54 should be staggered, preferably about 1 inch.

6. The taping of built-up struts hides the glued surfaces from inspection and, as it does not add to the strength, seems unnecessary.

7. The use of waterproof glue for built-up struts is recommended.

8. There is reason to believe that the construction of solid and routed built-up struts can be improved over present practice and over that here shown so as to more effectively relieve the internal stresses which tend to produce warping. It should be remembered, therefore, that while the information here presented is based on the most complete data now available on built-up struts the subject is one which has been but little studied and great improvements may consequently be expected.

Figure 55 shows various types of strut construction which have been used in machines or proposed for use.

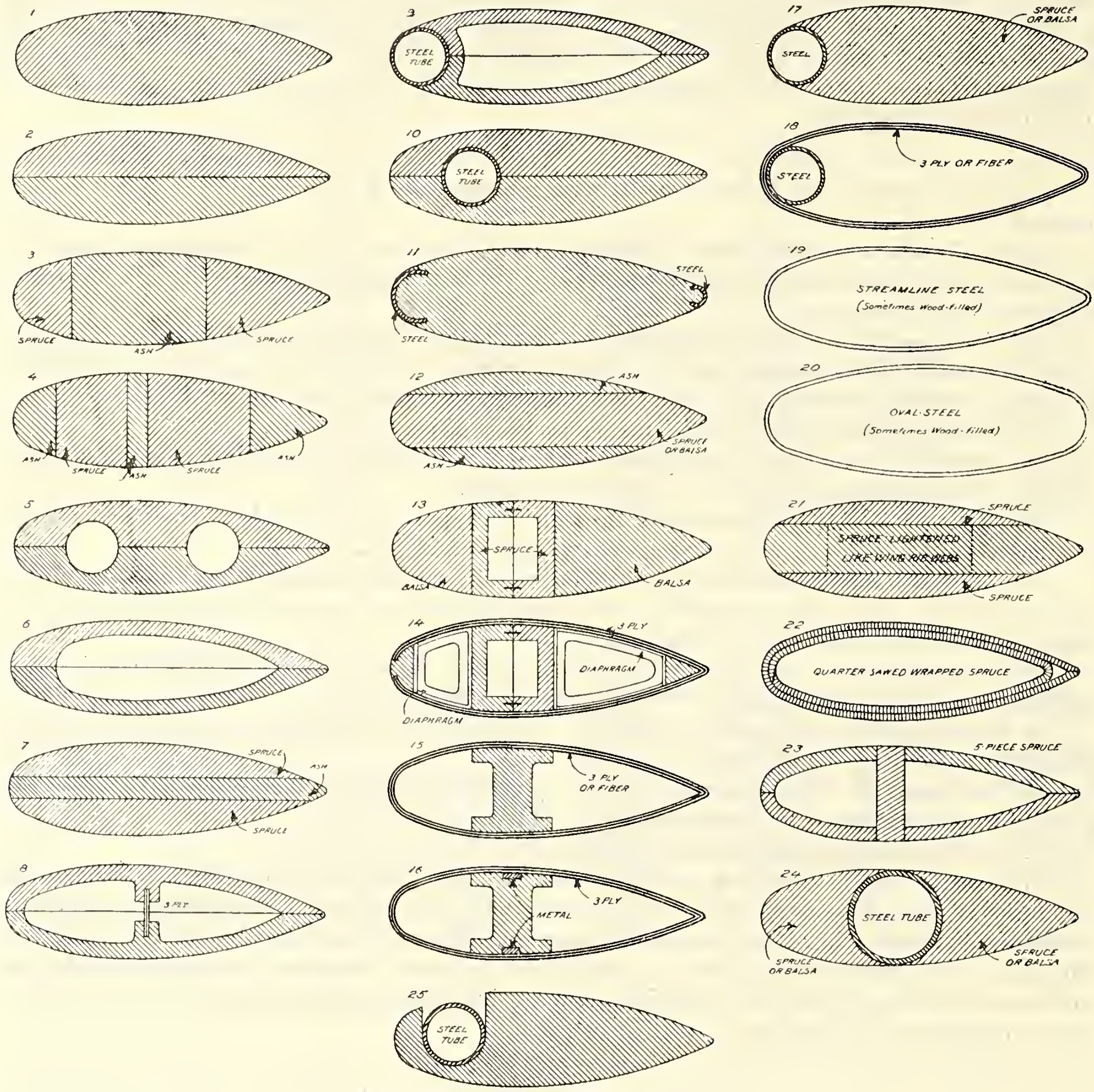

Fig. 55.—Typical built-up strut sections.

WING RIBS.

The construction and loading of wing ribs is of such a nature that it is practically impossible to calculate, with any reasonable degree of accuracy, the actual strength of any particular design. Further, it is quite impossible to determine without actual test the relative efficiency and strength of the various elements of the rib. As a result of these conditions it has been found necessary to develop a number of types through test. Some of the types which have been used or proposed for use are shown in figure 56. A number of these types have been tested, and several of them were developed as a result of the experiments.

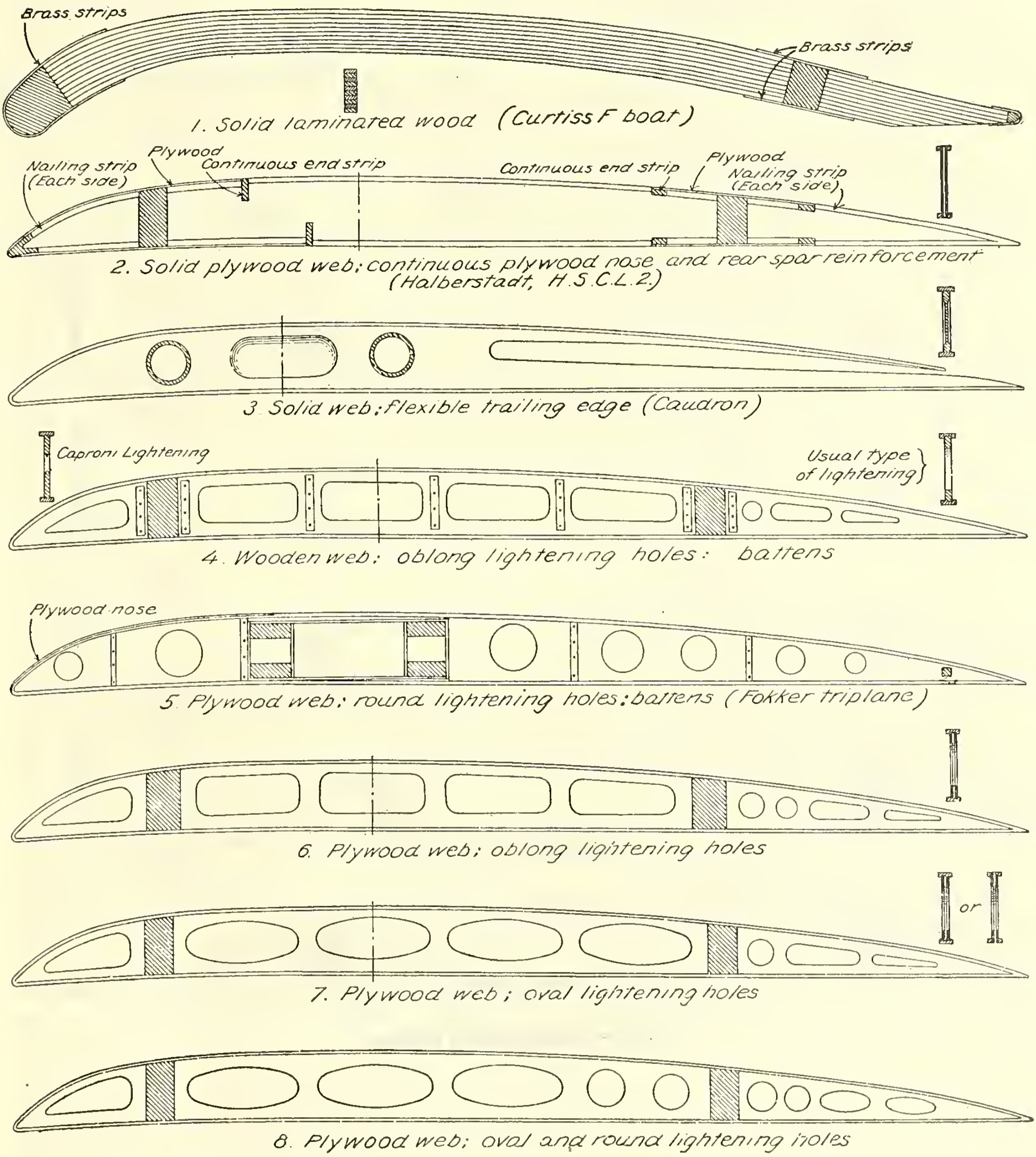

Fig. 56.—Typical wing-rib designs.

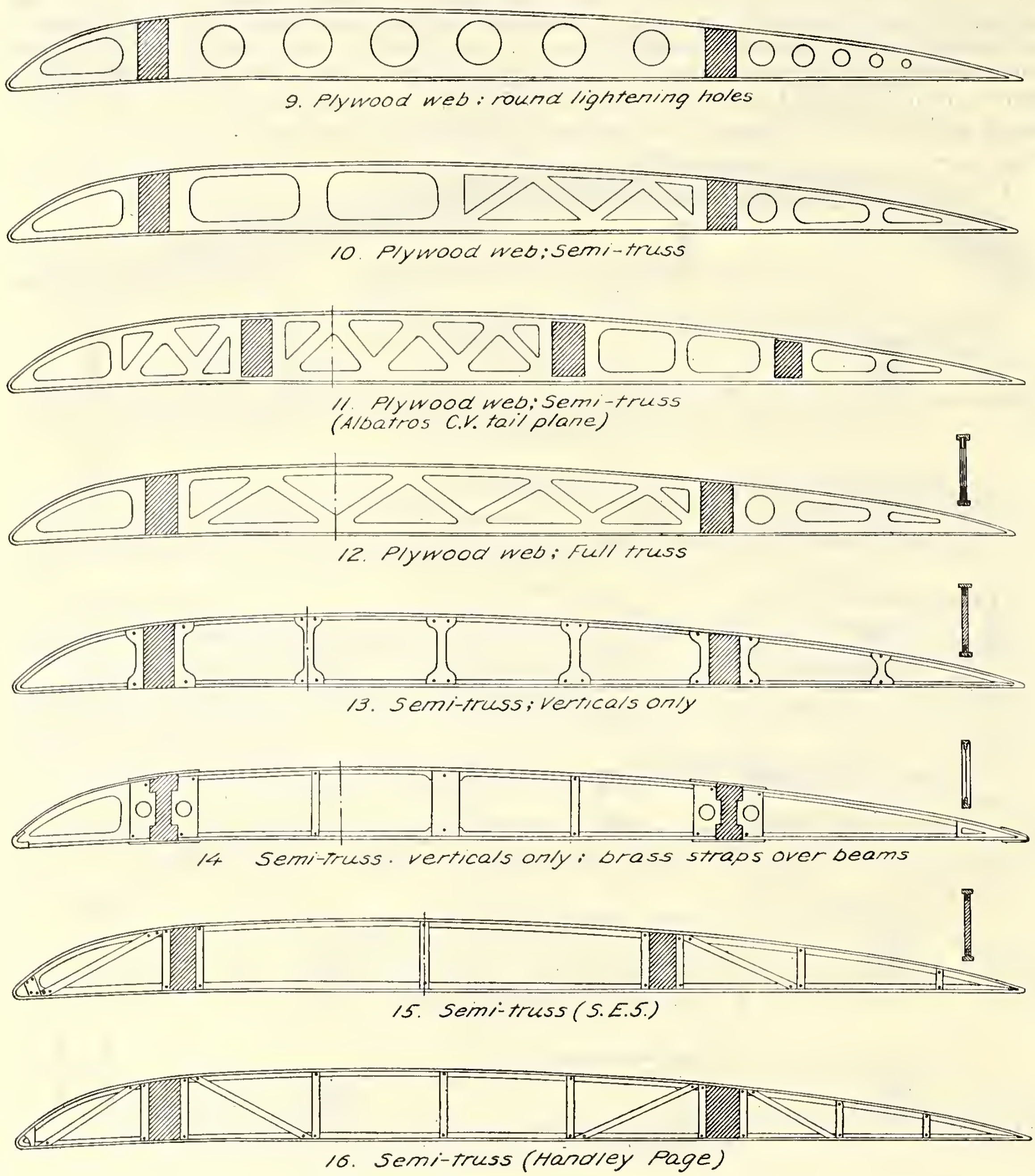

Fig. 56.—Typical wing-rib designs.

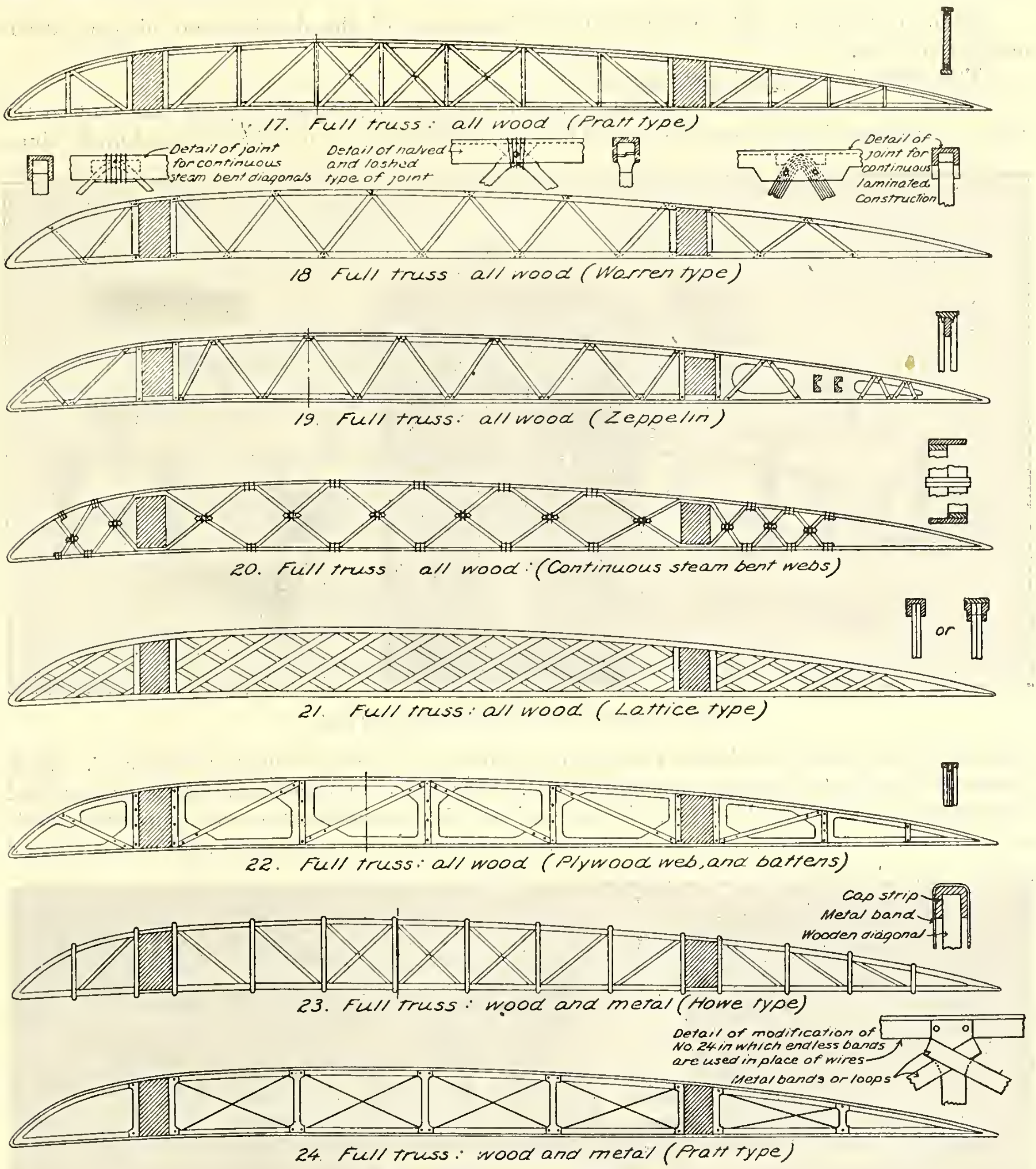

Fig. 56.—Typical wing-rib designs.

The two outstanding conclusions from the tests are: (1) The type of rib most suitable for small and medium chords, from the standpoint of the strength weight ratio combined with manufacturing ease, is the plywood web type, with oval and circular openings (fig. 56, case 8) and with vertical grain in the outer plies of the web.

(2) The type of rib most suitable for large chords is the full truss type. This has the greatest strength-weight ratio of all types, and the manufacturing difficulties are not overwhelmingly large in the case of large ribs.

Minor conclusions will be found in the discussions of the development of the various individual types.

The method of test is briefly as follows:

The ribs are mounted in a testing machine specially equipped to apply the load to the ribs at a number of points and the testing head is run down at a slow uniform speed until failure

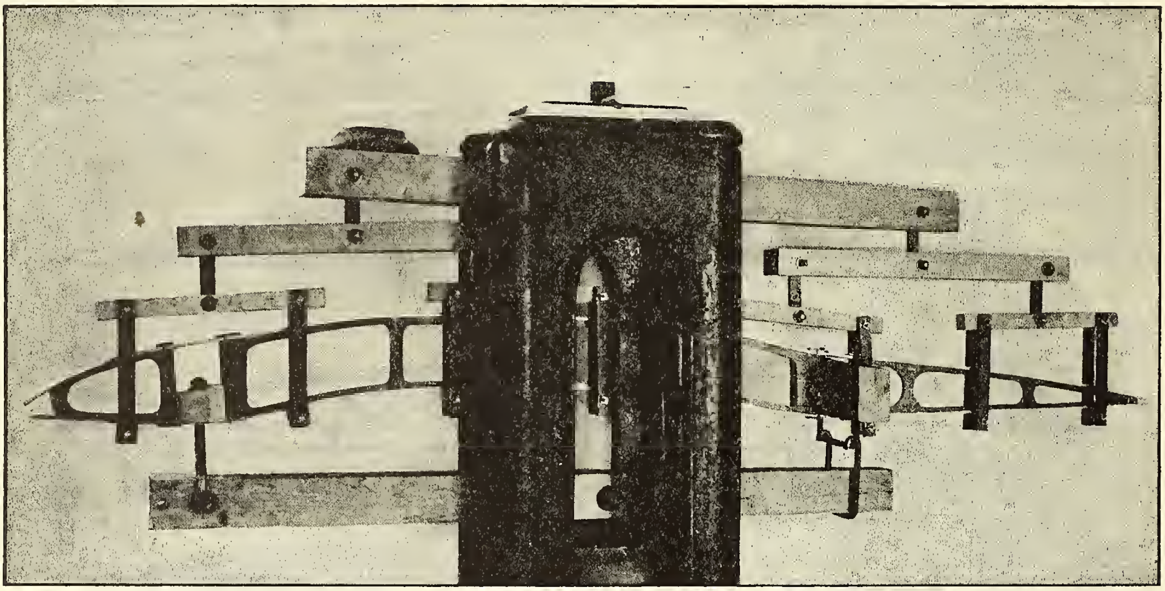

Fig. 57.—Apparatus for testing small wing ribs.

occurs. In the case of small ribs the load is applied at 8 points, as shown in figure 57. With the larger ribs 16-point loading is used (fig. 58). During the test the travel of the testing head is recorded at the various loads, and for some of the ribs the deformation at a number of points along the rib is measured. Figure 59 shows the relation between the total load in pounds and

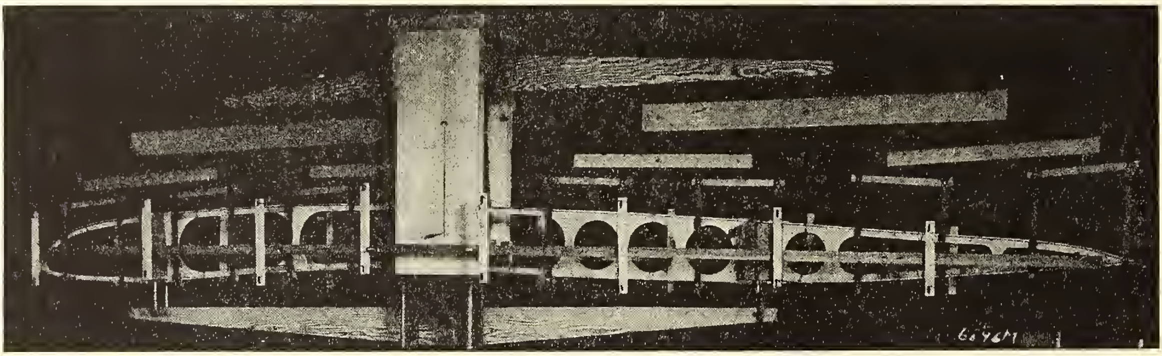

Fig. 58.—Apparatus for testing large wing ribs.

the travel of the testing head in inches. The strengthening and stiffening accomplished by judicious reinforcement are clearly shown.

The load distribution used in the first series of tests is shown in figure 60. Later a triangular distribution was adopted, in which the apex of the triangle is one-fourth of the chord from the leading edge (fig. 68).

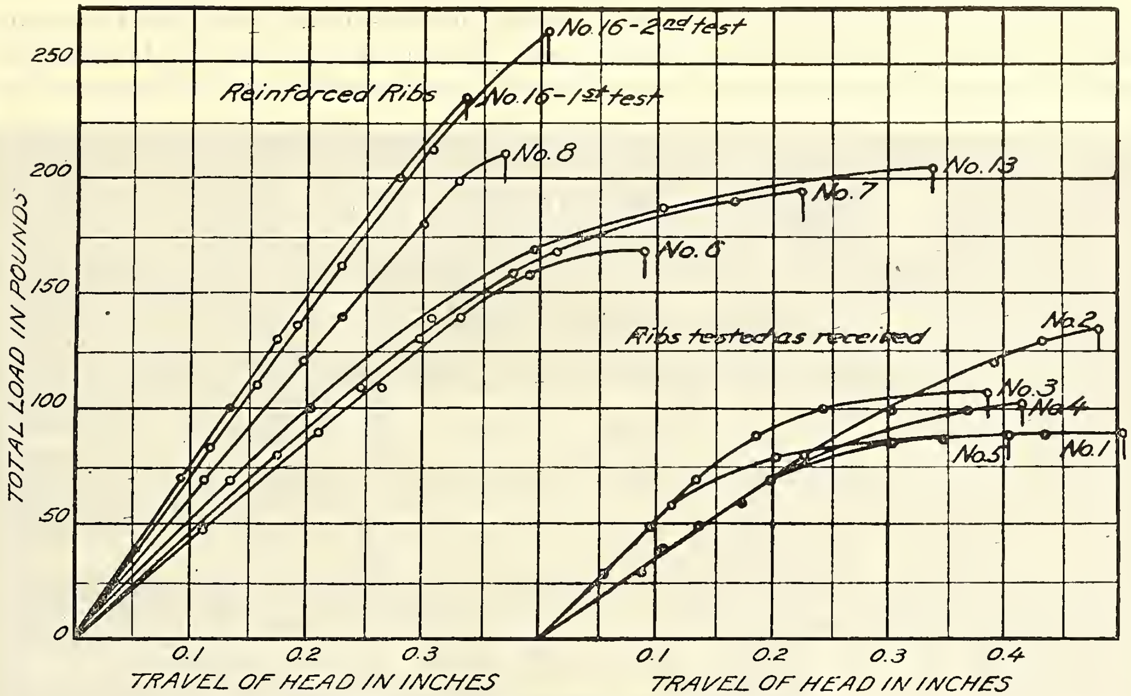

Fig. 59.—Wing rib load—deformation curves: DH-4 ribs.

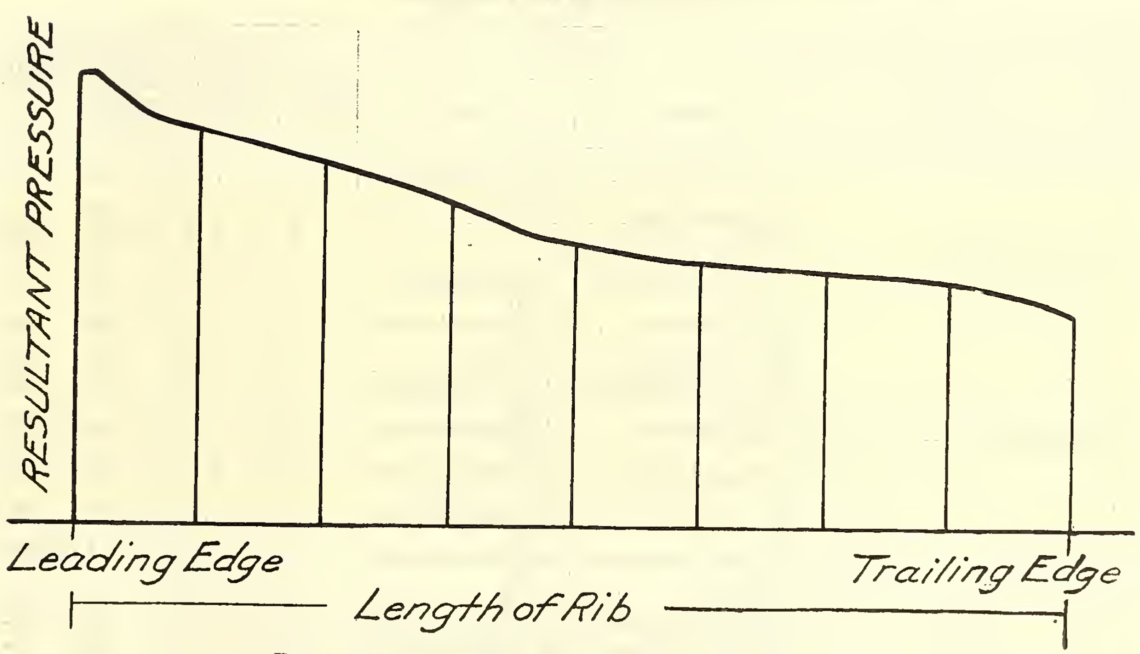

Fig. 60.—Low-speed load distribution used in wing rib tests.

TESTS ON DH–4 WING RIBS.

The first ribs upon which development work was undertaken were some DH–4 ribs submitted by one of the manufacturers. The original design is No. 1, figure 61. It was found that this rib, which has a plywood web, was lightened out too much near the spars, and the

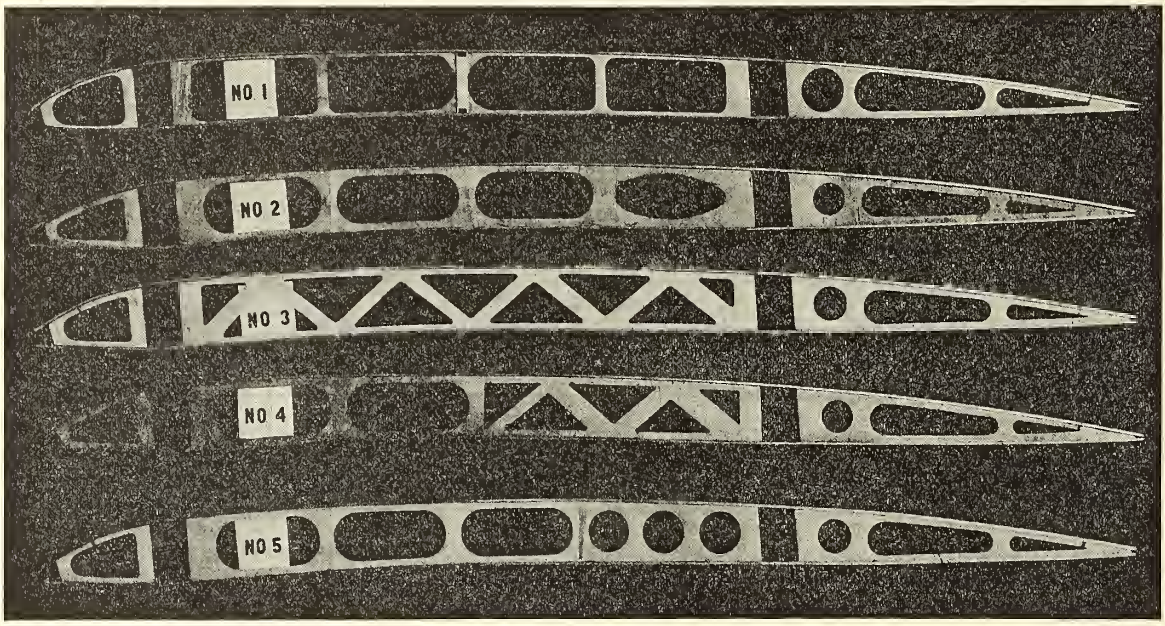

Fig. 61.—Tests on DH-4 wing ribs.

Rib. No.	Designation.	Description. Faces.	Description. Core.	Number of tests.	Net weight of rib, ounces, W.	Average total load sustained, pounds, P.	Ratio of strength to weight, $\frac{P}{W}$
1	Dayton-Wright	$\frac{1}{20}$-inch birch	$\frac{1}{16}$-inch yellow poplar.	4	7.71	136	17.7
		$\frac{1}{100}$-inch maple	do	2	5.23	232	44.4
2	Improved original	$\frac{1}{60}$-inch yellow poplar.	do	3	5.58	243	43.5
		$\frac{1}{64}$-inch Spanish cedar.	$\frac{1}{16}$-inch Spanish cedar.	3	5.26	274	52.1
		$\frac{1}{100}$-inch maple	$\frac{1}{16}$-inch yellow poplar.	2	5.59	232	41.5
		$\frac{1}{40}$-inch basswood	do	3	5.85	243	41.5
		$\frac{1}{64}$-inch Spanish cedar.	$\frac{1}{16}$-inch Spanish cedar.	5	5.06	253	50.0
3	Complete truss	$\frac{1}{70}$-inch birch	$\frac{1}{12}$-inch yellow poplar.	3	5.64	266	47.2
		$\frac{1}{40}$-inch basswood *	$\frac{1}{16}$-inch yellow poplar.	5	6.37	297	46.6
		do	$\frac{1}{12}$-inch yellow poplar.	4	6.12	300	49.0
4	Semitruss	$\frac{1}{60}$-inch yellow poplar.	$\frac{1}{16}$-inch yellow poplar.	4	5.46	274	50.2
		do	do	3	5.20	288	55.4
		$\frac{1}{40}$-inch basswood	do	3	5.61	325	57.9
5	Circular opening	$\frac{1}{70}$-inch birch	$\frac{1}{12}$-inch yellow poplar.	3	5.52	337	61.0
		$\frac{1}{40}$-inch Spanish cedar.	$\frac{1}{16}$-inch Spanish cedar.	2	5.40	346	64.0

* Core and face grain run parallel and perpendicular to diagonal members.

first improvement consisted in changing the shape and size of the lightening holes and incidentally reducing the weight by making the face veneer much lighter. The improvement in strength is shown in the last column of the table. Further development work led through the semitruss and full truss (plywood) designs to the design which was finally decided upon as the best obtainable (No. 5). This rib is shown drawn to scale in figure 62.

Several other types of DH-4 ribs were submitted for test, among them being several similar to case 13, figure 56. These were found to be very weak indeed, but stiffening and strengthening by means of wires, case 24, figure 56, produced a marked improvement. In fact, one rib developed as much as 42 pounds per ounce of weight.

Conclusions drawn from these tests, which included 150 ribs, are as follows:

(1) Plywood webs are superior to single-piece webs in strength, even if the latter are reinforced with vertical strips glued and nailed in position.

(2) Plywood webs with the face grain vertical are superior to plywood webs having the face grain longitudinal.

(3) Nails in the cap strips are practically useless in so far as contributing to the strength of the rib is concerned.

(4) Cap strips should be fastened rigidly to the spars.

(5) The circular-opening type of rib is superior to the other types tested.

(6) For the size of rib tested a core of one-sixteenth yellow poplar or Spanish cedar veneer with longitudinal grain is satisfactory. If high-density wood, like birch, is used for face veneer, the thickness should be from one-sixtieth to one-seventieth inch, while if low-density face veneer, such as yellow poplar, is to be used, a thickness of one-fortieth to one-fiftieth inch is required.

(7) Low-density face veneer is superior from the standpoint of manufacture of the plywood, and also gives somewhat greater stiffness for the same weight.

(8) Spruce cap strips $\frac{3}{16}$ by $\frac{7}{16}$ inch are satisfactory. They should be grooved and well glued.

TESTS ON SE-5 WING RIBS.

Table 17 presents the test data on a number of SE-5 ribs of the original design and of the design developed at the laboratory. The original ribs submitted for test were similar to case 15, figure 56, and consisted of 22 pieces. Under low-speed loading, figure 60, these ribs developed a strength of 25.3 pounds per ounce of weight, and under high-speed loading, figure 68, the average strength was 28.1 pounds per ounce of weight.

TABLE 17.—*Tests on SE-5 wing ribs.*

Rib number.	Type of rib.	Web construction.		Load distribution.	Net weight of rib, oz., W.	Total load sustained, lbs., P.	$\frac{P}{W}$
		Faces.	Core.				
Average of 1, 2, 6, 7	Original	Spruce braces and struts		Low speed	6.67	169	25.3
Average of 11, 12, 13, 14	do	do		High speed	6.59	185	28.1
Average of 3, 4, 5	Plywood No. 1	$\frac{1}{55}$-inch birch	$\frac{1}{16}$-inch basswood	Low speed	6.17	315	51.0
Average of 8, 9, 10	do	do	do	High speed	5.89	291	49.4
Average of 19, 20, 21, 22	Plywood No. 2	$\frac{1}{40}$-inch Spanish cedar.	$\frac{1}{16}$-inch Spanish cedar.	Low speed	4.61	270	58.5
Average of 15, 16, 17, 18	do	do	do	High speed	4.60	249	54.2
Average of 23, 24, 25, 26	do	do	do	Low speed	4.21	246	58.5
Average of 27, 28, 29, 30	do	do	do	High speed	4.23	275	65.0

Ribs No. 1 to 22, inclusive, loose in spars, bound by wire wound around rib at spars.
Ribs No. 23 to 30, inclusive, were glued to spars.
In all plywood web ribs the face grain was vertical.
Cap strips for ribs 1, 2, 6, 7, 11, 12, 13, and 14 were $\frac{3}{16}$ by $\frac{1}{2}$ inch spruce.
Cap strips for ribs 3, 4, 5, 8, 9, and 10 were $\frac{3}{16}$ by $\frac{7}{16}$ inch spruce.
Cap strips for ribs 15 to 30, inclusive, were $\frac{1}{8}$ by $\frac{7}{16}$ inch spruce.

The design finally developed is shown in detail in figure 63. Several types were made up, using different species and thicknesses of veneer in the web plywood. Of these the ribs having webs composed of one-fortieth-inch Spanish cedar faces and one-sixteenth-inch Spanish cedar core proved to be the strongest per unit of weight. The strength under low-speed loading was 58.5 pounds per ounce of weight, and under high-speed loading a strength of 65 pounds per ounce of weight was developed.

Besides being much stronger and lighter than the original ribs, the final design is decidedly stiffer.

TESTS ON HS WING RIBS.

The original HS ribs have a pine web, and are of the general type shown in case 4, figure 56. The final design is of the plywood web, oval and circular opening type, and is shown in detail in figure 64. Detailed results of the tests are presented in table 18. Attention is directed to the cap strips, which are patterned after the design used by Fokker in his recent biplane. Better cap-strip fastening is secured by this method when the web is thin. The basswood faces on the plywood web of the final design appear to be somewhat light, and it is anticipated that better results would be secured by the use of slightly heavier veneer.

TABLE 18.—*Tests on HS–1L wing ribs.*

Type of construction.	Load distribution.	Net weight of rib, ounces, W.	Total load sustained, pounds, P.	Ratio of strength to weight, $\frac{P}{W}$.	Remarks.
Present construction, single-ply pine web.	High speed....	16.80	410	....	$\frac{1}{4}$-inch, single-ply pine web; $\frac{1}{4}$ by $\frac{3}{4}$ inch spruce cap strips; 1 by $\frac{1}{16}$ inch stiffeners on each side of web between openings.
Do..........	.....do.........	17.28	440	....	
Do..........	.....do.........	16.31	350		
Average..........	.....do.........	16.80	400	24	
Present construction, single-ply pine web.	Low speed......	16.15	458	....	Construction same as above.
Do..........	.....do.........	16.31	530	....	
Do..........	.....do.........	16.80	590	....	
Average..........	.....do.........	16.42	526	32	
Circular opening, plywood web	High speed....	10.96	385	....	Ply-wood web; $\frac{1}{32}$-inch basswood faces; $\frac{1}{8}$-inch basswood core; $\frac{3}{16}$ by $\frac{1}{4}$ inch spruce cap strip on each side of web; grain of faces of web vertical.
Do..........	.....do.........	10.81	365	....	
Do..........	.....do.........	10.81	310	....	
Average..........	.....do.........	10.86	353	32	
Circular opening, plywood web	Low speed.....	10.68	490	....	Construction same as above.
Do..........	.....do.........	10.75	600	....	
Do..........	.....do.........	10.90	500	....	
Average..........	.....do.........	10.78	530	49	

TESTS ON F5–L WING RIBS.

The original F5–L ribs were of the general type of the HS ribs, case 4, figure 56. In developing the new ribs, it was thought that the use of a full truss type rib might be justified and, therefore, a rib of this type was designed and tested. Further, a truss type with plywood web was included in the series. The three designs are shown in figures 65, 66, and 67, and the results of the tests upon the three types with high-speed loading and low-speed loading are shown in table 19. Data on the strength of the original design are also included.

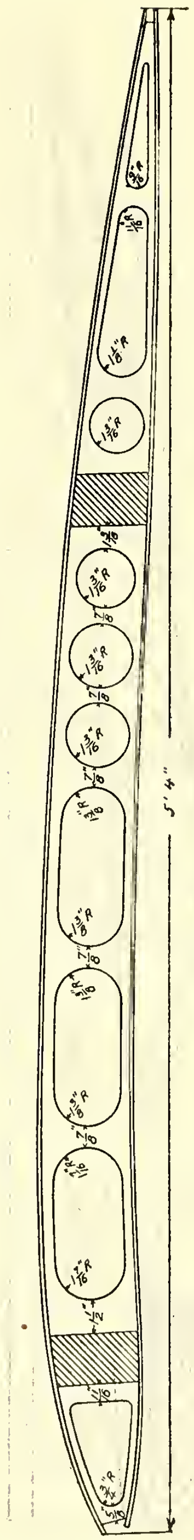

Fig. 62.—Circular opening type wing rib for DH-4 machine.

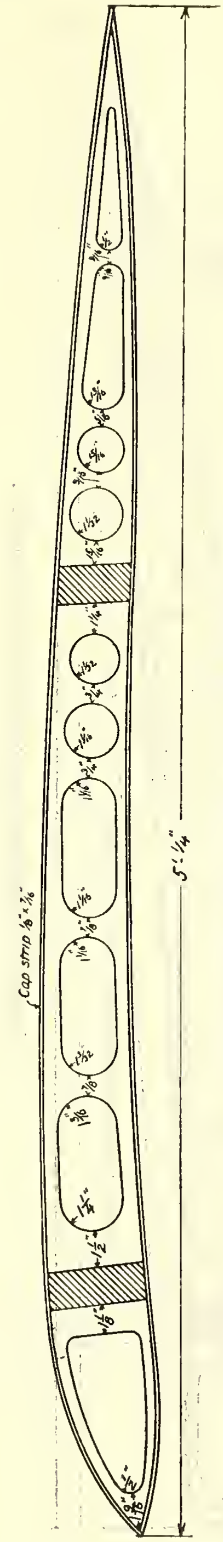

Fig. 63.—Circular opening type wing rib for SE-5 machine.

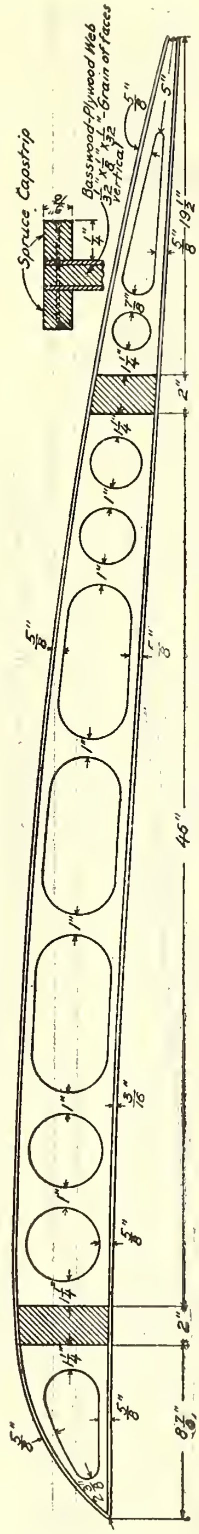

Fig. 64.—Circular opening type wing rib for HS-1L machine.

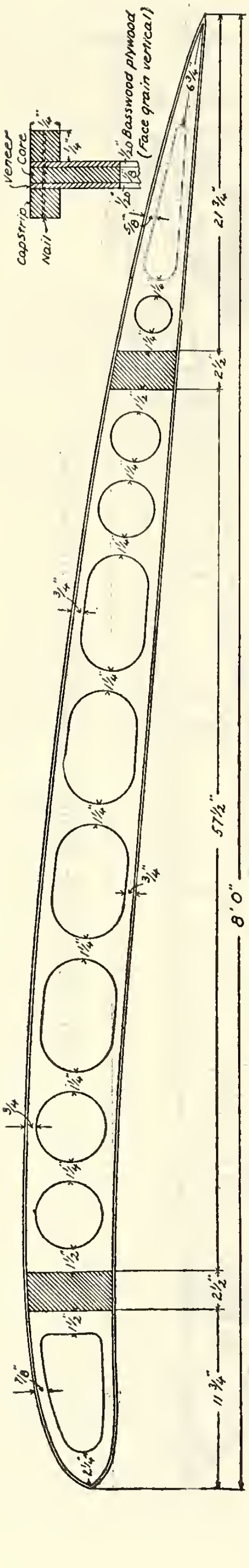

Fig. 65.—Circular opening type wing rib for F5-L machine.

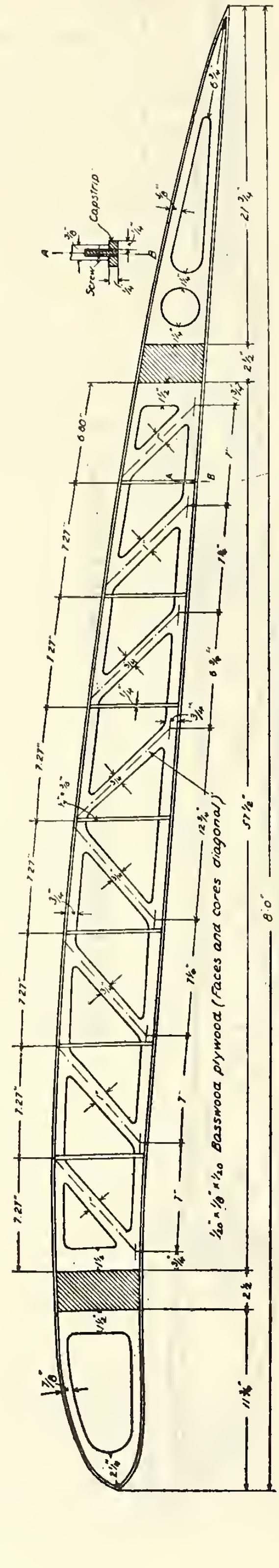

Fig. 66.—Plywood truss type wing rib for F5-L machine.

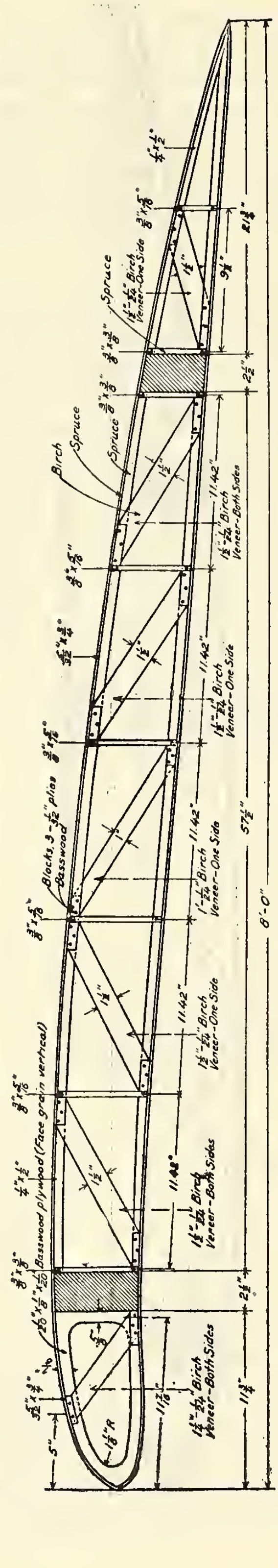

Fig. 67.—Pratt truss type wing rib for F5-L machine.

TABLE 19.—*Tests of F5–L wing ribs.*

No.	Design of rib.	Load distribution.	Net weight of rib, ounces. W.	Total load sustained, pounds. P.	Ratio of strength to Weight. $\frac{P}{W}$
1	Plywood, circular opening	High speed	15.5	540	....
2	.....do	.....do	15.5	485	....
3	.....do	.....do	15.7	400	....
	Average 1, 2, and 3		15.6	475	31
4	Plywood, circular opening	Low speed	15.5	592	....
5	.....do	.....do	15.7	498	....
6	.....do	.....do	15.5	642	....
	Average 4, 5, and 6		15.6	577	37
7	Plywood, truss	High speed	22.4	508	....
8	.....do	.....do	23.4	533	....
9	.....do	.....do	26.4	670	....
	Average 7, 8, and 9		24.0	570	23.7
10	Plywood, truss	Low speed	23.0	610	....
11	.....do	.....do	23.8	578	....
12	.....do	.....do	23.0	683	....
	Average 10, 11, and 12		23.3	624	26.7
13	Truss	High speed	12.5	580	....
14	.....do	.....do	12.5	505	....
15	.....do	.....do	12.3	520	....
	Average 13, 14, and 15		12.4	535	43
16	Truss	Low speed	12.5	665	....
17	.....do	.....do	12.9	710	....
18	.....do	.....do	12.6	610	....
	Average 16, 17, and 18		12.7	662	52
19	Original design	High speed	22.1	485	....
20	.....do	.....do	22.1	405	....
21	.....do	.....do	21.0	400	....
22	.....do	.....do	21.8	435	....
	Average 19, 20, 21, and 22		21.7	431	20
23	Original design	Low speed	22.8	593	....
24	.....do	.....do	23.4	585	....
25	.....do	.....do	23.5	550	....
26	.....do	.....do	24.2	590	....
	Average, 23, 24, 25, and 26		23.5	579	25

It will be seen from the data presented that the full truss type, figure 67, developed very much greater strength per unit weight than either of the other types and that the plywood truss type, figure 66, was by far the weakest of the three. Final choice between the full truss type and the plywood web type must be determined by the relative importance of weight saving and cost of production.

TESTS ON 15-FOOT WING RIBS.

The largest ribs so far tested have a 15-foot chord and were designed for a machine under contemplation but not yet built. Three general types of rib were first tested, a plywood web circular opening type, a semitruss type with reinforced plywood web, and a full truss type with vertical compression members and diagonal tension members (Pratt type). A glance at table 20 shows that the full truss was far superior to the other types in strength-weight ratio. The low-speed load distribution used is shown in figure 60 and the high-speed distribution in figure 68. The full truss design is shown in detail in figure 69. The stiffness of this design is illustrated in figure 70, which shows the relation between the travel of the testing head and the toal load in pounds. The uniformity in the properties of the three ribs is noteworthy. The need for thorough fastening of the cap strips and the verticals to the spars is emphasized.

TABLE 20.—*Tests on 15-foot wing ribs.*

No. of rib.	Type of rib.	Species of web.		Load distribution.	Cap strips.	Net weight of rib, pounds, W.	Total load sustained, pounds, P.	$\frac{P}{W}$ W= Weight in ounces.
		Faces.	Core.					
1	Circular opening	$\frac{1}{32}$-inch birch	$\frac{1}{16}$-inch Spanish cedar.	Low speed	$\frac{1}{4}$ by $\frac{3}{4}$ inch spruce.	2.42	251	6.5
2	do	$\frac{1}{45}$-inch birch	do	do	do	2.28	318	8.7
	Average values					2.35	285	7.6
7	Semitruss	$\frac{1}{30}$-inch birch	$\frac{1}{16}$-inch Spanish cedar.	High speed	$\frac{1}{4}$ by $\frac{3}{4}$ inch spruce.	2.92	286	6.1
8	do	$\frac{1}{45}$-inch birch	do	do	do	2.68	175	4.1
	Average values					2.80	231	5.1
10	Truss	Spruce compression members and web.		High speed	$\frac{3}{16}$ by $\frac{7}{8}$ inch spruce.	2.49	565	14.2
9	do	do		do	do	2.41	672	17.4
11	do	do		do	do	2.42	710	18.3
12	do	do		do	do	2.48	707	17.8
13	do	do		do	do	2.49	721	18.1
14	do	do		do	do	2.44	690	17.7
	Average values*					2.45	700	17.9

* (Rib No. 10 culled and omitted.)

After this series of tests was completed it was thought desirable to develop a truss type of rib which did not depend so largely upon glue for the security of the fastenings, and so a rib of the Warren type was designed and three built and tested. The design is shown in figure 71 and table 21 presents the results of the tests and also the results of the previous tests on the Pratt type for comparison. While the objects aimed at were attained, it was at the sacrifice of considerable weight, as will be seen from an inspection of the table. Tests have just been completed upon a number of modified ribs of the Warren type. These ribs showed a greater strength-weight ratio than any other 15-foot ribs tested at the laboratory.

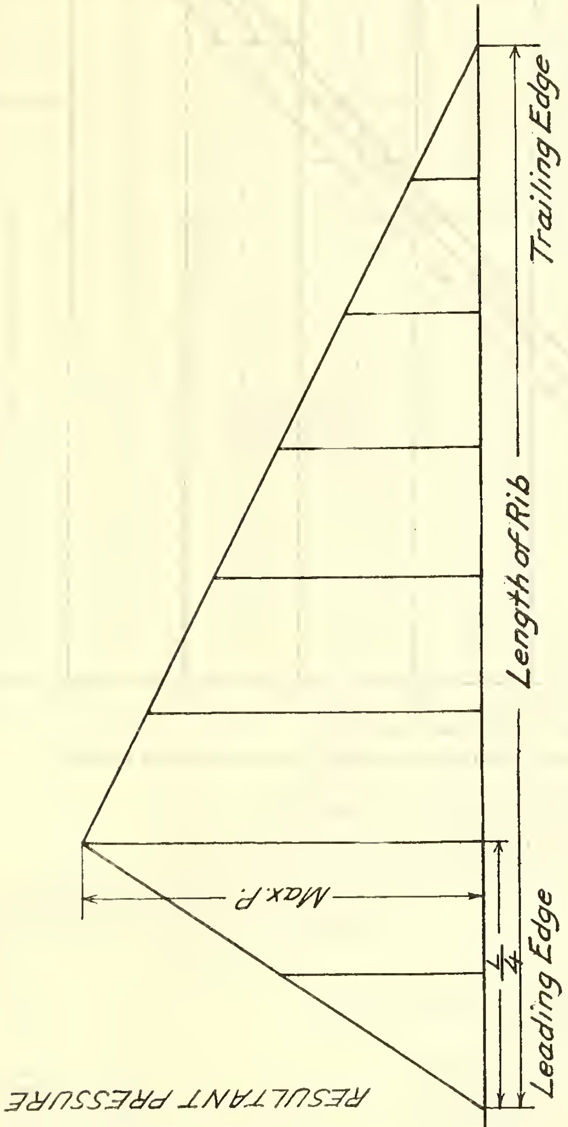

Fig. 68.—High-speed load distribution used in wing rib tests.

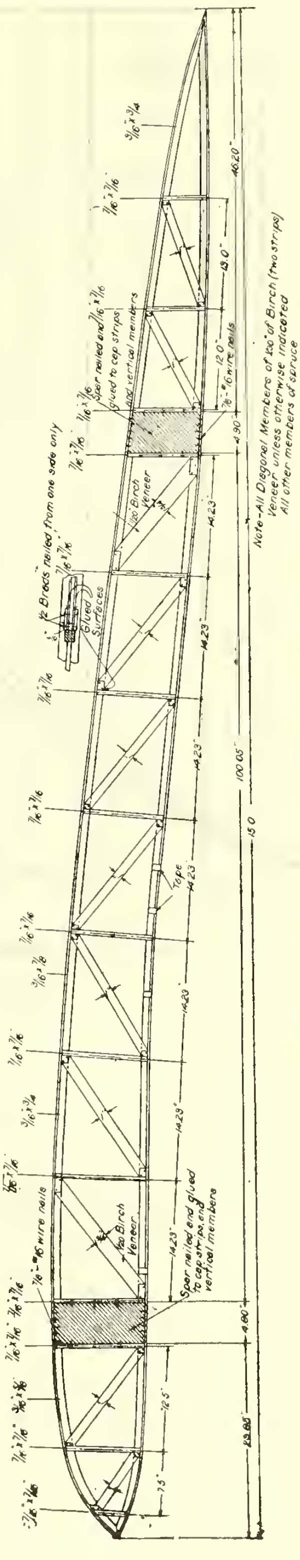

Fig. 69.—Pratt truss type wing rib for 15-foot chord. Modified R. A. F. 15 aerofoil.

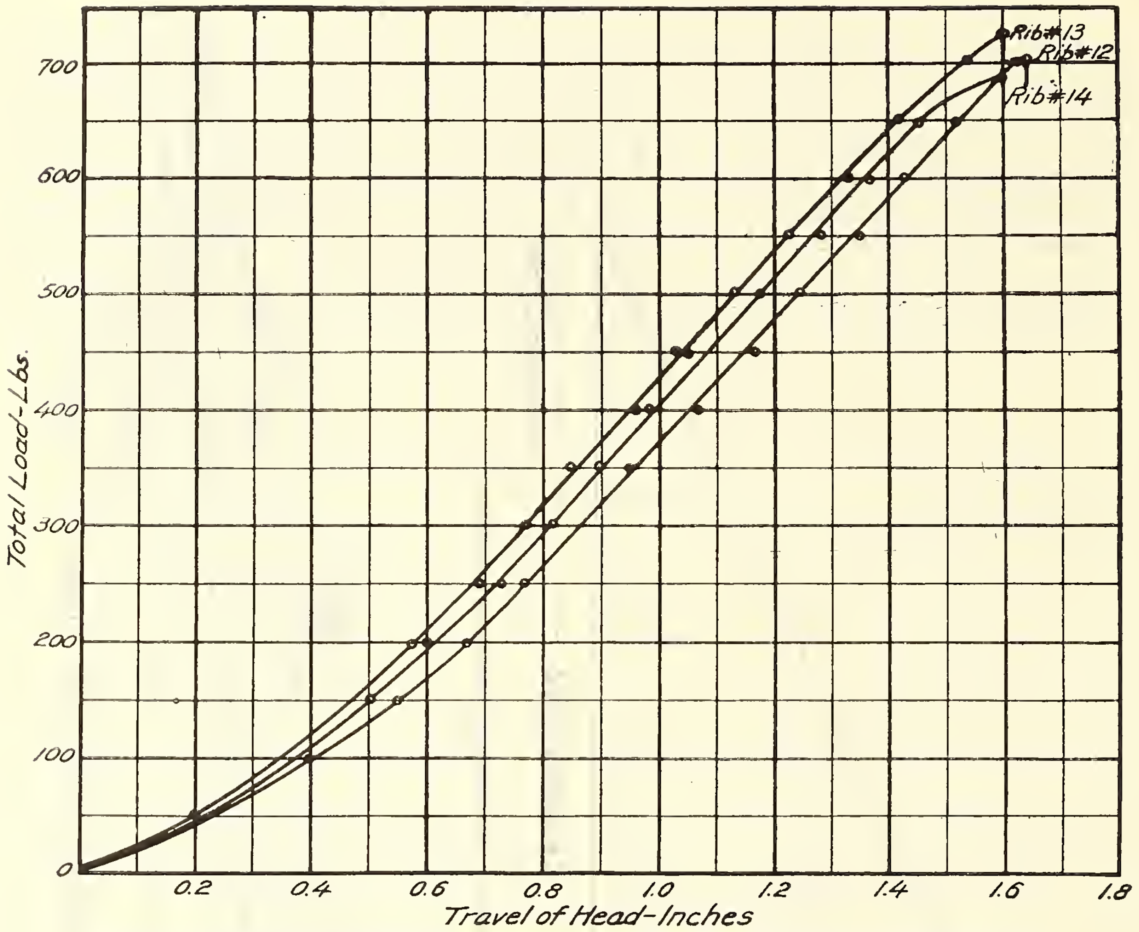

Fig. 70.—Wing rib load-deformation curves: Pratt truss type ribs for 15-foot chord machine.

TABLE 21.—*Tests on 15-foot wing ribs.*

Pratt truss and Warren truss type.

No. of rib.	Type of rib.	Construction.	Cap strips.	Net weight of rib pounds, W.	Total load sustained, pounds, P.	$\frac{P}{W}$ W=weight in ounces.
9	Pratt truss	Spruce compression members and birch veneer tension members.	$\frac{3}{16}$ by $\frac{7}{8}$ inch spruce	2.41	672	17.4
11	do	do	do	2.42	710	18.3
12	do	do	do	2.48	707	17.8
13	do	do	do	2.49	721	18.1
14	do	do	do	2.44	690	17.7
	Average values			2.45	700	17.9
15	Warren truss	Plywood members	Spruce channel, see sketch	3.72	770	12.9
16	do	do	do	3.54	855	15.1
17	do	do	do	3.63	830	14.3
	Average values			3.63	850	14.1

Ribs tested with high speed load distribution.

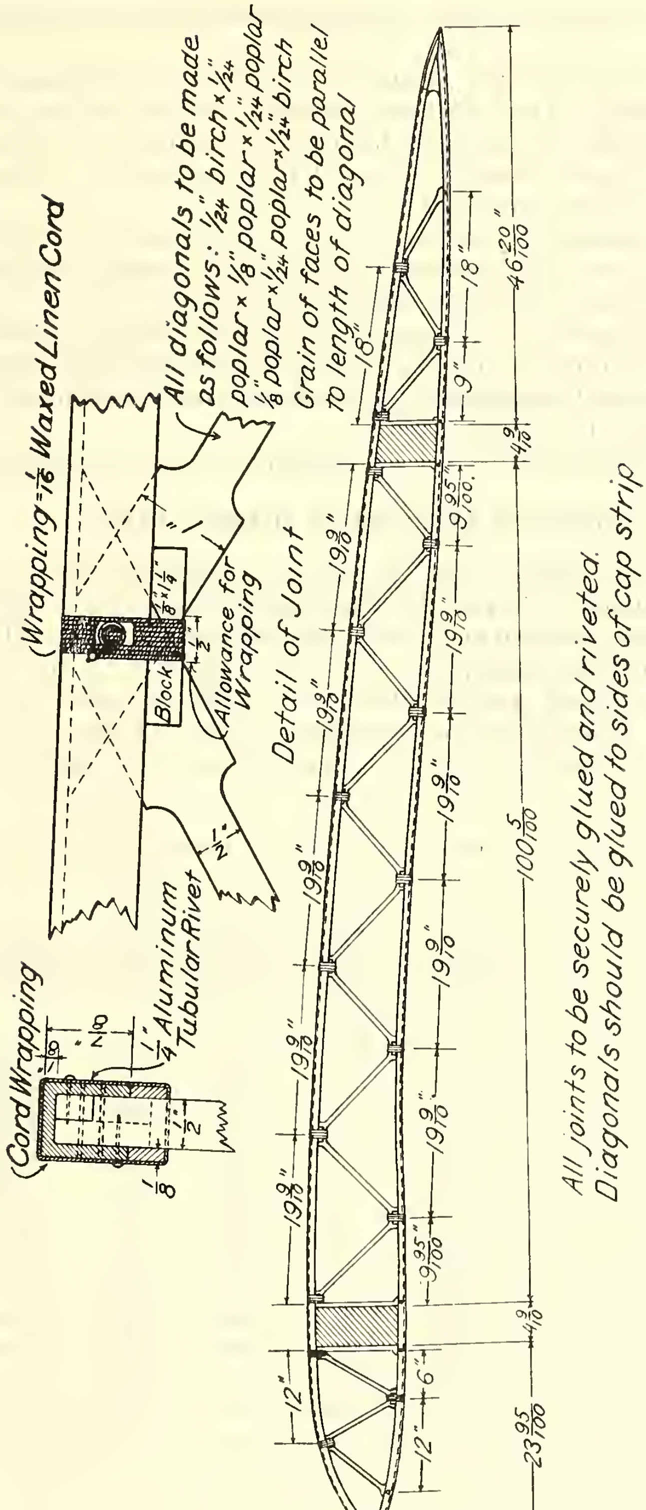

Fig. 71.—Warren truss type wing rib for 15-foot chord machine.

In addition to these types of 15-foot rib experiments were made upon three other types, as follows:

1. Full truss type, with vertical compression members and diagonal tension members running in both directions. These diagonal members consisted of two birch veneer bands wrapped continuously around the whole rib from end to end, one to the right and the other to the left. These bands passed around the caps at the panel points. These ribs developed a strength of 13 pounds per ounce of weight.

2. Full truss type, similar to 1, except that the veneer bands, instead of passing around the caps, passed between the caps and ends of the verticals, being given a twist at this point. The strength developed was 9 pounds per ounce of weight.

3. Full truss type, similar to 1, except that the veneer bands, instead of passing around the caps, were cut at these points and glued to the sides of the caps, which were of channel section. This type developed a strength of 13 pounds per ounce of weight and has the advantage of greater ease of assembly than types 1 and 2.

It is to be noted that none of these types developed as great strength as either the Pratt or Warren types.

TESTS ON ELEVATOR OR AILERON SPARS.

Comparatively little is known about the behavior of wood under torsion. This has not been of particular importance in the past, but the proper design of control surface spars demands such knowledge. Mention has been made, under Mechanical and Physical Properties of Wood, of a few torsion tests made on solid specimens of spruce and ash. A few tests have also been made on hollow dummy control spars of Sitka spruce. The individual results of the tests are given in table 22, and a comparison between these results and those on the solid specimens previously mentioned is shown in table 23. Details of the test specimens will be found in figure 72.

TABLE 22.—*Individual results of torsion tests on 15 hollow Sitka spruce elevator spars.*

Specimen No.	Moisture, per cent of oven-dry weight	Specific gravity (oven-dry weight and oven-dry volume).	Shearing stress at elastic limit (pounds per square inch).	Shearing stress at maximum load (pounds per square inch).	Shearing modulus of elasticity (pounds per square inch).	Work to elastic limit (inch pounds per cubic inch).	Work to maximum load (inch pounds per cubic inch).
1	12.6	0.44	500	1,000	92,100	1.12	7.1
2	15.0	.48	820	1,370	83,300	3.38	15.5
3	13.5	.38	950	1,000	79,500	4.75	6.4
4	12.8	.48	610	780	88,900	1.72	....
5	15.0	.45	930	1,260	76,100	4.74	11.4
6	14.2	.51	820	1,170	77,700	3.62	10.5
7	14.6	.34	710	940	55,300	3.84	9.1
8	13.2	.43	910	1,270	75,900	4.54	13.8
9	14.8	.50	820	1,070	77,800	3.62	8.0
10	13.6	.47	820	1,030	83,400	3.38	6.9
11	12.0	.52	740	1,040	73,800	3.06	7.3
13	13.4	.48	....	1,400	....	....	....
14	15.2	.37	910	1,350	80,600	4.27	15.8
15	13.4	.43	840	1,080	71,900	4.13	9.5
16	14.3	.49	....	970	....	....	....
Average	13.8	.455	800	1,110	78,200	3.55	10.11

Table 23.—*Summary of results of torsion tests on hollow Sitka spruce elevator spars and tests on solid circular specimens.*

	Tests on 15 hollow elevator spars, Sitka spruce (1).	Tests on 15 solid circular specimens Sitka spruce, (2).	Ratio of (1) to (2) in per cent.
Moisture, per cent of oven-dry weight	13.8	15.7	88
Specific gravity, based on oven-dry weight and oven-dry volume	0.46	0.39	118
Shearing stress at elastic limit (pounds per square inch)	[1] 800	1,090	73
Shearing stress at maximum load (pounds per square inch)	1,110	1,650	67
Shearing modulus of elasticity (pounds per square inch)	[1] 78,200	72,300	108
Work to elastic limit (inch pounds per cubic inch)	[1] 3.6	4.4	82
Work to maximum load (inch pounds per cubic inch)	[1] 10.1	19.7	51

[1] Based on 13 tests.

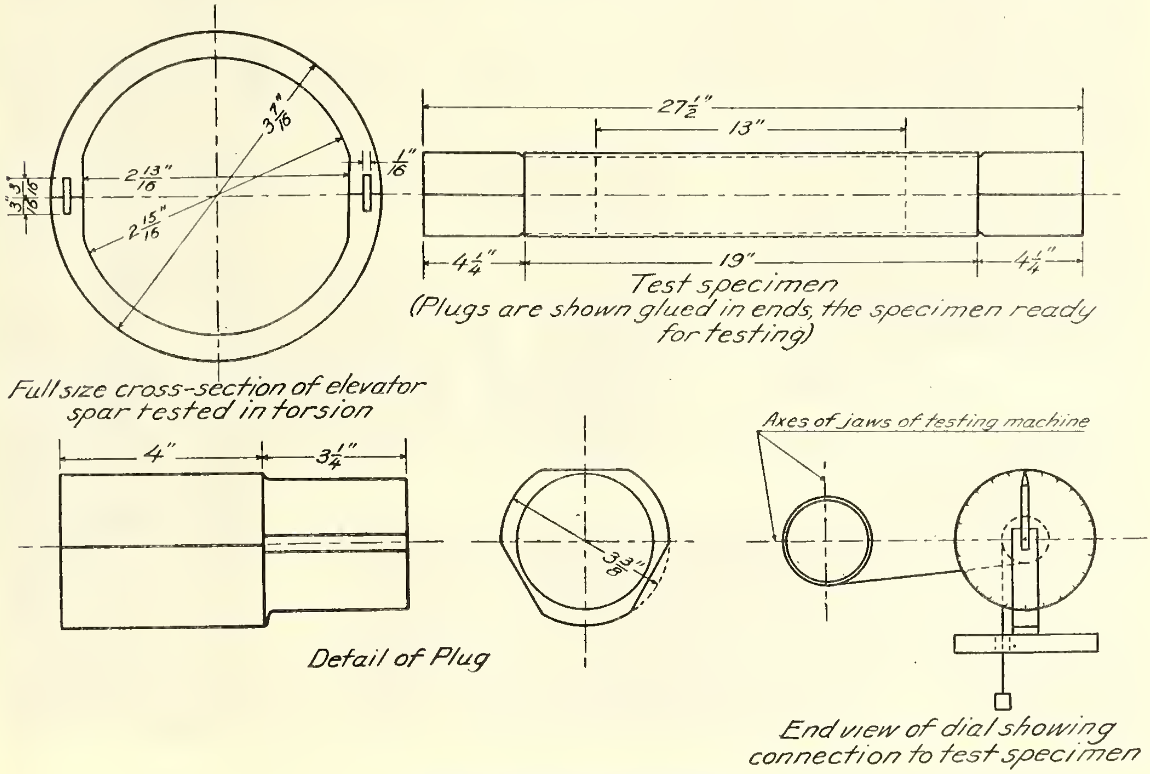

Fig. 72.—Torsion test specimen.

It is to be noted that 80 per cent of the specimens failed at or near the spline joint, indicating that the joint was a source of weakness in the specimens.

The relation between specific gravity and strength in shear is not definite enough to be used as a basis for selection of material to withstand shearing stresses.

These tests, as well as torsion tests in general, are subject to large variations. These variations are probably more pronounced in hollow spliced construction and will therefore necessitate using very large safety factors in order to obtain safe working stresses.

In addition to the tests already mentioned, a few tests have been made upon hollow spars with a hollow wooden core, around which veneer is wrapped in right and left spirals. The indications are that both the ultimate strength in torsion and torsional stiffness can be doubled by this method of construction.

TESTS ON AIRCRAFT ENGINE BEARERS.

A short series of tests was made to determine the relative merits of engine bearers built of all veneer and those built with a spruce filler. A preliminary series indicated the desirability of making a few modifications in the arrangement of the material which were embodied in the bearers here reported. The details of the veneer and spruce filler types are shown in figures 75 and 76, respectively, and the methods used in thrust loading and in vertical loading are illustrated in figures 73 and 74, respectively. The results of the tests are shown in table 24:

TABLE 24.—*Tests on modified engine bearers (second series).*

Engine bearers No.	Type.	Weight, pounds.	Moisture content at test.	Deflection at maximum thrust load, in inches.	Maximum thrust load, in pounds.	Deformation at maximum vertical load, in inches.	Maximum vertical load, in pounds.
1	All-veneer (grain of faces horizontal)	6.88	13.0	2.81	1,430	0.63	11,560
2		7.27	13.4	1.75	1,360	.61	12,260
3		7.30	13.2	2.25	1,580	.49	11,800
	Average	7.15	13.2	2.27	1,457	.58	11,873
4	All-veneer (grain of faces vertical)	7.54	12.8	2.12	1,940	.40	11,360
5		7.11	11.8	2.42	1,850	.56	11,540
6		7.40	13.6	1.65	2,000	.45	10,500
	Average	7.35	12.7	2.06	1,930	.47	11,133
7	Plywood with spruce filler (grain of faces horizontal).	7.10	12.3	2.04	1,850	.38	12,500
8		7.14	12.5	1.82	1,720	.48	17,000
9		7.26	12.2	1.84	1,690	.50	16,000
	Average	7.17	12.3	1.90	1,753	.45	15,167
10	Plywood with spruce filler (grain of faces vertical).	7.12	11.6	1.88	1,960	.60	16,500
11		7.20	11.7	1.62	1,910	.49	16,500
12		7.02	12.1	2.45	2,150	.67	15,430
	Average	7.11	11.8	1.98	2,007	.59	16,143

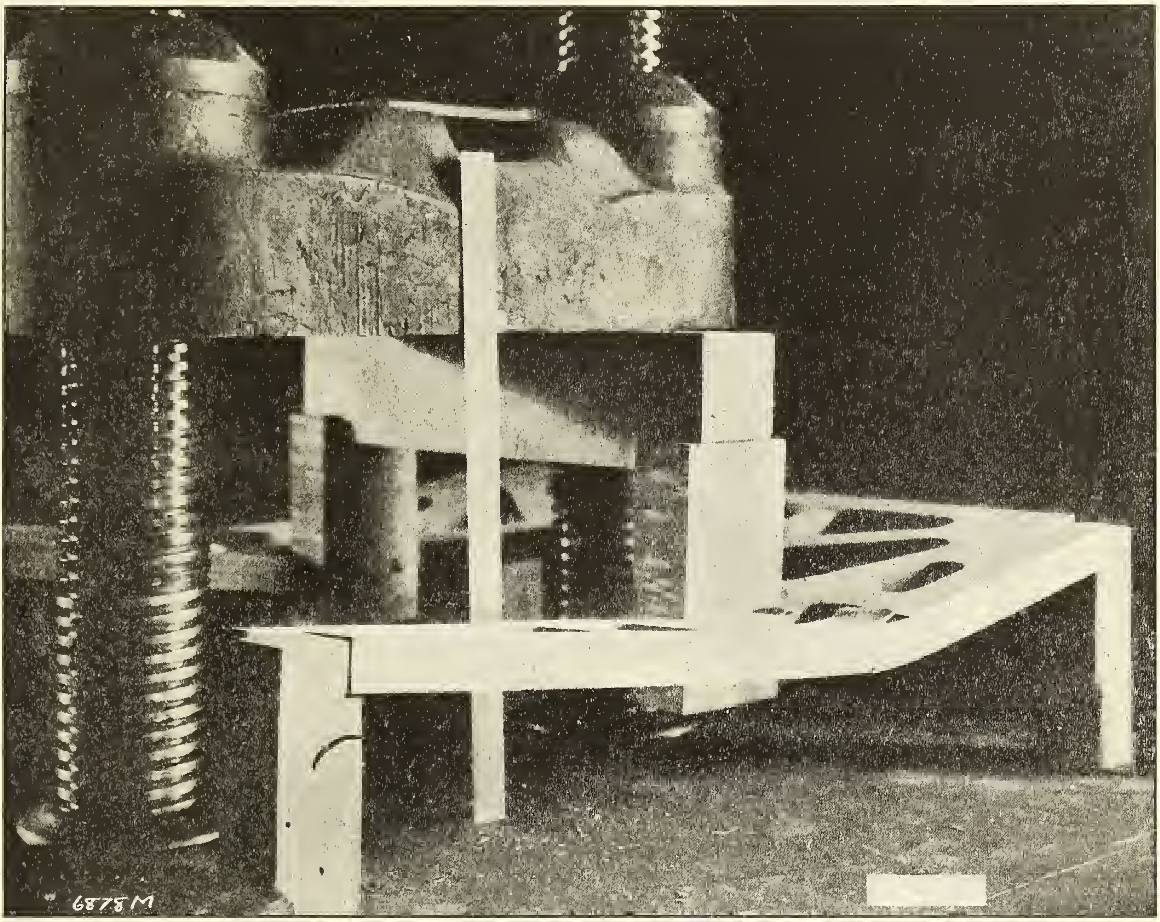

Fig. 73.—Strength tests of engine bearers: Method of testing for thrust loading.

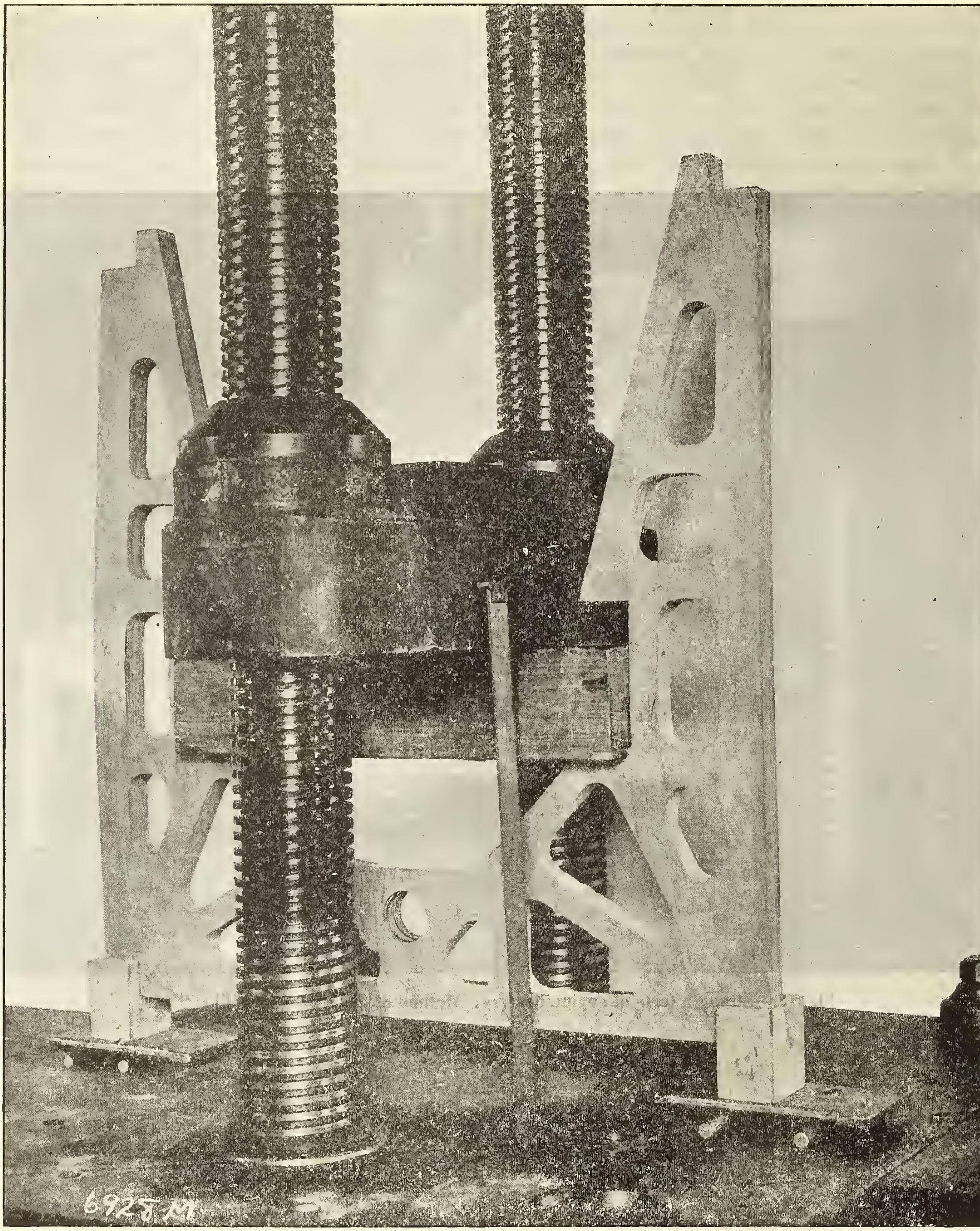

Fig. 74.—Strength tests of engine bearers: Method of testing for vertical loading.

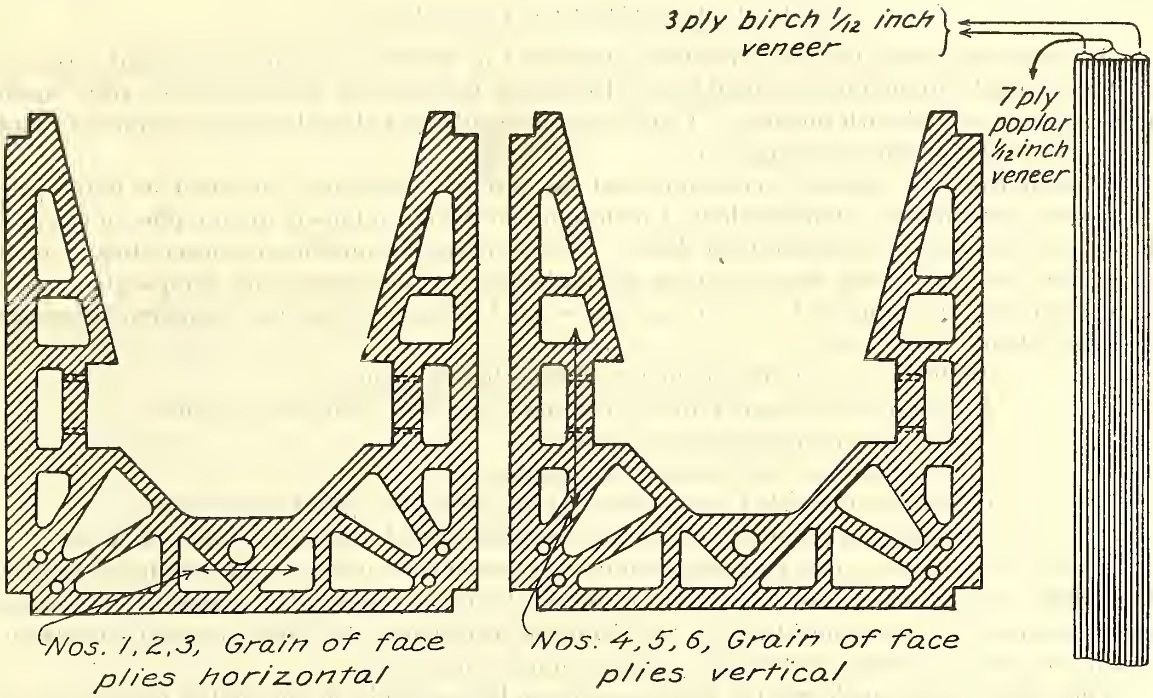

Fig. 75.—Engine bearers, all-veneer type.

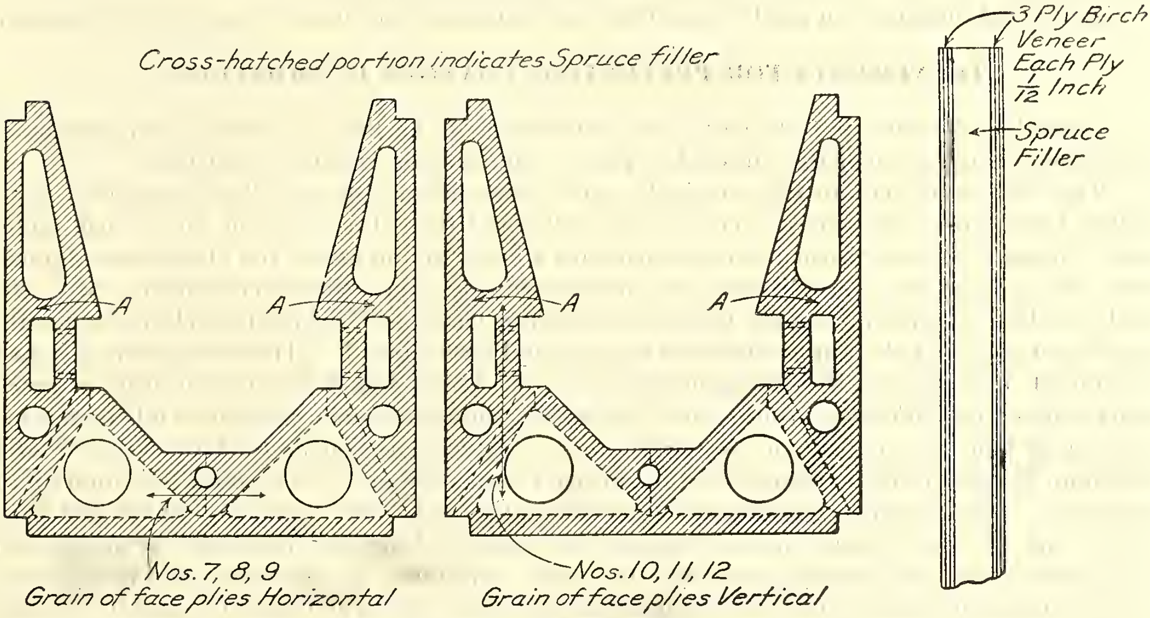

Fig. 76.—Engine bearers, spruce-filler type.

It is to be noted that the spruce-filler type was somewhat superior to the other in thrust loading and much superior in vertical loading; also that the bearers of the former type with the face grain of the plywood vertical were superior to those with the face grain horizontal. It may be mentioned that these particular bearers were designed to take vertical loading only.

TESTS ON BAKELIZED CANVAS (MICARTA).

In connection with tests on substitutes for wood in aircraft construction several series of tests were made upon micarta members. Reference has already been made to tests upon spruce struts covered with micarta. Tests were also made on hollow longerons of micarta and on a few samples of micarta wing spars.

Several tubes of micarta were submitted for test as substitutes for wood in longerons. These tubes were hollow, 36 inches long, 1 inch square outside, and made up of 6 plies of canvas, the walls being about one-eighth inch thick. For comparison a number of spruce sticks 1 inch square and 36 inches long were cut from a plank selected at random and comparative tests made upon the tubes and sticks. The tests show the following properties, compared to spruce (moisture about 10 per cent):

1. Modulus of elasticity about two-thirds that of spruce.
2. Fiber stress at elastic limit in bending about four-fifths that of spruce.
3. Tensile strength half that of spruce.
4. Specific gravity three times that of spruce.
5. Compression parallel to the grain, elastic limit, one-half that of spruce.
6. Compression parallel to the grain, maximum load, three times that of spruce.

Impact tests upon several longerons and impact tests made upon several samples of I-beam and hollow section wing spars of micarta indicate that this material is superior to average spruce (about 10 per cent moisture) in the following properties: (*a*) Fiber stress at the elastic limit in impact; (*b*) elastic resilience in impact (much superior).

The cause of the much greater elastic resilience lies not only in the higher fiber stress at the elastic limit but also in the lower modulus of elasticity in impact.

In general, micarta can not be considered as a substitute for spruce in aircraft construction.

TREATMENTS FOR PREVENTING CHANGES IN MOISTURE.

Several long series of tests have been conducted in the hope of finding some means of preventing changes of moisture in finished parts with changing weather conditions.

The first series had to do principally with varnishes of the so-called waterproof type. Yellow birch blocks were given a coat of silex filler and then three coats of the varnish under test. In some cases the varnish was applied with a brush and in others the blocks were dipped. Some of the specimens were dried in the air between coats and others were baked. After the final coat had set the blocks were hung in a humidity chamber in which the relative humidity was 95 per cent and the temperature between 75 and 80 degrees F. The blocks were weighed at intervals to determine the absorption of moisture. It was found that the absorption varied widely among the different varnishes and that baking improved some varnishes while increasing the rate of moisture transmission through others. The absorption in 17 days varied from a minimum of 4.36 grams per square foot of surface to a maximum of 26.8 grams per square foot of surface. The specimen showing the least absorption happened to be one which had been dipped and air dried, while the one showing the greatest absorption happened to be brushed and baked. The tests showed not only the great variability in moisture resistance among good varnishes but showed also that the moisture resistance was in all cases increased by increasing the number of coats of varnish applied. Table 25 shows the absorption of water by specimens given various miscellaneous treatments. The absorptions at 17 days are comparable with the figures just quoted. None of the treatments furnished the desired water resistance.

TABLE 25.—*Humidity tests of miscellaneous treatments.*

Wood, yellow birch: Average thickness, 0.6 inch; average width, 4 inches; average length, 8 inches; average surface area, 0.54 square foot; average weight, 0.49 pound air dry; average volume, 0.011 cubic foot.

Treatment.	Number of specimens averaged.	Average absorption in grams per square foot of surface in—		
		3 days.	10 days.	17 days.
1. Muslin glued with Le Page's Cold Glue, 4 coats of airplane dope, and 2 coats of airplane gray enamel	2	1.85	4.36	7.39
2. Three brush coats of orange shellac	2	1.68	4.49	7.98
3. Paste filler (silex), 1 coat airplane gray undercoat, 3 coats airplane gray enamel (Adams & Elting Co.)	2	1.80	4.79	8.23
4. Paste filler (silex), 2 brush coats orange shellac, 2 brush coats Lowe Bros. Finishing Varnish V 801	2	1.43	5.36	8.30
5. Paste filler (silex), 2 coats of Hampden's W. P. Varnish No. 1 and 1 coat of Lowe Bros. Marine Spar	2	1.55	5.11	8.35
6. Paste filler, 2 coats of white lead, linseed oil, and lampblack, 1 coat of rubbing varnish, 4 coats of spar varnish	5	2.60	6.21	10.23
7. Two brush coats of orange shellac and 2 brush coats of Lowe Bros. Finishing Varnish V 801	2	1.97	6.75	10.36
8. Two brush coats of orange shellac and 3 brush coats of Lowe Bros. Finishing Varnish V 801	2	2.22	5.79	9.93
9. Wood dyed alternating the two following solutions: No. 1—100 gr. aniline hydrochloride, 40 gr. ammonium chloride, 650 gr. water. No. 2—100 gr. copper sulphate, 50 gr. potassium chlorate, 615 gr. water; washed with soap and water and thoroughly rubbed with vaseline; 3 coats of Lowe Bros. Marine Spar Varnish were added	2	4.20	10.31	15.44
10. Four brush coats of Toch Bros. 1017 Marine Varnish thinned with turpentine	2	3.56	11.90	17.88
11. One-half hour vacuum and 1 hour atmospheric pressure (Special Varnish, Adams & Elting Co.)	1	4.86	13.40	20.7
12. One-half hour vacuum and 1 hour atmospheric pressure (Toch Bros. No. 1017 M. S. Preservative	2	6.04	16.72	25.4
13. Hot and cold treatment with paraffin dissolved in gasoline	2	6.55	23.9	....
14. Five applications of hot boiled linseed oil and 2 coats of prepared wax, each coat applied at intervals of not less than 4 hours and each thoroughly rubbed	2	14.0	36.7	47.6
15. Same as 9 except that no varnish was applied	1	21.6	....	....
16. Plain yellow birch panels, no treatment	10	20.5	42.5	51.9

Some conclusions not already mentioned follow:

(1) A more effective coating may be secured by dipping than by hand brushing.

(2) Cellulose varnishes are not as durable as oil varnishes.

(3) Linseed oil and wax treatments are not effective in keeping out moisture.

(4) All the varnishes tested were somewhat affected by water, including those that do not turn white as well as those which do.

(5) Very resistant coatings may be secured by using certain rubbing varnishes followed by top coats of spar varnish as a protection, also by using certain linseed oil varnishes, covered by a more durable China wood oil varnish.

Tests were also conducted on electroplated metal coatings and on vulcanized rubber coatings. Both of these types of coating are extremely resistant to the penetration of moisture, so long as they remain intact. The metal coating in particular, however, is rather delicate and does not adhere to the wood. The vulcanized rubber coatings were about an eighth of an inch thick and would probably be quite satisfactory from the standpoint of durability.

Of all the coatings upon which experiments were made, an aluminum leaf coating appears to be the most satisfactory from the standpoint of resistance to moisture penetration combined with general feasibility. This coating consists, in effect, of aluminum leaf laid over the surface between layers of varnish, just as sign painters lay on leaf over size. The leaf itself has no wearing strength and the coating has just the durability and wear resistance of the coats of varnish and enamel placed over the leaf. The resistance of the leaf coating to the passage of moisture is very remarkable indeed, as will be seen from a study of table 26 and figure 81, which present comparable data on several kinds of aluminum leaf coatings and several common kinds of finish.

TABLE 26.—*Humidity tests of metal leaf coatings.*

Treatment.	Number of specimens averaged.[1]	Average absorption in grams per square foot of surface for—				
		3 days.	10 days.	17 days.	24 days.	31 days.
Silex filler, gold size, aluminum leaf, and 3 coats of E. P. black lacquer	2	−0.210	0.084	0.168	−0.042	0.461
Silex filler, 1 coat of rubbing varnish, gold size, imitation gold leaf, 1 coat Valspar	3	0.218	0.445	0.805		
Silex filler, 1 coat of rubbing varnish, gold size, aluminum leaf, 1 coat Valspar	1	0	0.252	0.420		
Silex filler, 3 brush coats Hampden's waterproof varnish No. 2	10	1.28	4.56	7.29		
5 applications of linseed oil applied hot and 2 coats of wax	2	14.0	36.7	47.6		
No treatment	10	20.5	42.5	51.9		

[1]*Average data on yellow birch panels.*

Thickness	inch	0.60
Width	inches	3.960
Length	do	8.000
Surface	square feet	.540
Weight (air-dry)	pound	.490
Volume	cubic feet	.011

It has been found, in actual practice, that the process is entirely workable, and very good results have already been secured from its use.

The following instructions explain in detail the method of applying aluminum leaf to propellers. The same method could be used in coating other aircraft parts if it were found desirable to do so.

INSTRUCTIONS FOR APPLYING ALUMINUM LEAF TO AIRCRAFT PROPELLERS.

The leaf used in this process is exceedingly thin and light, there being probably 12,000 to 15,000 leaves per inch which makes it appear difficult to handle. If the instructions are carefully followed, however, the leaf may be easily and thoroughly applied.

Preparation.—It is important to provide a perfectly smooth surface over which to apply the coating. The surface should be sanded perfectly smooth and be free from all tool marks or other imperfections. The bolt holes at the hub should be plugged with corks which should be cut off flush and finished in the same manner as the rest of the surface.

Filling.—For open-grained woods a coat of filler consisting of 83 per cent liquid and 17 per cent silex should be used. The liquid should consist of 77 per cent airplane spar varnish and 23 per cent turpentine. The silex should pass a 200-mesh sieve.

The filler should be applied to the wood and allowed to flatten, after which it should be rubbed off across the grain so as to thoroughly fill the pores. The filler should dry at least 24 hours, after which it should be sanded lightly.

Shellac varnish undercoating.—The shellac varnish should consist of four and one-half pounds of orange shellac gum in one gallon of clean, neutral, denatured alcohol.

This varnish should be applied evenly over the surface of the propeller and allowed to dry three or four hours, after which it should be sanded lightly.

Size.—The size should consist of 75 per cent airplane spar varnish and 25 per cent turpentine. It is suggested that a small amount of Prussian blue in Japan be added to the varnish to give it a color, so that spots subsequently left uncovered by the leaf will be readily visible.

This size should be brushed evenly over the surface as sparingly as possible and allowed to dry until a tack is reached, which will permit the handling of the propeller immediately after the application of the leaf. The time will vary with the varnish and the kind of a day. The varnish should probably dry an hour and a half on a light dry day or in a heated building in the winter time, but a longer time may be required on cloudy or damp days. This is a very important point and should be carefully considered as the coating hardens very slowly after the leaf is applied.

Care should be exercised so as not to produce fatty edges or runs in applying the size. If they occur, the leaf will be easily rubbed from the surface in handling the blade.

It has been found convenient to size one side of the blade at a time; that is, the front or back of the blade. This is a convenience in applying the leaf later.

Aluminum leaf.—After the size has reached the right tack the leaf should be applied very rapidly over the surface, and after the sized surface has been entirely covered the leaf should be patted down with the palm of the hand or with a pad of cotton, after which the rough edges should be rubbed away (see fig. 79b). Any points not covered with leaf should be coated by applying a small piece of leaf to the spot with the fingers. The coating should be rubbed well with a piece of cotton which has been dipped in aluminum powder. This will insure the leaf sticking securely over the entire surface and will fill any small holes not already filled.

Aluminum leaf comes in packs containing 500 leaves. The pack is divided up into 10 or 20 books containing 50 or 25 leaves, respectively. The metal leaf is placed between the pages of these books and comes in 4-inch, $4\frac{1}{2}$-inch, 5-inch, or $5\frac{1}{2}$-inch squares.

It has been found best to apply the leaf directly from the book by turning back the first page of the book halfway, holding the same between the first and second fingers of the right hand (see fig. 78a). The book itself should be held between the thumb and fingers and in such a way that the back of the hand will be toward the work when the leaf is applied, the book being given a slight bend to prevent the corners of the leaf from drooping. The end of the leaf exposed by turning back the first page of the book should be placed against the surface to be coated and held securely in place by the left hand (see fig. 78b). The sheet held between the first and second fingers should be drawn back so as to allow the whole leaf to come in contact with the surface (see fig. 79a). The next sheet should be applied in a like manner, lapping edges with the first, and so on. The best results will be obtained if the gilder works in one direction with each row of leaf; that is, from left to right. If this be done, it will aid considerably in completing and smoothing off the surface.

It is suggested that in turning the pages of the books the back of the book be held between the first two fingers of the left hand (see fig. 77a). The leaves from which the leaf has been removed should be turned back and held between the thumb and first finger of the left hand. The next sheet of paper may then be turned back exposing one-half of the next leaf. The operation of changing the book from left to right hand is shown in figure 77b.

Large hub hole.—The large hub hole should receive the same treatment as the rest of the propeller. In applying the leaf to the hub hole it has been found convenient to cut the books

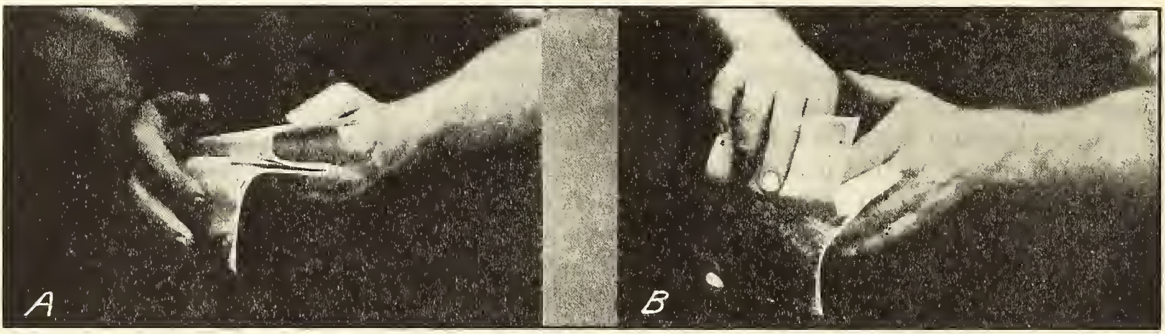

Fig. 77.—Aluminum leaf coating. (a) Method used in turning page of book. (b) Transferring book from left to right hand.

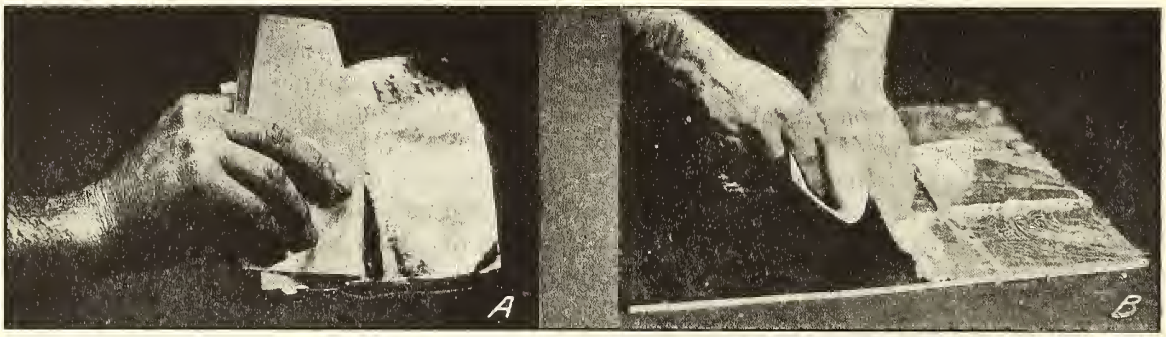

Fig. 78.—Aluminum leaf coating. (a) Method of holding book when applying leaf. (b) First operation in laying leaf.

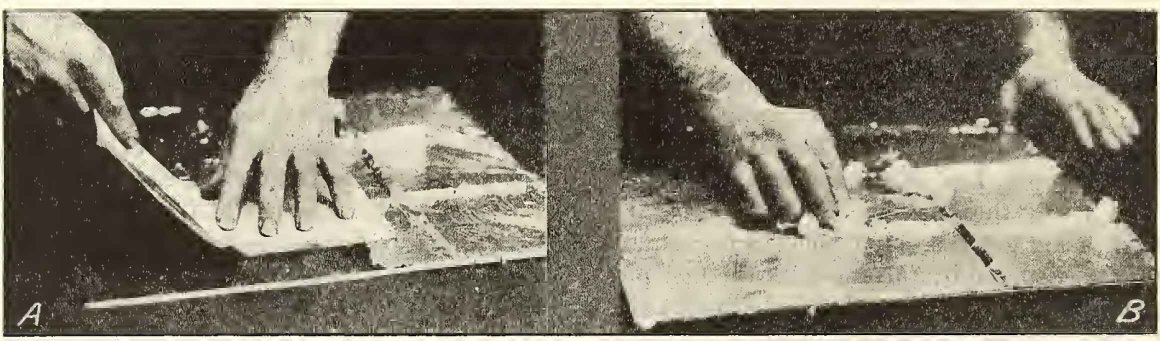

Fig. 79.—Aluminum leaf coating. (a) Second operation in laying leaf. (b) Smoothing off surface after application of leaf.

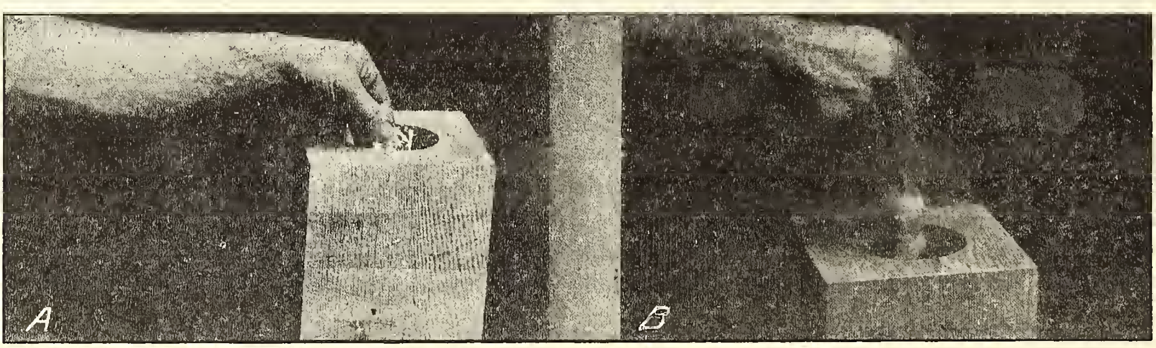

Fig. 80.—Aluminum leaf coating. (a) Applying leaf to large hub hole. (b) Smoothing off leaf in large hub hole.

of leaf up into about 1-inch strips of leaf and paper and drop them vertically into the opening and bring into contact with the size (see fig. 80a). After the entire surface of the hole has been covered the leaf should be patted into place with a wad of cotton attached to the end of a stick (see fig. 80b).

Small hub holes.—These holes should be simply corked up with ordinary corks, the tops of which should be cut off flush with the surface of the propeller and covered with the regular finish.

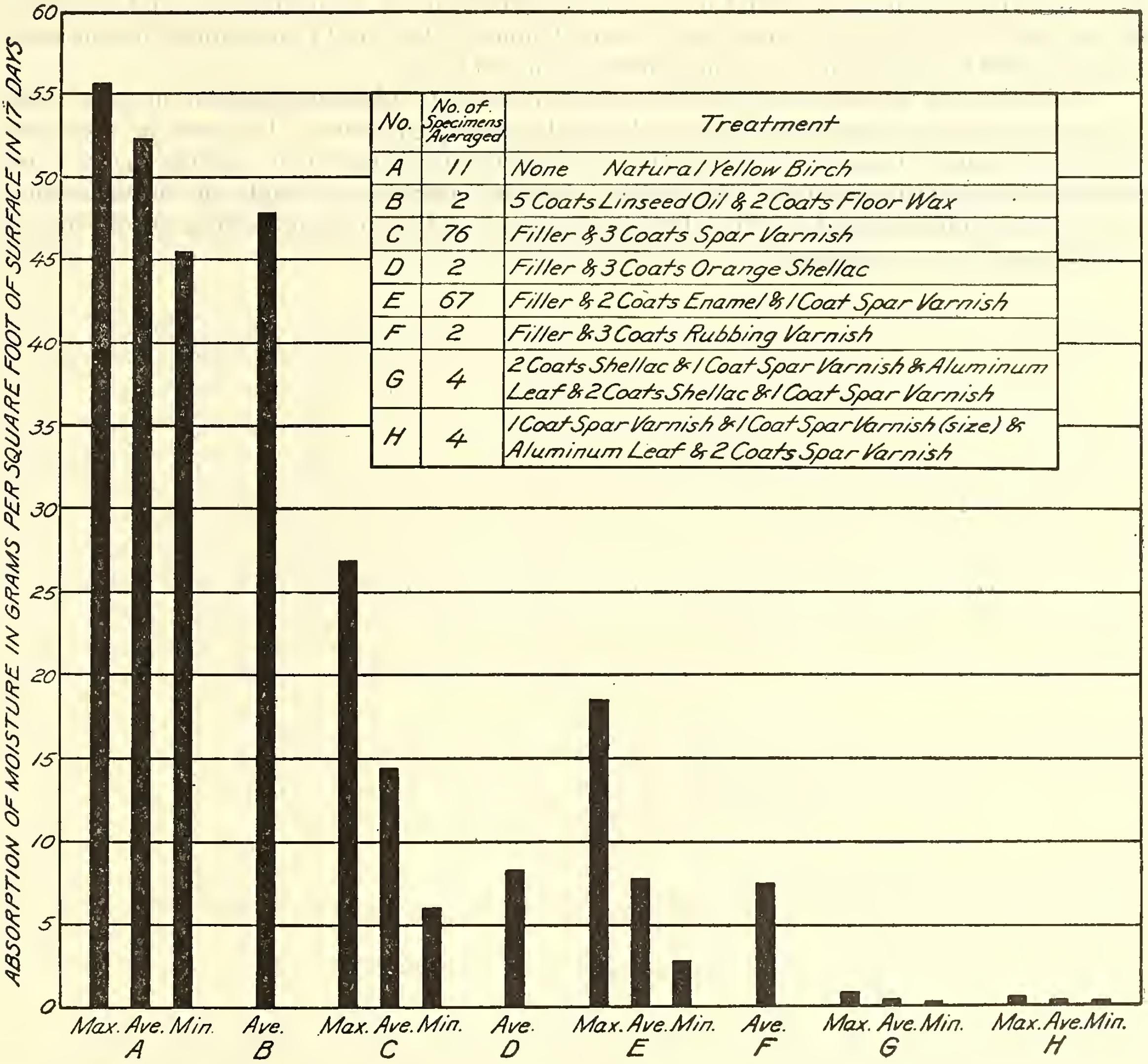

Fig. 81.—The comparative effectiveness of various coatings in moisture-proofing wood. Humidity of air during tests, 95 to 100 per cent.

Shellac color varnish.—After the application of the leaf two coats of shellac color varnish should be applied. This varnish should be made as described under the heading of "Shellac varnish undercoat," except that enough color should be added to produce a mahogany color. Four or 5 per cent of Bismark brown in the shellac varnish gives about the right color. The amount of this material to get the best results should be determined by trial. The varnish should dry three or four hours before rubbing or recoating.

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Each coat of shellac should be rubbed down lightly between coats without the use of oil.

Finishing varnish.—A final flowing coat of airplane spar varnish should be applied and allowed to dry about 48 hours. This coating should not be rubbed or sanded.

Estimated time required to coat a propeller.—The time required to apply the leaf to a propeller should not be more than 40 or 50 minutes. This time could be reduced after the finisher becomes more experienced. The estimated time required for applying the complete finish described in the foregoing paragraphs would be in the neighborhood of 8 or 10 hours, and the total time required for drying the various coats about 90 hours. The total time required for the total operation would probably be in the neighborhood of 100 hours.

Modification of aluminum leaf spirit varnish process.—It might be desirable in some cases to use oil varnishes or enamels in lieu of the shellac described above. This may be done and satisfactory results obtained. In case oil varnishes are substituted, it is possible that a more durable coating will be obtained. It requires a much longer time to apply the finish because of the greater time required for the oil varnishes to dry. Each coat of varnish should dry at least 72 hours before recoating.

APPENDIX.

For convenient reference, specifications for the determination of the moisture content in wood and for the determination of the specific gravity of wood are embodied in the appendix.

THE DETERMINATION OF MOISTURE CONTENT IN WOOD.

SELECTION OF TEST SPECIMENS.

1. Short pieces of wood dry out much more rapidly than longer ones. In order to reduce the time required for drying, the length of the test specimen in the direction of the grain should usually be about 1 inch or not more than enough to give a volume of from 5 to 25 cubic inches.

TESTS.

2. Having selected a representative piece of material for a test specimen, the procedure for determining the moisture content is as follows:

3. Immediately after sawing remove all loose splinters and weigh the test specimen. It is important that the weight be taken immediately after sawing, since the wood is subject to moisture changes on exposure to the air. The degree and rapidity of change are dependent on the moisture content of the piece and the conditions of the air to which it is exposed.

4. Put the test specimen into a drying oven and dry at approximately 212° F. (100° C.) to constant weight. This usually requires three to five days. Specimens placed in the oven for drying must be open piled to allow free access of air to all parts of each piece.

5. Weigh the test specimens immediately after removing from the oven.

6. The loss in weight expressed in per cent of the dry weight is the percentage moisture content of the wood from which the test specimen was cut.

$$\text{Percentage moisture} = \frac{(W-D)}{D} \times 100$$

W = original weight as found under paragraph 3.
D = oven-dry weight as found under paragraph 5.

ACCURACY.

7. In order to insure good results, the weight should be correct to within at least one-half of 1 per cent.

THE DETERMINATION OF SPECIFIC GRAVITY OF WOOD.

GENERAL.

1. The specific gravity (or density) of all woods used in aircraft construction shall be determined, when required, in accordance with this specification. Method A shall be used whenever possible.

SELECTION OF TEST SPECIMENS.

2. Short pieces of wood dry out much more rapidly than longer ones. In order to reduce the time required for drying, the length of the test specimen in the direction of the grain should usually be about 3 centimeters.

METHOD A.

3. Having selected a representative piece of material for a test specimen, the procedure is as follows:

4. Immediately after sawing remove all loose splinters and put the test specimen into a drying oven and dry at about 212° F. (100° C.) to constant weight. This usually requires three to five days. Specimens placed in the oven for drying must be open piled to allow free access of air to all parts of each piece.

5. Weigh the test specimen.

6. Determine the volume of the oven-dry specimen preferably by the method described in paragraphs 9 to 12.

7. Specific gravity $= \frac{M}{V}$

M = oven-dry weight in grams as determined under paragraphs 4 and 5.
V = oven-dry volume in cubic centimeters as determined under paragraph 6.

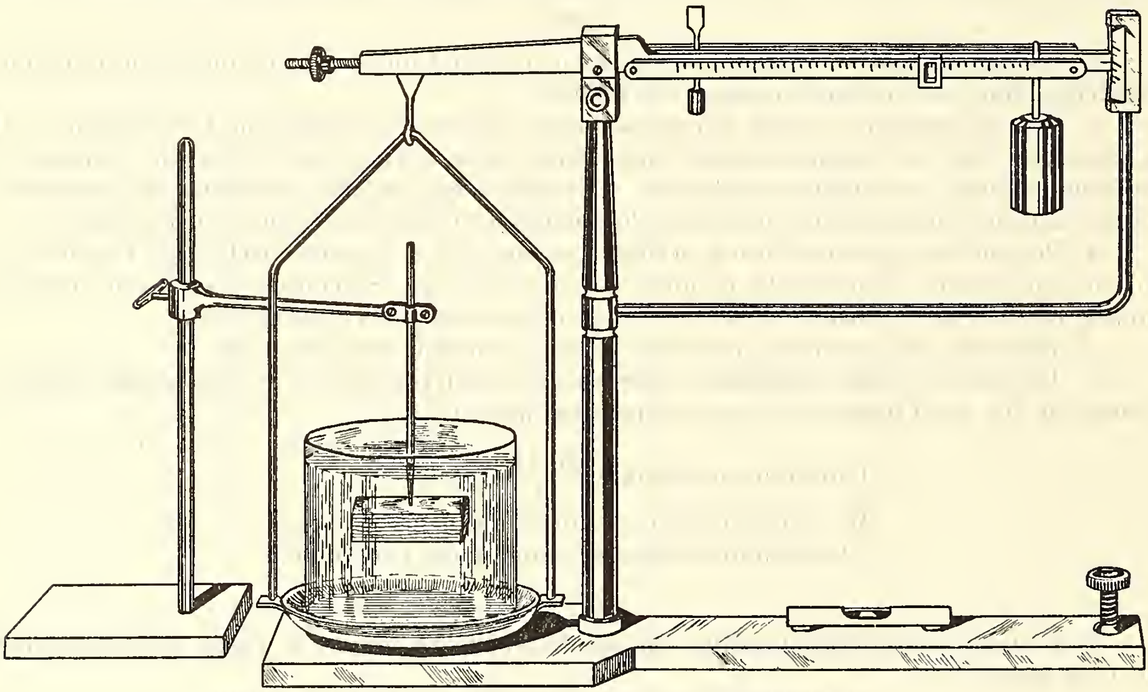

Fig. 82.—Determination of specific gravity of wood.

REDUCTION FACTORS.

8. One inch = 2.54 centimeters; 1 ounce = 28.4 grams; 1 cubic inch = 16.4 cubic centimeters; 1 pound = 454 grams.

DETERMINATION OF VOLUME.

9. After the oven-dry weight has been obtained dip the test specimen in hot paraffin and allow it to cool. Scrape off any surplus paraffin which adheres to the specimen.

10. The volume of the test specimen is found by determining the weight of water it displaces when immersed, as shown in figure 82. This weight in grams is numerically equal to the volume of the specimen in cubic centimeters.

11. It is important that the determination of the volume by weighing be made as quickly as possible after the immersion of the specimen, since any absorption of water by the specimen directly influences the aeeuraey of the result. By estimating the volume of the specimen and plaeing approximately the required weights on the plan before the specimen is immersed the time necessary for balancing may be reduced to a minimum.

12. To determine the volume, a container holding sufficient water for the complete submergence of the speeimen is placed on one pan of a balanee scale. The container and water are then balanced with weights added to the other scale pan. By means of a sharp-pointed rod, shown in figure 82, the specimen is held eompletely submerged and not touching the container while the scales are again balanced. The weight required to balance is the weight of water displaced by the specimen, and, if in grams, is numerically equal to the volume of the speeimen in eubic centimeters.

13. The sharp-pointed rod, by means of whieh the speeimen is held in position, should be of as small diameter as possible. Care should be taken not to lower the specimen into the water to a mueh greater depth than required to completely submerge it; otherwise the weight of water displaced by the rod will affect the accuracy of the result.

ACCURACY.

14. In order to insure good results, the weights and volumes should be correct to within at least one-half of 1 per cent.

METHOD B.

15. The following method of determining the speeific gravity may be used when the apparatus required by test A is not available.

16. Select the test specimen as in paragraph 2.

17. Dry the specimen as in paragraph 4.

18. Cut the oven-dried specimen while hot to a standard volume of not less than 80 cubic centimeters so that its volume may be accurately determined by measurement.

19. Weigh the oven-dried speeimen while hot and record its weight in grams. This weight must be accurate to within one-half of 1 per eent.

20. Determine the volume in cubic centimeters of the oven-dried specimen while hot by measuring each edge in centimeters and taking measurements to the nearest one-half millimeter.

20. Speeific gravity $= \frac{M}{V} = \frac{\text{weight in grams}}{\text{volume in cc}}$.

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AIRCRAFT DESIGN DATA. NOTE 13.

PROPELLERS.

II—DESIGN OF TANDEM PROPELLERS.

FRONT.

The front propeller is designed as though it were to act alone and has the following characteristics:

Number of blades, 2.
Diameter, large as possible.
Revolutions per minute, low as possible; a geared engine is necessary.

REAR.

The rear propeller is designed to operate with an air speed equal to the speed of advance plus six-tenths of the added slip stream velocity plus a correction for indraft.

Slip stream velocity, due to front propeller, is computed by distributing uniformly over the effective disk area the momentum necessary to produce the thrust assumed. The effective disk area is taken as the full disk area less the amount blanketed by the engine nacelle; that is,

$$T = M(V_s - V) = \frac{\rho}{g} A V_s (V_s - V)$$

Where

A = Effective disk area in square feet,
V = Forward speed of advance in feet per second,
V_s = Velocity in slip stream in feet per second,
ρ = Weight of air, pounds per cubic foot,
g = Acceleration of gravity in feet per second squared,
T = Thrust in pounds,

and everything is known except V_s.

Allowance for indraft.—Compute the slip velocity due to the rear propeller and allow two-thirds of this velocity as inflow taking place before the air strikes the blades.

The rear propeller has the following characteristics:

Number of blades, 4.
Diameter, large as possible, but not to exceed 85 per cent of diameter of front.
Revolutions per minute, in general, same as for front propeller.

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GENERAL.

The distance between the two propellers should be equal to the diameter of the front propeller. It is bad practice to permit any obstructions between them and, in particular, radiators should not be placed in the slip stream of the front propeller.

EFFICIENCY.

Tandem propellers are considered effective at high speed, but are much less so when climbing, and are comparatively inefficient when planing on the water at an air speed of approximately 40 per cent of the maximum speed.

Assuming a constant throttle opening, which allows the engine revolutions per minute to fall out at the lower speeds, the following results are obtained from an analysis of recent trials:

	Maximum speed.		0.7 maximum speed.		0.4 maximum speed.	
	Thrust horse-power.	Efficiency.	Thrust horse-power.	Efficiency.	Thrust horse-power.	Efficiency.
		Per cent.		*Per cent.*		*Per cent.*
Front	269	76. 5	223	63. 4	150	42. 6
Rear	260	71. 0	198	56. 3	118	33. 5
Combination	519	73. 7	421	59. 8	268	38. 1

The above data is plotted in figure 1.

The English Handley-Page uses two Rolls-Royce motors in tandem. These motors are geared down and give 375 brake horsepower. The propellers are as follows:

Propeller.	Diameter.	Number of blades.	Blade with ratio.	Face pitch.	Revolutions per minute.	Brake horse-power.	Efficiency.	Speed miles per hour.
	Feet.			*Feet.*			*Per cent.*	
Front	13. 12	2	5. 94	8. 99	1, 140	375	78	100
Rear	10. 50	4	5. 76	9. 58	1, 140	375	....	....

Distance between propellers, 13.38 feet.

Plan form of blades, Watts No. 1, both front and rear.

Ratio $\frac{\text{diameter of front}}{\text{diameter of rear}}=0.80$.

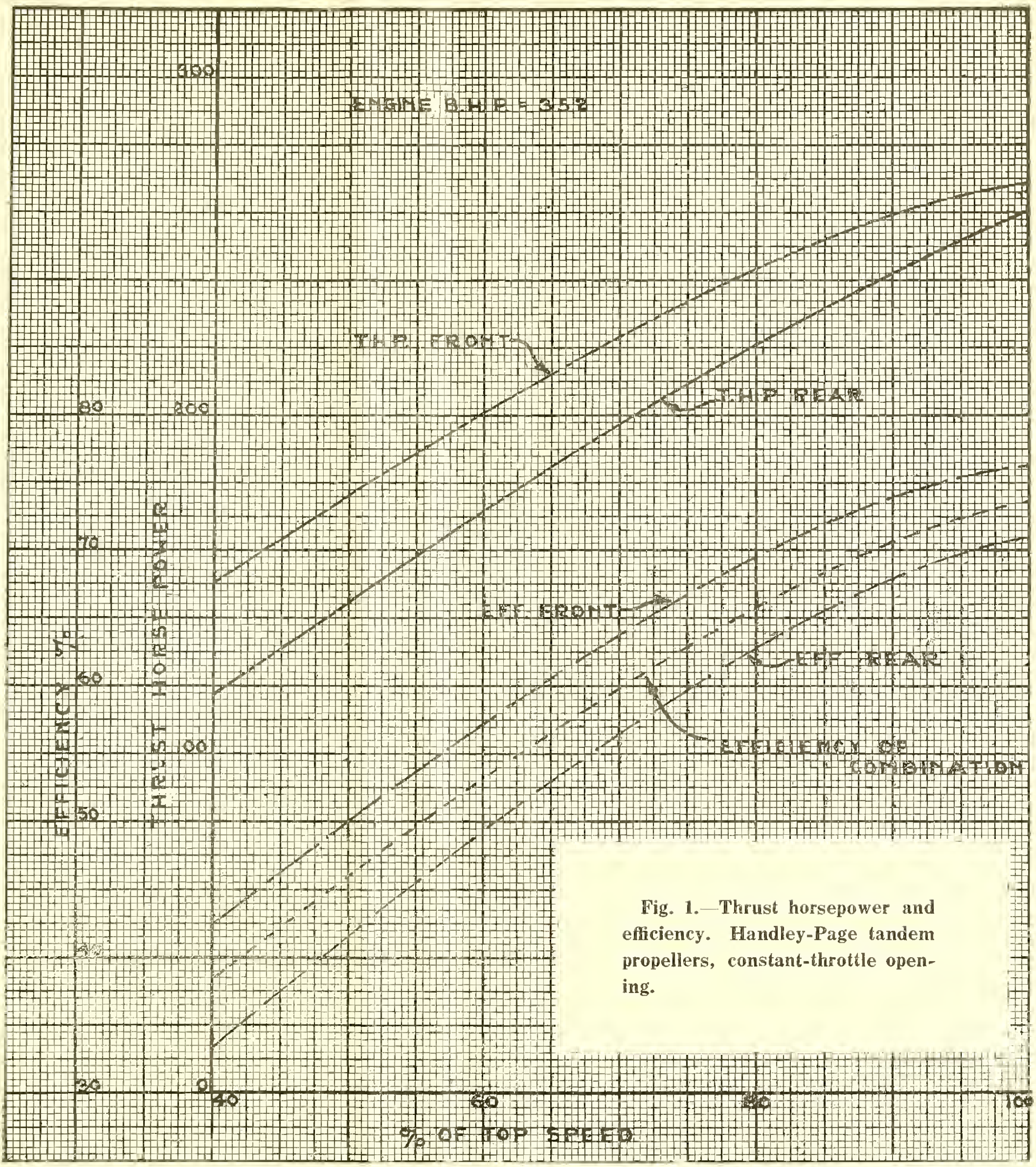

Fig. 1.—Thrust horsepower and efficiency. Handley-Page tandem propellers, constant-throttle opening.

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